Spatial Climate-Economic Models in the Design of Optimal Climate Policies across Locations

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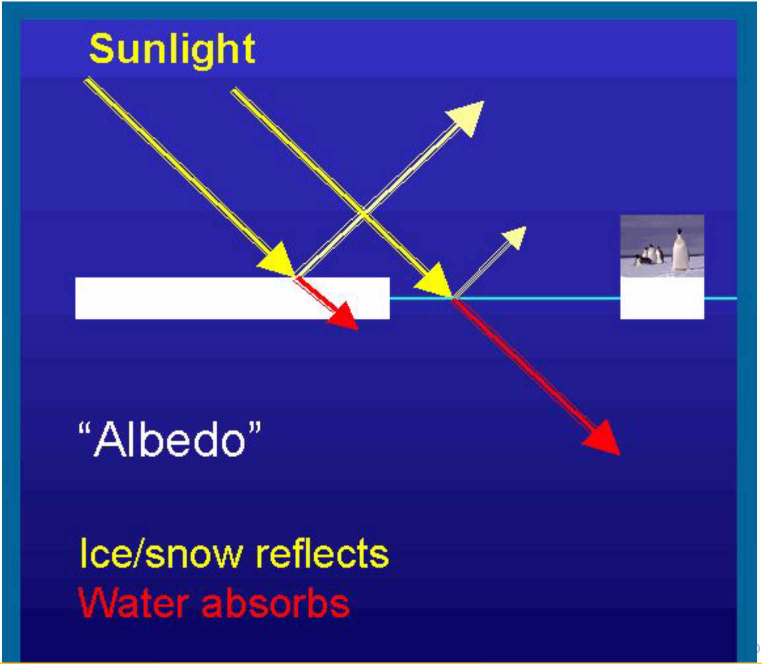
The spatial dimension of damages from climate change can be associated with two main factors:

1. Natural mechanisms which produce a spatially *non-uniform* distribution of the surface temperature across the globe.
    - Heat transport that balances incoming and outgoing radiation.
    - Differences among the local heat absorbing capacity - the local albedo - which is relatively higher in ice covered regions
2. Economic related forces which determine the damages that a regional (local) economy is expected to suffer from a given increase of the local temperature.
    - Production characteristics (e.g. agriculture vs services), or
    - Local natural characteristics (e.g. proximity to the sea and elevation from sea level).


Sunlight
"Albedo"
Ice/snow reflects
Water absorbs

- IAMs with a carbon cycle and no spatial dimension are "zero-dimensional" models and do not include spatial effects due to heat transportation across space.
- Existing literature (e.g. the RICE model) provides a spatial distribution of damages where the relatively higher damages from climate change are concentrated in the zones around the equator. Spatial distribution due to economic forces.
- Energy balance climate models (EBCMs) are "one- or two-dimensional" models which incorporate heat transport across latitudes or across latitudes and longitudes (e.g. Budyko 1969; Sellers 1969,1976; North 1975 a,b; North et al. 1981; Kim and North 1992; Wu and North 2007).
- Alternative spatial models: Pattern scaling, emulation theory. More detailed spatial patterns but may not be as useful for incorporating economic forces and nonlinear feedbacks.

- One-dimensional EBCMs predict a concave temperature distribution across latitudes with the maximum temperature at the equator.

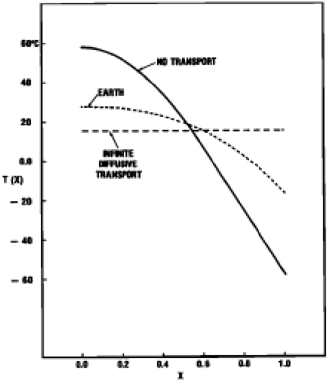


Fig. 4. Temperature (degrees Celsius) versus sine of latitude for the cases of no transport, infinite transport, and earth (schematic).

North, Cahalan and Coakley (1981)

The main contribution of our paper is to couple spatial climate models with economic models, and use these models to achieve three objectives:

First Objective: To show the role of heat transport across latitudes in the prediction of the spatial distribution and the corresponding temporal evolution of temperature and damages.

- In pursuing this objective we endogenously derive temperature and damage distributions.
- To our knowledge, this is the first time that the spatial distribution of surface temperature and damages, and their temporal evolutions, are determined endogenously in the conceptual framework of a coupled EBCM - economic growth model.

Second Objective: To provide insights regarding the optimal spatial and temporal profile of policy instruments (carbon taxes), when thermal transport across latitudes is taken into account.

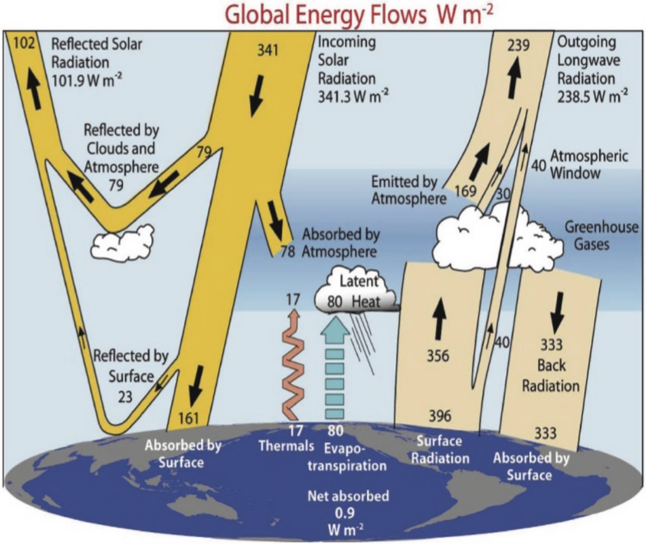
- Regarding the spatial profile of fossil fuel taxes, our results suggest higher tax rates for wealthier geographical zones due to the practical inability of implementing without cost the international transfers needed to implement a competitive equilibrium associated with the Pareto optimum, or when Negishi welfare weights are not used.
    - One-dimensional model provides a basis for exploring the impact of heat transport on the spatial differentiation of fossil fuel taxes between poor and wealthy regions.
    - Our results provide new insights into a result (non-uniform optimal mitigation) that was first noted by Chichilnisky and Heal (1994) by characterizing the spatial distribution of fossil fuel taxes and linking the degree of spatial differentiation of optimal fossil fuel taxes to the heat transport.

Second Objective (Continued):

- Regarding the temporal profile of optimal mitigation, among economists dealing with climate change on the mitigation side, the debate has basically settled on whether to increase mitigation efforts that is, carbon taxes, gradually (e.g. Nordhaus 2007, 2010, 2011), or whether to mitigate rapidly (e.g. Stern 2006, Weitzman 2009 a,b).
- In this paper we locate sufficient conditions for profit taxes on fossil fuel firms to be decreasing over time and for unit taxes on fossil fuels to grow over time more slowly than the rate of return on capital (also in Golosov et al. 2011). We also locate sufficient conditions for the tax schedule to be increasing according to the gradualist approach.

Third Objective: The third objective is to introduce spatial EBCMs with heat transport and endogenous albedo into economics as a potentially useful alternative to simple carbon cycle models in studying the economics of climate change.

- Latitude dependent climate models can address **damage reservoirs**. Damage reservoirs are sources of climate damages which will eventually cease to exist when the source of the damages is depleted, for example **Ice lines** and **permafrost**
- By deriving the spatiotemporal profile for carbon taxes, we show how the spatial EBCMs can contribute to the current debate regarding:
  1. how much to mitigate now,
  2. whether mitigation policies should be spatially homogeneous or not, and
  3. how to derive geographically specific information regarding damages and policy measures.


Global Energy Flows W m-2
102
Reflected Solar Radiation 101.9 W m-2
341
Incoming Solar Radiation 341.3 W m-2
239
Outgoing Longwave Radiation 238.5 W m-2
Reflected by Clouds and Atmosphere 79
79
40
Atmospheric Window
Emitted by Atmosphere
169
30
Greenhouse Gases
Absorbed by Atmosphere
78
Latent Heat
80
17
Reflected by Surface 23
356
40
333 Back Radiation
161
396
333
17 Thermals
80 Evapo-transpiration
Absorbed by Surface
Surface Radiation
Absorbed by Surface
Net absorbed 0.9 W m-2

Characteristics

1. Explicit incorporation of the spatial dimension into the climate model in the form of heat transport across latitudes.
   - one (latitude only) or , two (latitude and longitude) dimensional models.
2. The presence of an endogenous ice line where latitudes north (south) of the ice line are solid ice and latitudes south (north) of the ice line are ice free.
3. The underlying spatially dependent temperature function.

The use of a spatial EBCM allows us to:

- Estimate a spatial temperature distribution.
- Estimate spatial effects of temperature changes by deriving a damage function that depends not on the mean global temperature but on local temperature (i.e. the distribution of temperature across latitudes).
- Introduce the concept "damage reservoirs" like ice-lines and permafrost as feedback mechanisms generating in general damages in locations different from the location of the damage reservoir.

$I(x,t)$: infrared radiation to space in W/m^2 at latitude x at time t, $T(x,t)$: surface (sea level) temperature in $^\circ C$

$$I(x,t) = A + BT(x,t), \text{ empirical approximation}$$

$$A\left[(W/m^2)\right],\ B\left[W/(m^2)(^\circ C)\right], \text{empirical coefficients}$$

$$C_c \frac{\partial T(x,t)}{\partial t} = QS(x,t)\,\alpha(x,x_s(t)) - [I(x,t) - h(x,t)] + D\frac{\partial}{\partial x}\left[\left(1-x^2\right)\frac{\partial I(x.t)}{\partial x}\right]$$

x : the *sine* of the latitude $x = 0$ denotes the Equator, $x = \pm 1$ the N-S Pole; Q is the solar constant $S(x)$ is the mean annual meridional distribution of solar radiation; $\alpha(x, x_s(t))$ is the absorption coefficient which is one minus the albedo of the earth-atmosphere system (co-albedo), with $x_s(t)$ being the latitude of the ice line at time t; D is a thermal transport coefficient $Wm^{-2\circ}C^{-1}$; and C_c is heat capacity per unit area. Outgoing radiation is reduced by the human input $h(x,t)$.

The heat transport is modelled by the term:
$D\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial I(x,t)}{\partial x}\right]$
The ice line is determined dynamically by the condition:

$$\begin{array}{ll} T > -\widetilde{T}^{\circ}\mathrm{C} & \text{no ice line present at latitude } x \\ T < -\widetilde{T}^{\circ}\mathrm{C} & \text{ice present at latitude } x \end{array}$$

$-\widetilde{T}$ is empirically determined (e.g.-10°C)

Specifications for the co-albedo function:

$$\alpha(x, x_s) = \begin{cases} \alpha_0 = 0.38 & |x| > x_s \\ \alpha_1 = 0.68 & |x| < x_s \end{cases}$$

$$\alpha\left(x, T\left(x,t\right)\right) = c_0 + c_1 \tanh\left(T\left(x,t\right)+10\right)$$
$$a\left(x\right) = 0.681 - 0.202 P_2\left(x\right)$$

- Human input: $h(x,t) = \xi \left(1 + \ln \frac{M(t)}{M_0}\right)$ where M_0 denotes the preindustrial and $M(t)$ the time t stock of carbon dioxide in the atmosphere, ξ is a temperature-forcing parameter ($^\circ$C per W per m^2)
- The stock of the atmospheric carbon dioxide:

$$\dot{M}(t) = \int_{x=-1}^{x=1} \beta q(x,t)\, dx - mM(t), \;\; M(0) = M_0$$

  Emissions are proportional to the amount of fossil fuels used.
- The total stock of fossil fuel available is fixed, or

$$\int_{x=-1}^{x=1} q(x,t)\, dx = q(t) \;\; , \;\; \int_{0}^{\infty} q(t) = R_0,$$

  where $q(t)$ is total fossil fuels used across all latitudes at time t, and R_0 is the total available amount of fossil fuels on the planet.

$$
\begin{aligned}
C_c \frac{\partial T(x,t)}{\partial t} &= QS(x,t)\,\alpha(x, x_s(t)) - [I(x,t) - h(x,t)] + \\
& \quad D \frac{\partial}{\partial x}\left[\left(1-x^2\right)\frac{\partial I(x.t)}{\partial x}\right] \\
I(x,t) &= A + BT(x,t)
\end{aligned}
$$

$$
h(t) = \xi \ln\left(1 + \frac{M(t)}{M_0}\right)
$$

$$
\dot{M}(t) = \int_{x=-1}^{x=1} \beta q(x,t)\,dx - mM(t), \quad M(0) = M_0
$$

$$
\int_{x=-1}^{x=1} q(x,t)\,dx = q(t), \quad \int_0^{\infty} q(t) = R_0
$$

Approximating the temperature PDE

$$C_c \frac{\partial T(x,t)}{\partial t} = QS(x)\alpha(x,x_s) - [(A + BT(x,t)) - h(t)] + D\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial T(x,t)}{\partial x}\right]$$

Approximation

A satisfactory approximation of the solution can be obtained by the so-called two-mode solution where $n = \{0,2\}$, $P_n(x)$ Legendre polynomials.

$$\hat{T}(x,t) = \sum_{n \text{ even}} T_n(t) P_n(x)$$

$$\hat{T}(x,t) = T_0(t) + T_2(t)P_2(x)$$

$$C_c \frac{dT_0(t)}{dt} = -A - BT_0(t) +$$

$$\frac{1}{2}\int_{-1}^{1} QS(x)\alpha(x,x_s)dx + \xi \ln\left(1 + \frac{M(t)}{M_0}\right)$$

$$C_c \frac{dT_2(t)}{dt} = (-B + 6D)T_2(t) +$$

$$\frac{5}{2}\int_{-1}^{1} QS(x)\alpha(x,x_s)P_2(x)dx$$

$$T_0(0) = T_{00},\ T_2(0) = T_{20},\ P_2(x) = \frac{(3x^2-1)}{2}$$

$$S(x) = 1 + S_2P_2(x)\ ,\ S_2 = -0.482$$

Proposition

Assume that $\int_{-1}^{1} QS(x)\alpha(x, x_s)P_2(x)dx = \Phi(t) \leq UB < \infty$, *and that* $D \to \infty$. *Then the solution* $T_2(t)$ *of the two-mode solution vanishes.*

- Thus for a given transport $D < \infty$, the relative contribution of $T_2(t)$ to the solution $\hat{T}(t)$ can be regarded as a measure of whether the heat transport is important in the solution of the problem. This result suggests that the use of the global mean temperature alone in IAMs may introduce a bias.

$a(x) = a_0 - a_1 P_2(x);\ S(x) = 0.5\left[1 - s_0 P_2(x)\right]$,
The two-mode approximating ODEs become

$$C_c \frac{dT_0}{dt} = -A - BT_0(t) +$$
$$\frac{1}{2}\langle QS(x)\alpha(x), P_0(x)\rangle + \xi \ln\left(1 + \frac{M(t)}{M_0}\right)$$
$$C_c \frac{dT_2}{dt} = -(B + 6D)\, T_2(t) + \frac{5}{2B}\langle QS(x)\alpha(x), P_2(x)\rangle$$

Set $\frac{dT_0}{dt} = \frac{dT_2}{dt} = 0$. Then

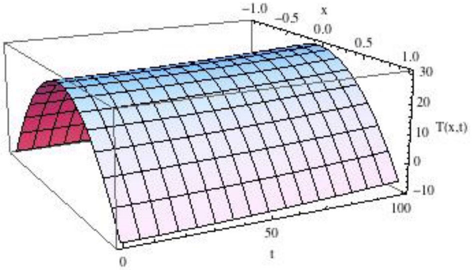
$$T(x, t; D) = C_0 + C_1 \ln \frac{M(t)}{M_0} - \frac{C_2}{(B + 6D)} P_2(x)\ , C_0, C_1, C_2 > 0$$

Table 1: Parametrization*

Parameter	**Value**	**Parameter**	**Value**
a_0	0.681	Q	340 W/m^2
a_1	0.202	M_0	596 GtC
s_0	0.477	$M\,(2011)$	831 GtC
A	221.6 W/m^2	ξ	5.35 °C (W/m^2)
B	1.24	g	1.178%
D	0.3	m	0.83%

(*)Values for the dimensionless α_0, a_1, s_0 have been obtained by North et al. (1981). Values for A, B,and D $\left[W/(m^2)(^\circ C)\right]$ have been obtained by calibration so as to reproduce current global temperature. C_c has been absorbed into the empirical coefficients. $g = 1.178\%$ is the average annual growth of total CO_2 emissions corresponding to the IPCC scenario A1F1 (http://www.ipcc-data.org/sres/ddc_sres_emissions.html)

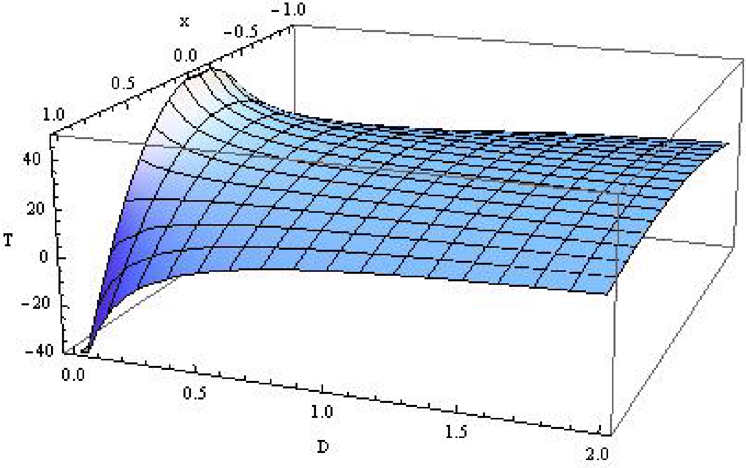
Current temperature: Equator 27°C, N-S Poles -9.5°C
Predicted temperature change $t = 100$: $+3.2$°C



Temperature function

$D \to \infty$ mean temperature around 14.8°C for 2011 and 14.4°C for the period 1951-1980. NASA's estimate of the mean temperature for the base period 1951-1980 is 14°C.

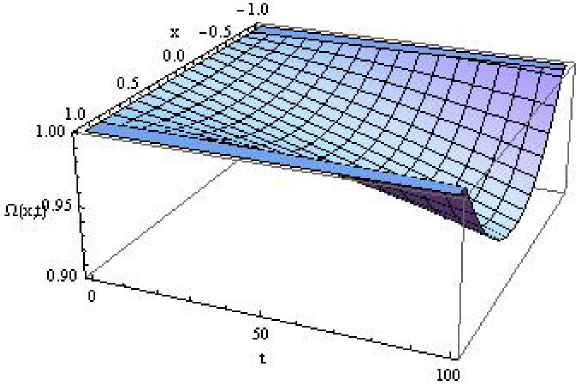
The impact of thermal transport



$$\Omega\left(x,t;D\right)=\exp\left(-\gamma t\hat{T}\left(x,t;D\right)\right)$$

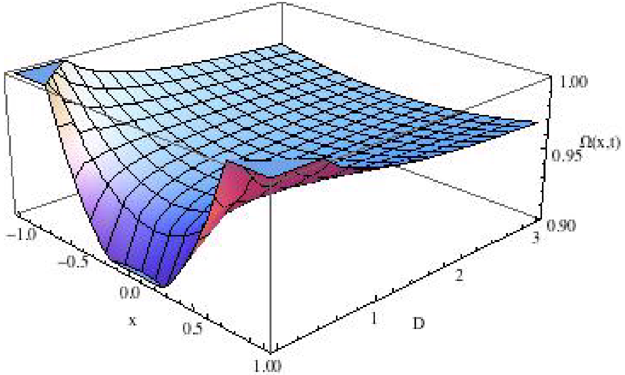
$$\gamma=0.0000252$$

Calibration based on Nordhaus and Boyer(1999,pp.4-44)



The damage function

The impact of heat transport on damages



$$
\begin{aligned}
dT_0(t) &= \left(\frac{dT_0}{dM}\right) dM(t), \;\; dT_2(t) = \left(\frac{dT_2}{dM}\right) dM(t) \\
dT(t,x) &= dT_0(t) + P_2(x)\, dT_2(t) = \\
&\left[\frac{dT_0}{dM} + \frac{dT_2}{dM} P_2(x)\right] dM(t)
\end{aligned}
$$

The impact on damages will then be determined as:

$$
\begin{aligned}
d\Omega(T(x,t)) &= \Omega'_T\left[dT_0(t) + P_2(x)\, dT_2(t)\right] = \\
&\Omega'_T\left[\frac{dT_0}{dM} + \frac{dT_2}{dM} P_2(x)\right] dM(t)
\end{aligned}
$$

$$
\begin{aligned}
Y(t,x) &= A(x,t)\Omega(T(x,t))F(K(x,t),L(x,t),q(x,t)) \\
&\equiv e^{(a+n\alpha_L)t}\Psi(x,T(x,t))K(x,t)^{\alpha_K}q(x,t)^{\alpha_q} \\
& \quad F_{total}(K(t),q(t),\{T(x,t)\}_{x=-1}^{x=1};x,t)
\end{aligned}
$$

"*Potential world GDP at date t*". The maximum output that the whole world can produce, given total world capital $K(t)$ available and total world fossil fuel $q(t)$ used, for a given distribution of temperature $T(x,t)$

$$
C(t)+\dot{K}(t)+\delta K(t) = F_{total}(K(t),q(t),\{T(x,t)\}_{x=-1}^{x=1};x,
$$

$$
j(t) = \int_{x=-1}^{x=1} j(x,t)\,dx, j = C, K, q
$$

$$
F_{total}(K(t),q(t),T;t) = \left[e^{(a+\alpha_L n)t}K(t)^{\alpha_K}q(t)^{\alpha_q}\right]J(t;D)
$$

$$
J(x,t;D) = \frac{\Psi(x,T(x,t))^{1/\alpha_L}}{\left[\int_{x'}\Psi(x',T(x',t)^{1/a_L}dx'\right]^{a_K+a_q}}
$$

- The Cobb-Douglas specification allows the "separation" of the climate damage effects on production across latitudes, as the "index" $J(t;D)$, which depends on thermal diffusion coefficient D, that multiplies a production function that is independent of x.
- Thus population growth and technical change affect the "macrogrowth component" $e^{(a+\alpha_L n)t}K(t)^{\alpha_K}q(t)^{\alpha_q}$, while changes in the size of D have a direct effect on the "climate component".
- The combination of the macrogrowth and the climate component determine the potential world input.

The Problem of the Social Planner

$$\max \int_0^{\infty} e^{-\rho t} \int_X \upsilon(x) L(x,t) \left[U\left(\frac{C(x,t)}{L(x,t)} \right) - \Omega_C(T(x,t)) \right] dxdt$$

subject to:

- Climate dynamics
- Resource constraint for the economy
- Total consumption and total fossil fuel constraints
- States: $\mathbf{v} = (K(t), R(t), M(t), T(t,x))$
- Controls: $\mathbf{u} = (C(t), C(x,t), q(t), q(x,t))$, $x \in X = [-1, 1]$

$$
\begin{aligned}
\mathcal{H} = & \int_X \upsilon(x) L(x,t) \left[U\left(\frac{C(x,t)}{L(x,t)} \right) - \Omega_C(T(x,t)) \right] dx + \\
& \lambda_K(t) \left[F_{total}(K(t), q(t), T; t) - C(t) - \delta K(t) \right] \\
& -\mu_R(t) q(t) + \lambda_M(t) \left[\int_X \beta q(x,t) \, dx - mM(t) \right] \\
& +\lambda_T(t,x) \left[\frac{1}{C_c} \left[QS(x) \alpha(x, T(x,t)) - (A + BT(x,t)) \right. \right. \\
& \left. \left. +\xi \ln \left(1 + \frac{M(t)}{M_0} \right) + D \frac{\partial}{\partial x} \left[(1 - x^2) \frac{\partial T(x,t)}{\partial x} \right] \right] \right. \\
& +\mu_C(t) \left[C(t) - \int_X C(x,t) \, dx \right] + \\
& \mu_q(t) \left[q(t) - \int_X q(x,t) dx \right].
\end{aligned}
$$

$$C(t), C(x,t) \quad : \quad \lambda_K(t) = \mu_C(t) = \upsilon(x)\, U'\left(\frac{C(x,t)}{L(x,t)}\right)$$

$$\text{or } F'_{total,q} = \frac{\mu_R(t) - \lambda_M(t)\beta}{\lambda_K(t)},$$

For equal weights, per capital consumption should be equated across locations.

$$\dot{\lambda}_K(t) = \left[\rho + \delta - F'_{total,K}(K(t), q(t), T; t)\right] \lambda_K(t)$$

$$\dot{\mu}_R(t) = \rho \mu_R(t)$$

$$\dot{\lambda}_M(t) = (\rho + m)\lambda_M(t) - \frac{\xi}{C_c\left(1 + \frac{M(t)}{M_0}\right)} \int_X \lambda_T(t,x)\, dx$$

$$\dot{\lambda}_T(t,x) = (\rho + 1)\lambda_T(t,x) + \upsilon(x) L(t,x) \Omega'_{c,T}(T(t,x))$$
$$-\lambda_K(t) F'_{total,T}(K(t), q(t), T; t) -$$
$$QS(x) \frac{\lambda_T(t,x)}{C_c} \frac{\partial \alpha(x, T(x,t))}{\partial T} - \frac{D}{C_c} \frac{\partial}{\partial x} \left[\left(1 - x^2\right) \frac{\partial \lambda_T(x,t)}{\partial x} \right]$$

- A solution of the welfare maximization problem, provided it exists and satisfies the desirable stability properties, will determine the optimal temporal and latitudinal paths for the states, the controls and the costates.
- The optimal time paths will be dependent on the thermal diffusion coefficient D. Denoting optimality by a $(^*)$, these paths can be written as:

$$\left\{K^*(t;D), K^*(t,x;D), M^*(t;D), T^*(t,x;D)_{x=-1}^{x=1}\right\}$$

$$\left\{C^*(t;D), C^*(x,t;D), q^*(t;D), q^*(x,t;D)\right\}_{x=-1}^{x=1}$$

$$\left\{\lambda_K^*(t;D), \lambda_R^*(t;D), \lambda_M^*(t;D), \lambda_T^*(t,x;D)\right\}_{x=-1}^{x=1}$$

- We consider a global market economy in which each latitude x can be considered as a country.
- In each country the representative consumer maximizes utility subject to a permanent income constraint by considering as parametric damages due to climate change.
- The representative consumption-good-producing firm maximizes profits by considering as parametric fossil fuel prices and taxes on fossil fuel use.
- Fossil fuel firms maximize profits by considering as parametric taxes on their profits.

$$\max_{\{C(x,t)\}} \left\{ \int_0^\infty e^{-\rho t} \left[L(x,t) U\left(\frac{C(x,t)}{L(x,t)} \right) - \bar{\Omega}_C(T(x,t;D) \right] dt \right.$$

$$\text{s.t} \int_{t=0}^\infty e^{-\Gamma(t)} p^s(t)\, C(x,t) dt =$$

$$K_0(x) + \int_{t=0}^\infty e^{-\Gamma(t)} p^s(t)\, I(x,t) dt$$

$$\lambda U' \left(\frac{C(x,t))}{L(x,t} \right) = \Lambda(x) e^{\rho t} p^C(t)$$

$$p^C(t) = e^{-\Gamma(t)} p^s(t),\ \Gamma(t) = \int_{s=0}^t r(s) ds$$

$\Lambda(x)$ the marginal utility of capitalized income at location x.

1st Welfare Theorem

Exact correspondence between the equilibrium problem and the planner's problem with optimality condition,

$$v(x)\,U'\left(\frac{C(x,t))}{L(x,t}\right) = \lambda_K(t;D)\,, \tag{1}$$

is obtained by letting $p^C(t) = e^{-\rho t}\lambda_K(t;D)$. The welfare weights are the reciprocal of marginal utility, or the so-called Negishi weights, $v(x) = 1/\Lambda(x)$.

2nd Welfare Theorem

A solution to the planner's problem resulting for a specific choice of welfare weights can be implemented as a competitive equilibrium with transfers across locations. The choice of zero transfers corresponds to the case of using the Negishi weights as welfare weights.

$$\max \; p^C(t)\,[\mathbb{A}(x,t)\Omega(T(x,t))F(K(x,t),L(x,t),q(x,t)) - (r(t)+\delta)K(x,t) - w(x,t)L(x,t) - (p(x,t)+\tau(x,t))\,q(x,t) - (p(x,t)+\tau(x,t))\,q(x,t)]$$

Optimality conditions:

$$\mathbb{A}(x,t)\Omega(T(x,t;D))F_K'(K(x,t),L(x,t),q(x,t)) = r(t)+\delta$$

$$\mathbb{A}(x,t)\Omega(T(x,t;D))F_q'(K(x,t),L(x,t),q(x,t)) = p(x,t)+\tau(x,t)$$

$$\mathbb{A}(x,t)\Omega(T(x,t;D))F_L'(K(x,t),L(x,t),q(x,t)) = w(x,t).$$

$$\max_{q(x,t)} \int_{t=0}^{\infty} e^{-\Gamma(t)} p^C(t) \left(p(x,t) q(t)(1-\theta(t))\right] dt,$$

$$\text{subject to} \quad \int_{t=0}^{\infty} \int_X q(x,t) dx dt \leq R_0$$

FONC

$$p(t)(1-\theta(t)) = \mu_0 e^{\Gamma(t)} =$$
$$\left[\mathbb{A}\Omega F_q' - \tau(x,t)\right](1-\theta(t)).$$

In any decentralized problem, consumption goods firms at latitude x will choose demands $K(x,t)$ and $q(x,t)$ to set

$$r(t)+\delta = \mathbb{A}\Omega F_K', \;\; p(x,t)+\tau(x,t) = \mathbb{A}\Omega F_q'$$

Market clearing requires

$$\begin{aligned}
\int_X B(x,t)\,dx &= 0, \int_X K(x,t)\,dx = K(t), \\
\int_X q(x,t)\,dx &= q(t) \\
\int_X C(x,t)\,dx &= C(t), \int_X Y(x,t)\,dx = Y(t).
\end{aligned}$$

Firms take temperature and taxes as parametric:

$$\left\{C^e(x,t;D,\tau,\theta,p), K^e(x,t;T,\tau,\theta,p), q^e(x,t;T,\tau,\theta,p)\right\}_{x=-1}^{x=1}$$

- For a given set of welfare weights $\upsilon(x)$, the social planner solves the Pareto optimum problem, denoted as $\mathrm{PO}^*(\upsilon)$. The solution produces the optimal paths $(^*)$.
- Implementation by competitive markets implies that each actor in the economy, i.e. consumers and firms, is faced with a tax on fossil fuels equal to the social marginal cost $\tau^*(x,t)$ of using fossil fuels at each x, t.
- This tax will induce the consumers and firms to produce a competitive equilibrium equal to the optimal quantities, provided that the firms' problems are concave and the consumers' problems are concave.

Implementation of $\text{PO}^*(\upsilon)$ requires that social and private marginal products for K and q be equated. The regulator can obtain a spatially uniform tax by:

1. Carrying out without cost the necessary adjustments to intertemporal endowment flows across locations so that $\Lambda(x) = \Lambda(x') = \bar{\Lambda} = 1/\bar{\upsilon}$ for all x, x'. Per capita consumption will be equated across latitudes

$$\tau^*(t; D) = \frac{\mu_R^*(t; D) - \beta\lambda_M^*(t; D)}{\bar{\upsilon}U'\left(C_{\bar{\upsilon}}^*(t)/L(t)\right)} - p(t). \qquad (2)$$

2. Using Negishi weights to implement a competitive equilibrium with zero transfers so that $\upsilon(x)\, U'\left(\frac{C^*(x,t)}{L(x,t)}\right) = 1$,

3. Making appropriate transfers so that $\hat{\upsilon}(x)\, U'\left(\frac{C_{\hat{\upsilon}}^*(x,t)}{L(x,t)}\right)$ is the same across locations for any arbitrary set of welfare weights $\hat{\upsilon}(x)$.

We use two-mode dynamics and the co-albedo function independent of the temperature field or $\alpha(x, T(x,t)) = \alpha(x)$.

$$\hat{T}(x,t) = T_0(t) + T_2(t,D) P_2(x)$$

$$C_c \dot{T}_0 = -A - BT_0 - \frac{A}{B} + \int_{x=-1}^{x=1} QS(x)\,\alpha(x)\,dx + \xi \ln\left(1 + \frac{M(t)}{M_0}\right)$$

$$C_c \dot{T}_0 = Z_0 - BT_0 + Z_1 \ln\left(1 + \frac{M(t)}{M_0}\right),$$

$$Z_0 = -A + \int_{-1}^{1} QS(x)\,\alpha(x)\,dx,\ \ Z_1 = \xi.$$

Optimal carbon taxes in closed economies

All locations are closed economies which have their own isolated capital markets, fossil fuel reserves and fossil fuel markets.

Proposition

The optimal full social price of fossil fuels for each closed economy across latitudes is:

$$p^*(x,t) = p(x,t) + \tau^*(x,t) =$$
$$\frac{\mu_R^*(x,t;D) - \beta\lambda_M^*(t;D)}{\lambda_K^*(x,t;D)} = \frac{\mu_R^*(x,t;D) - \beta\lambda_M^*(t;D)}{\upsilon(x)\, U'\left(\frac{C^*(x,t)}{L(x,t)}\right)}.$$

If the planner makes no international transfers and uses Negishi weights so that $\upsilon(x)\, U'\left(\frac{C^*(x,t)}{L(x,t)}\right) = 1$ for all x, then $p^*(x,t;D) = \mu_R^*(x,t;D) - \beta\lambda_M^*(t;D)$.
Assume $\mu_R^*(x,0;D) = \mu_R^*(x',0;D)$ for all x, then

$$\frac{p^*(x,t)}{p^*(x',t)} = \frac{\upsilon(x')\,U'\left(\frac{C^*(x',t)}{L(x',t)}\right)}{\upsilon(x)\,U'\left(\frac{C^*(x,t)}{L(x,t)}\right)}.$$

If $\upsilon(x')\,U'\left(\frac{C^*(x',t)}{L(x',t)}\right) \neq \upsilon(x)\,U'\left(\frac{C^*(x,t)}{L(x,t)}\right)$ for all $x' \neq x$, then the optimal full social price of fossil fuels is different across locations.

Proposition

When welfare weights across latitudes are equal and independent of x, *a latitude located at the equator* $x = 0$ *will pay a lower social price for fossil fuels relative to a latitude located at latitude* $x \neq 0$, *if* $U'\left(\frac{C^*(x,t)}{L(x,t)}\right) < U'\left(\frac{C^*(0,t)}{L(0,t)}\right)$.

Since latitudes around the equator are expected to be poorer, with relatively lower per capita consumption which implies $\frac{C^*(x,t)}{L(x,t)} > \frac{C^*(0,t)}{L(0,t)}$, these latitudes will pay a lower social price for fossil fuels relative to a richer latitude located away from the equator. For example with logarithmic utility and equal welfare weights,

$$\frac{p^*(x,t)}{p^*(0,t)} = \frac{C^*(x,t)/L(x,t)}{C^*(0,t)/L(0,t)}.$$

Optimal carbon taxes with costly international transfers

The regulator can transfer endowments across locations.

Transfers across locations are however costly (Chichilnisky and Heal 1994; Chichilnisky, Heal and Starrett 2000; Sandmo 2006; Anthoff 2011).

Cost of transfers

$$\int_X \left[C(x,t) + \dot{K}(x,t) + \delta K(x,t)\right] dx =$$

$$\int_X Y(t,x)\, dx - \frac{C_0}{2}\Theta(t)$$

$$\Theta(t) = \int_X \left[y(t,x) - C(t,x)\right]^2 dx,$$

$$y(t,x) = Y(t,x) - \delta K(t,x) - u(t,x)$$

$$\dot{K}(t,x) = u(t,x),$$

$$Y(t,x) = \mathbb{A}(x,t)\Omega(\hat{T}(x,t))F(K(x,t), L(x,t), q(x,t))$$

$y(t,x)$ can be interpreted as private consumption available out of the production of location x at time t.

Proposition

Assume that the difference between private consumption available out of local production and local private consumption is approximately constant over time, or $\frac{d[y(x,t)-C(y,t)]}{dt} \simeq 0$. *Then the optimal spatially non-uniform full social price for fossil fuels is*

$$p(x,t) + \hat{\tau}(x,t) = \frac{\mu_R^*(t;D) - \beta\lambda_M^*(t;D)}{\lambda_K^*(t;D)\left[1 - C_0\left[y^*(x,t) - C^*(x,t)\right]\right]}$$

$$\frac{p^*(x,t)}{p^*(0,t)} = \frac{\left[1 - C_0\left[y^*(0,t) - C^*(0,t)\right]\right]}{\left[1 - C_0\left[y^*(x,t) - C^*(x,t)\right]\right]}.$$

Proposition

If $[y^*(x,t) - C^*(x,t)] > [y^*(0,t) - C^*(0,t)]$, *then* $p^*(x,t) > p^*(0,t)$.

- Since locations around the equator are poor relative to higher latitude locations, it is expected that $[y^*(x,t) - C^*(x,t)] > [y^*(0,t) - C^*(0,t)]$, for $x \gg 0$.
- Poor locations should pay a smaller social price for fossil fuel relative to rich locations, which is similar to the result obtained above.
- If $p(x,t)$ is approximately equal across locations, the proposition implies that poor locations around the equator should pay a lower carbon tax.

- Spatially uniform taxes emerge as an optimal solution only when:
  1. transfers across locations equalize per capita consumption or marginal social valuations,
  2. Negishi welfare weights are used and distribution across latitudes does not change.
- Negishi weights - being the inverse of marginal utility - assign relatively larger welfare weights to locations with higher per capita consumption.
- The RICE model adopts Negishi weights and produces spatially uniform carbon taxes with invariant regional distribution.
- Our results suggest that the spatial structure of the optimal carbon tax is sensitive to welfare weights.
- When intertemporal distribution is treated as fixed or it is costly to change it, and welfare weights are not Negishi weights, poor locations could, under plausible assumptions, pay lower carbon taxes.

Assume optimal spatially uniform taxes. Hotelling's rule:

$$\frac{d\left[p(t)(1-\theta^*(t))\right]/dt}{p(t)(1-\theta^*(t))} = r(t) = \mathbb{A}\Omega F_K' - \delta$$

$$\frac{(\dot{p}(t)-\dot{\tau}^*(t))}{(p(t)-\tau^*(t))} = r(t) \quad \text{for } \theta^*(t) = 0$$

The policy ramp under the gradualist approach suggests that $\dot{\tau}^*(t) > 0, \dot{\theta}^*(t) > 0$.
To have a declining tax schedule through time:

$$r(t) - \frac{\dot{p}(t)}{p(t)} > 0,$$

Lemma

$\zeta(t) \equiv \int_x \lambda_T^*(t,x;D)dx < 0,\ \lambda_M^*(t;D) < 0.$

- Thus $\zeta(t)$ is the global shadow cost of temperature at time t across all latitudes.
- $\lambda_M^*(t;D) < 0$ means that an increase in atmospheric accumulation of CO_2 at any time t will reduce welfare.

Proposition

If $m < \delta$, then the optimal profit tax decreases through time, or $\dot{\theta}^*(t) < 0$. Furthermore, the optimal unit tax on fossil fuels grows at a rate less than the rate of interest, or $\frac{\dot{\tau}^*(t)}{\tau^*(t)} < r^*(t)$.

Proposition

If $m > \delta$ and $\lambda_M^*(m-\delta) - \left(\frac{\xi}{BM(t)}\right)\int_X \lambda_T^* dx > 0$ then $\dot{\theta}^*(t) > 0$ and $\frac{\dot{\tau}^*(t)}{\tau^*(t)} = \frac{\dot{p}^*(t) - r^*(t)p^*(t)}{\tau^*(t)} + r^*(t) > r^*(t)$.

- Thus, we have sufficient conditions for rapid ramp-up of profit taxes and for unit carbon to rise at a rate less than the net of depreciation rate of return $r^*(t)$ on capital.
- The gradualist tax schedule requires rapid decay of the atmospheric carbon dioxide, and a relatively small global shadow cost of temperature at time t across all latitudes.

Simplifying assumptions: Assume no technical change, constant population, no fossil fuel constraint at each latitude, logarithmic utility function with no damages in utility due to temperature increase, a constant returns to scale production function at each location, and an exponential damage function associated with output $\Omega(\hat{T}(x,t)) = \exp\left(-\gamma \hat{T}(x,t)\right)$.

Let

$$\begin{aligned} A_1 &= \left\{x : -1/\sqrt{3} < x < 1/\sqrt{3}\right\} ,\ A_2 = \left\{x : x = \pm 1/\sqrt{3}\right\} \\ A_3 &= \left\{x : 1/\sqrt{3} < x \leq 1 \text{ and } -1 \leq x < -1/\sqrt{3}\right\}. \end{aligned}$$

Proposition

Under the simplifying assumptions above, an increase in the heat transport coefficient D will have the following effects on the steady state Pareto optimal solution of the planner's problem in closed economies with nonnegative welfare weights: (i) in A_1 it will reduce temperature and damages, increase per capita capital and consumption, and increase the social cost of fossil fuels, (ii) in A_2 it will have no effect, and (iii) in A_3 it will increase temperature and damages, reduce per capita capital, consumption, and the social cost of fossil fuels.

Is there any bias from ignoring heat transport or equivalently ignoring cross latitude externalities?

Not accounting for D implies that the Pareto optimal solution will underestimate temperature and damages, overestimate per capita capital and consumption and underestimate the social price of fossil fuels at low latitudes in A_1. The opposite applies to high latitudes in A_3.

- The use of fossil fuels in production generates emissions which block outgoing solar radiation, thus increasing the temperature. Heat moves across latitudes.
- We derive latitude dependent temperature, damage and climate response functions, as well as optimal fossil fuel taxes, which are all determined endogenously through the interaction of climate dynamics with optimizing forward-looking economic agents.
- We show the links between welfare weights and international transfers across locations and the spatial structure of optimal taxes.
- When per capita consumption across latitudes can be adjusted through costless transfers for any set of nonnegative welfare weights, so that marginal valuations across latitudes are equated, or transfers are zero and Negishi welfare weights are used, then optimal carbon taxes are spatially homogeneous.

- When marginal valuations across latitudes are not equated, due to institutional/political constraints or the cost of transfers, optimal carbon taxes are spatially differentiated.
- If the international transfers are costly and the planner is not constrained to using Negishi weights, then taxes on fossil fuels could be lower in relatively poorer geographical zones.
- Comparative static analysis suggests that since heat transport affects local damages and local economic variables, ignoring it - by using mean global temperature as the central state variable for climate - introduces a bias.

- If the decay of atmospheric CO_2 is lower than the depreciation of capital, then profit taxes on fossil fuel firms will decline over time and unit taxes on fossil fuels will grow at a rate less than the interest rate.
- These results, which can be contrasted with the gradually increasing policy ramps derived by IAM models like DICE or RICE, indicate that mitigation policies should be stronger now relative to the future.
- Increasing policy ramps require rapid decay of the atmospheric carbon dioxide, and a relatively small global shadow cost of temperature increase.

- Augment our EBCM with a deep ocean component that redistributes vertically the heat energy via uniform vertical diffusion
- Use the one-dimensional EBCM with feedback mechanisms for the co-albedo to introduce latitude dependent damage reservoirs like endogenous ice-lines and permafrost
- Since reservoir damages are expected to arrive relatively early and diminish in the distant future - because the reservoir will be exhausted - the temporal profile of the policy ramp could be declining, enforcing the result obtained for profit taxes, or even U-shaped