Structural change in a multi-sector model of the climate and the economy

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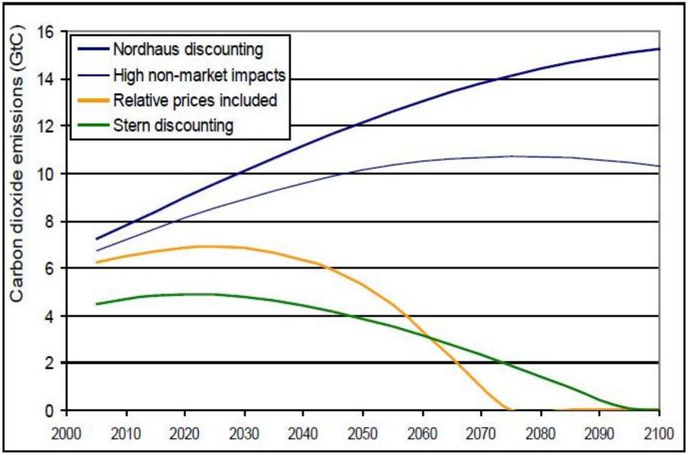
Motivation (Sterner and Persson (2008))

$$U(C) = [(1-\gamma)C^{1-1/\sigma} + \gamma E^{1-1/\sigma}]^{(1-\alpha)\sigma/(\sigma-1)}/(1-\alpha) \qquad E(t) = E_0 / [1 + aT(t)^2]$$

Figure 2: *Optimal carbon dioxide emission paths in the DICE model for four different cases:*



We assume an elasticity of 0.5, implying that if, hypothetically, the relative price of the environmental good would increase by 1%, then the purchase of environmental goods would decline by 0.5% relative to the purchase of other goods.

Moving forward

- The results in Sterner and Persson (2008) are dramatic and show how relative prices can play a role for optimal mitigation policies!
- Has received considerable attention from the research community (178 cites according to Google scholar).
- How can their results be applied in practice when constructing IAMs?
- What is the environmental good? How do we measure it and how does it relate to other commodities in the economy?
- What about the role of adaptation?
- What to do? Why not develop a multi-sector macroeconomic growth model where each sector is impacted differently by climate change and where resources in the economy can be transfered to counteract the heterogeneous impacts?

Outline

- This paper develops a multi-sector growth model of the *climate-economy* interaction featuring i) endogenous saving ii) emissions from fossil fuel use in production and iii) inter-sectoral factor allocation decisions and iv) allow each sector to be impacted differently by climate change.
- The purpose of this exercise is to explore (as in Sterner and Persson) the role of *relative prices* for optimal mitigation policies, but within the context of a multi-sector growth model which allows for calibration to currently available data.
- By allowing for reallocation of input factors across sectors over time this approach also introduces adaptation into the climate-economy interaction, which reveals a direct relationship between adaptation and optimal fossil fuel taxes.

Structural Change

- Structural change refers to the reallocation of production factors (typically labor) across different sectors of the economy over time.
- During the process of development, strong structural change takes place with movements of labor and other resources from agriculture to manufacturing and then to services (*Kuznets Facts*).
- On the aggregate level growth rate of per-capita output, real interest rate, capital-output ratio and the labor income share has remained fairly constant (*Kaldor facts*).
- Challenge to theory has been to reconcile the non-balanced characteristics of growth at the sector level with balanced picture of growth at the aggregate level.

Multi-sector growth models reconciling *Kuznets* and *Kaldor* facts of growth

- Kongsamut, Rebelo and Xie (2001)
  - Structural change is driven by non-homothetic preference which leads to differences in income elasticity of demand across goods.
  - Engel's law: as a household's income increases, fraction that it spends on food (agricultural products) declines.
- Ngai and Pissarides (2007)
  - Structural change is driven by differences in technological growth rates.
- Acemoglu and Guerrieri (2008)
  - Differences in capital intensities as a driver of structural change through capital deepening.

Model description

- Description of the economy
- The social planning problem
  - Static efficiency
  - Dynamic efficiency
- The competitive equilibrium
- Numerical calibration and simulations
- Conclude

Description of an n-sector economy

Representative households preferences are given by

$$\sum_{t=0}^{\infty} \beta^t U(C_t) \tag{1}$$

The economy produces a unique final good which can be thought of as an aggregate/composite good consisting of the n intermediaries

$$Y_t = \left(\sum_{i=1}^{n} w_i Y_{i,t}^{(\epsilon-1)/\epsilon} \right)^{\epsilon/(\epsilon-1)} \quad ; \quad \sum_{i=1}^{n} w_i = 1 \tag{2}$$

Production in sector i

$$Y_{it} = \Omega_i(S_t) A_{i,t} K_{i,t}^{\alpha_1} L_{i,t}^{\alpha_2} E_{i,t}^{\alpha_3}; \quad \forall i \tag{3}$$

Factor inputs can be allocated *free of charge* across all sectors

$$K_t = \sum_{i=1}^{n} K_{i,t}; \quad L_t = \sum_{i=1}^{n} L_{i,t}; \quad E_t = \sum_{i=1}^{n} E_{i,t} \tag{4}$$

Description of an n-sector economy

The economy's aggregate budget constraint is given by

$$K_{t+1} + C_t = Y_t + (1 - \delta)K_t \tag{5}$$

Fossil fuel use E_t of a finite stock R_t is governed by

$$R_0 \geq \sum_{t=0}^{\infty} E_t \quad \Rightarrow \quad R_{t+1} - R_t = -E_t \tag{6}$$

Human induced carbon dioxide accumulates in the atmosphere according to

$$S_{t+1} = (1 - \varphi)S_t + \xi E_t \tag{7}$$

Planning problem

$$\max_{\{K_{t+1},R_{t+1},S_{t+1}E_t,C_t,\{K_{i,t},L_{i,t},E_{i,t}\}\forall i\}} \sum_{t=0}^{\infty} \beta^t U(C_t)$$

subject to

$$K_{t+1} = Y_t - C_t + (1-\delta)K_t$$
$$S_{t+1} = (1-\varphi)S_t + \xi E_t$$
$$R_{t+1} = R_t - E_t$$
$$K_t = \sum_{i=1}^{n} K_{i,t}; \quad L_t = \sum_{i=1}^{n} L_{i,t}; \quad E_t = \sum_{i=1}^{n} E_{i,t}$$

where

$$Y_t = \left(\sum_{i=1}^{n} w_i Y_{i,t}^{(\epsilon-1)/\epsilon}\right)^{\epsilon/(\epsilon-1)} ; \quad Y_{i,t} = \Omega_i(S_t) A_{i,t} K_{i,t}^{\alpha_1} L_{i,t}^{\alpha_2} E_{i,t}^{\alpha_3}; \quad \forall i$$

Static efficiency

From the planing problem the following static efficiency conditions follow

$$\frac{p_{i,t}}{p_{j,t}} = \frac{\partial Y_{j,t}}{\partial K_{j,t}} / \frac{\partial Y_{i,t}}{\partial K_{i,t}} = \frac{\partial Y_{j,t}}{\partial L_{j,t}} / \frac{\partial Y_{i,t}}{\partial L_{i,t}} = \frac{\partial Y_{j,t}}{\partial E_{j,t}} / \frac{\partial Y_{i,t}}{\partial E_{i,t}}; \quad \forall i, j \qquad (8)$$

where $p_{i,t} \equiv w_i \left(Y_t / Y_{it} \right)^{\frac{1}{\epsilon}}$ denotes the price of good i in the decentralized market equilibrium. The following proposition follows

Proposition 1

Given equal factor income shares across sectors and constant returns to scale production functions the intra temporal resource allocation at time t is determined by

$$\frac{K_{i,t}}{K_{j,t}} = \frac{L_{i,t}}{L_{j,t}} = \frac{E_{i,t}}{E_{j,t}} = \left(\frac{w_i}{w_j} \right)^{\epsilon} \left(\frac{\Omega_i(S_t)}{\Omega_j(S_t)} \frac{A_{i,t}}{A_{j,t}} \right)^{\epsilon - 1} \equiv \Psi_{i,j}(S_t), \quad \forall i, j$$

Static efficiency

By proposition 1 we can also show that relative factor inputs shares also equal the expenditure share of production in sector i relative to sector j.

$$\frac{p_{i,t}Y_{i,t}}{p_{j,t}Y_{j,t}} = \left(\frac{w_i}{w_j}\right)^{\epsilon}\left(\frac{\Omega_i(S_t)}{\Omega_j(S_t)}\frac{A_{i,t}}{A_{j,t}}\right)^{\epsilon-1} = \Psi_{i,j}(S_t), \quad \forall i,j \tag{9}$$

This corresponds exactly to condition (10) of Ngai and Pissarides (2007) which in their model corresponds to the ratio of consumption expenditure in good i relative to sector m.
Relative prices between sector i and j will be determined by

$$\frac{p_{i,t}}{p_{j,t}} = \frac{\Omega_j(S_t)}{\Omega_i(S_t)}\frac{A_{j,t}}{A_{i,t}}, \quad \forall i,j \tag{10}$$

Static efficiency

Define, $\tilde{A}_{i,t} \equiv \Omega_i(S_t)A_{i,t}$ as the climate externality adjusted TFP growth rate. Then by proposition 1, taking the logs of both sides and differencing

$$\ln\left(\frac{L_{i,t+1}}{L_{i,t}}\right) - \ln\left(\frac{L_{j,t+1}}{L_{j,t}}\right) = (1-\epsilon)\left(\ln\frac{\tilde{A}_{j,t+1}}{\tilde{A}_{j,t}} - \ln\frac{\tilde{A}_{i,t+1}}{\tilde{A}_{i,t}}\right), \quad \forall i,j \quad (11)$$

which gives us

Proposition 2

Necessary and sufficient conditions for structural change are that $\epsilon \neq 1$ and that $\ln(\tilde{A}_{j,t+1}/\tilde{A}_{j,t}) \neq \ln(\tilde{A}_{i,t+1}/\tilde{A}_{i,t})$ for some i. Let the climate adjusted TFP growth rate be smaller in sector i compared to sector j when $\epsilon < 1$, or alternatively let i have a larger climate adjusted TFP growth rate when $\epsilon > 1$. In both case this implies that sector i expands faster over time relative to sector j and that prices of good i increase at a faster pace relative to good j.

Static efficiency

Finally by proposition 1 get the *maximized value* of current output $\tilde{Y}_t$ given the capital, fossil fuel and carbon dioxide stock at time t

Proposition 3

Based on proposition 1 and the market clearing conditions the composite production function, or final good, can be written as

$$\tilde{Y}_t = \Gamma_t(S_t) K_t^{\alpha_1} L_t^{\alpha_2} E_t^{\alpha_3} \tag{12}$$

with

$$\Gamma_t(S_t) \equiv \left(\sum_{i=1}^{n} w_i \left(\Omega_i(S_t) A_{it} \Psi_{i,j}(S_t) \right)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \frac{1}{\hat{\Psi}_j}; \quad \hat{\Psi}_j \equiv \sum_{i=1}^{n} \Psi_{i,j} \tag{13}$$

Dynamic efficiency

The static solution simplifies the planning problem.

$$
\begin{aligned}
\max_{\{C_t, K_{t+1}, R_{t+1}, E_t, S_{t+1}\}} & \sum_{t=0}^{\infty} \beta^t U(C_t) \\
\text{s.t} \quad & K_{t+1} = \tilde{Y}_t - C_t + (1-\delta)K_t; \\
& R_{t+1} - R_t = E_t; \\
& S_{t+1} = (1-\varphi)S_t + \xi E_t \\
\text{with} \quad & R_0;\ K_0;\ S_0; R_t \geq 0
\end{aligned}
\tag{14}
$$

First order conditions w.r.t C_t, K_{t+1}

$$
\frac{U'(C_t)}{\beta U'(C_{t+1})} = \alpha_1 \frac{\tilde{Y}_{t+1}}{K_{t+1}} + (1-\delta) \tag{15}
$$

Dynamic efficiency

The necessary optimality conditions w.r.t. S_{t+1} implies

$$\frac{U'(C_t)}{\beta U'(C_{t+1})} = \frac{\alpha_3 \frac{\tilde{Y}_{t+1}}{E_{t+1}} + \xi \frac{\lambda_{S,t+1}}{U'(C_{t+1})}}{\alpha_3 \frac{\tilde{Y}_t}{E_t} + \xi \frac{\lambda_{S,t}}{U'(C_t)}} \tag{16}$$

which denotes the present value of the marginal damages (Lagrangian multiplier of S_t)

F.o.c. for R_{t+1} and E_t results in

$$\lambda_{S,t} = \sum_{s=1}^{\infty} (1-\varphi)^{s-1} \beta^s U'(C_{t+s}) \frac{\partial \Gamma_{t+s}}{\partial S_{t+s}} \frac{\tilde{Y}_{t+s}}{\Gamma_{t+s}} \tag{17}$$

which is an externality adjusted Hotelling type formula where $\lambda_{S,t}$

Dynamic efficiency

Assuming log utility and a full capital depreciation as in Golosov et.al. (2012), consumption and investment rates are constant i.e. $C_t = (1-\beta\alpha)\tilde{Y}_t$ and $K_{t+1} = \beta\alpha\tilde{Y}_t$ satisfies the Euler equation and capital budget constraint.

The externality adjusted Hotelling rule is thus simplified and given by

$$\frac{1}{\beta} = \frac{\alpha_3 \frac{1}{E_{t+1}} + \sum_{s=1}^{\infty} \xi(1-\varphi)^{s-1}\beta^s \frac{\partial \Gamma_{t+1+s}}{\partial S_{t+1+s}} \frac{1}{\Gamma_{t+1+s}}}{\alpha_3 \frac{1}{E_t} + \sum_{s=1}^{\infty} \xi(1-\varphi)^{s-1}\beta^s \frac{\partial \Gamma_{t+s}}{\partial S_{t+s}} \frac{1}{\Gamma_{t+s}}} \tag{18}$$

which together with our definition of Γ_t and the dynamics $S_{t+1} = (1-\varphi)S_t + \xi E_t$ and $R_0 \geq \sum_{t=0}^{\infty} E_t$ solves the problem of optimal fossil fuel consumption.

Decentralized competitive equilibrium with taxes

The representive household problem

$$\max_{C_t, K_{t+1}} \sum_{t=0}^{\infty} \beta^t U(C_t),$$
$$s.t. \;\; C_t + K_{t+1} = r_t K_t + w_t L_t + \Pi_t^e + G_t \tag{19}$$

The representative intermediate goods firm within the each sector solves

$$\max_{K_{i,t}, L_{i,t}, E_{i,t}} p_{y_{i,t}} Y_{i,t} - r_t K_{i,t} - w_t L_{i,t} - p_{E_t} E_{i,t}, \quad \forall i$$

Final good production implies that the marginal product of each good will equals its price

$$\max_{Y_{i,t}} P_t Y_t - \sum_{i=1}^{n} p_{y_{i,t}} Y_{i,t}$$

where as before $Y_t = \left(\sum_{i=1}^{n} w_i Y_{i,t}^{(\epsilon-1)/\epsilon} \right)^{\epsilon/(\epsilon-1)}$.

Decentralized competitive equilibrium with taxes

Given log utility and full capital depreciation the f.o.c. of households imply

$$\frac{C_{t+1}}{C_t} = \beta r_{t+1} \tag{20}$$

the f.o.c. for final goods production yield

$$p_{y_{it}} = w_i \left(\frac{Y_t}{Y_{it}} \right)^{\frac{1}{\epsilon}}; \quad \forall i \tag{21}$$

Making use of (21) the f.o.c. for intermediate firms can be written as

$$r_t = w_i \left(\frac{Y_t}{Y_{i,t}} \right)^{\frac{1}{\epsilon}} \frac{\partial Y_t}{\partial K_{i,t}}; \quad w_t = w_i \left(\frac{Y_t}{Y_{i,t}} \right)^{\frac{1}{\epsilon}} \frac{\partial Y_t}{\partial L_{i,t}}; \quad p_{E_t} = w_i \left(\frac{Y_t}{Y_{i,t}} \right)^{\frac{1}{\epsilon}} \frac{\partial Y_t}{\partial E_{i,t}}$$

From market clearing and prop. 1 we can show that factor input prices simplify to

$$r_t = \alpha_1 \Gamma_t(S_t) K_t^{\alpha_1 - 1} L_t^{\alpha_2} E_t^{\alpha_3}; \quad w_t = \alpha_2 \Gamma_t(S_t) K_t^{\alpha_1} L_t^{\alpha_2 - 1} E_t^{\alpha_3};$$

$$p_{E_t} = \alpha_3 \Gamma_t(S_t) K_t^{\alpha_1} L_t^{\alpha_2} E_t^{\alpha_3 - 1}$$

Decentralized competitive equilibrium with taxes

The representative resource extraction firm solves the problem given *ad-valorem* (τ_t) or *per-unit taxes* (θ_t)

$$\max_{R_{t+1}} \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t} r_s \right)^{-1} (p_{E_t} - \theta_t)(1 - \tau_t) E_t$$

$$s.t.\ R_0 \geq \sum_{t=0}^{\infty} E_t, \quad R_0 \geq 0$$

Once again the externality adjusting Hotelling type formula

$$r_{t+1} = \frac{(p_{E_{t+1}} - \theta_{t+1})(1 - \tau_{t+1})}{(p_{E_t} - \theta_t)(1 - \tau_t)} \tag{22}$$

Decentralized competitive equilibrium with taxes

Define $\Lambda_{s,t} \equiv \xi \lambda_{S,t} / U'(C_t)$. The optimal tax can then be implemented by either setting

$$\theta_t = -\Lambda_{s,t} \text{ and } \tau_t = \tau \ \ \forall t$$

or by setting

$$\tau_t = -\frac{\Lambda_{s,t}}{\partial \tilde{Y}_t / \partial E_t} \text{ and } \theta_t = 0$$

By setting the rental price of capital from (22) equal to the marginal product of capital from the planning problem (16)

$$\frac{(p_{E_{t+1}} - \theta_{t+1})(1 - \tau_{t+1})}{(p_{E_t} - \theta_t)(1 - \tau_t)} = \frac{\alpha_3 \frac{\tilde{Y}_{t+1}}{E_{t+1}} + \Lambda_{S,t+1}}{\alpha_3 \frac{\tilde{Y}_t}{E_t} + \Lambda_{S,t}}$$

from this expression, if $\tau_t = \tau$ then $\theta_t = \Lambda_{st}$ this implements the planner optimum. Likewise, if $\theta_t = 0$ then $\tau_t = \frac{\Lambda_{st}}{\partial \tilde{Y}_t / \partial E_t}$ implements the optimum.

Numerics - data and calibration

Calibrate a two-sector model, consisting of an agricultural and a non-agriculture sector for the U.S. and India separately.

- Nordhaus (2007) impact estimates for the U.S. and Indian economy based on a 2.5 degree warming.
  - U.S. economy estimates an economic impact of 0.03% of GDP from the agricultural sector and 0.88% for the rest.
  - Indian economy he estimates an economic impact of 0.32% of GDP from the agricultural sector and 2.75% for the rest.
- Calibrate damage functions $\Omega_a(T_t) = \frac{1}{1+\theta_a T_t^2}$, $\Omega_m(T_t) = \frac{1}{1+\theta_m T_t^2}$
- Martin and Mitra (2001) estimate overall growth rate of TFP in manufacturing varies between 1.13% and 1.86% between 2.34% and 2.91% for agriculture for a sample of 50 countries between 1967-92.
- As in Golosov et.al. (2012) we set $\beta = 0.985^{10}$, $\alpha_1 = 0.3$, $\alpha_2 = 0.67$ and $\alpha_3 = 0.03$
- Rogner (1997) estimates current fossil fuel reserves at $\approx 5000 GtC$
- Set $T_t = \lambda \ln\left(1 + \frac{S_t}{\bar{S}}\right) / \ln 2$ and $\lambda = 3$, $\xi = 0.5$ and $\varphi = 0.05$

Numerics - data and calibration

Sector data on nominal and real value added attained from the Groningen Growth and Development Centre (GGDC) 10-sector database (1950-2005) for the U.S. and Indian economy. Assume competitive markets and nominal output defined as $Y^n_{i,t} \equiv p_{y_{i,t}} Y_{i,t}$. Following, Acemogulo and Guerrieri (2008) ϵ can be estimated by the log of nominal sectoral output ratios

$$\ln\left(\frac{Y^n_{m,t}}{Y^n_{a,t}}\right) = \ln\left(\frac{w_m}{w_a}\right) + \frac{\epsilon - 1}{\epsilon} \ln\left(\frac{Y_{m,t}}{Y_{a,t}}\right) \tag{23}$$

This yields an estimate $\epsilon \approx 1.62$ for the U.S. and $\epsilon \approx 2.13$ for the Indian economy. Using 2005 as a benchmark year we calibrate the intercept in (23) so as to match the data for 2005.

By proposition 1 we can then calibrate A_{m0} and A_{a0} by the following expression

$$\frac{A_{a0}}{A_{m0}} = \frac{w_m}{w_a} \frac{\Omega_m(T_t)}{\Omega_a(T_t)} \left(\frac{Y_{a0}}{Y_{m0}}\right)^{1/\epsilon} \tag{24}$$

Results - India

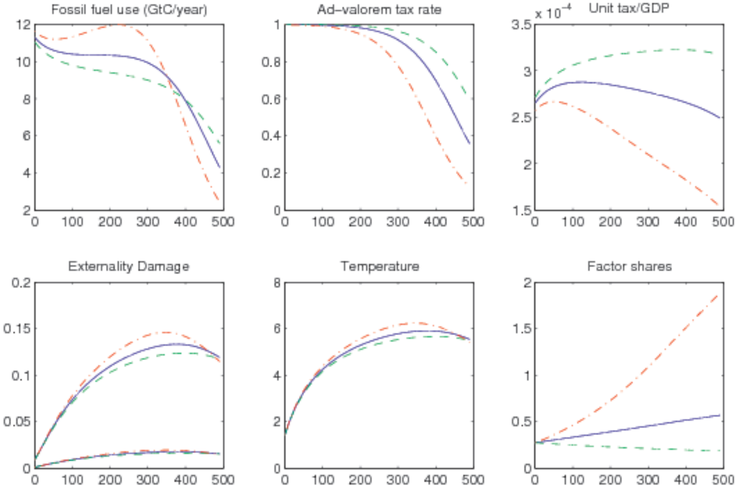


Figure: Indian economy: blue solid lines correspond to the benchmark calibration. Red dashed lines $\epsilon = 4$. Green dashed lines $\epsilon = 0.4$. Factor shares $= (L_{a,t}/L_{m,t})$.

Results - U.S.

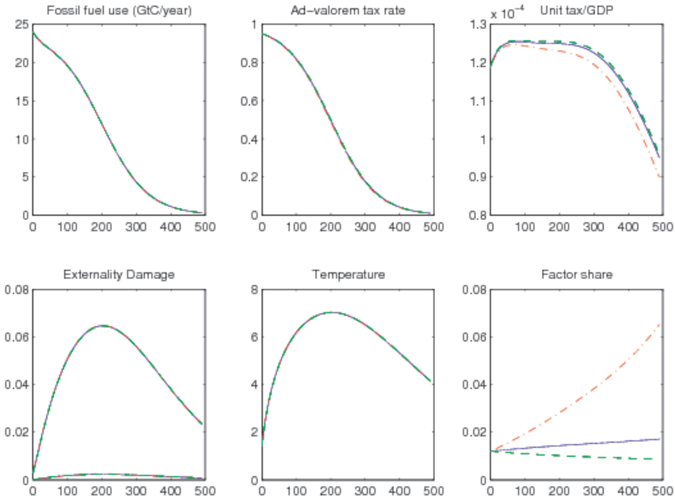


Figure: U.S. economy: blue solid lines correspond to the benchmark calibration. Red dashed lines $\epsilon = 4$. Green dashed lines $\epsilon = 0.4$. Factor shares $= (L_{a,t}/L_{m,t})$.

Concluding remarks

- A climate-economy model which can capture heterogeneous impacts across different sectors of the economy.
- Explored the role of relative price, within a multi-sector growth framework and derived explicit expressions for optimal tax rates related to relative prices and adaptation.
- Showed how these model can be calibrated based on economic data and how substitutability among goods may impact on optimal fossil fuel use.
- From a climate-economy perspective this framework can be seen as allowing not only for mitigation but also for adaptation when sectors are impacted heterogeneously by climate change.
- Caveat/Future research - movement of input factors across sectors is costly in the real world.