Lecture 2
Dynamic stochastic general equilibrium (DSGE) models

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Neoclassical growth model

- Goal of modern macro research is to provide a model that is consistent with the "trend" facts, but can also replicate the "cyclical properties."
- Model combines ingredients of firm behavior and household behavior and includes a well-specified definition of equilibrium.
- Model can be "quantified" (analyzed and evaluated numerically) and its predictions can be compared to the data.

Outline

Key features of the model:

1. Firms maximize profits by demanding capital and labor and supplying output. Firms take the price of capital (r) and the price of labor (w) as outside their control.

2. Households maximize utility subject to their budget constraint by demanding output (to be split into consumption and savings) and supplying labor and capital. Households take the price of capital (r) and the price of labor (w) as given and outside their control.

3. An equilibrium is a set of prices, r and w, such that firms maximize profits, households maximize utility, and markets clear: Output supplied by firms is equal to output demanded by households, and capital and labor demanded by firms is equal to capital and labor supplied by households.

Sources of fluctuations

We can view booms and busts of real GDP as reflective of relatively high and low levels of technology or policy.

- Technology increases at a relatively fixed rate over time, consistent with the long-run growth observations.
- The level of technology can persistently deviate from its growing trend, explaining the cycle:
  - A shock to technology causes an increase in the marginal product of capital.
  - Households want to save rather than consume when the marginal product of capital (and interest rates) are high.
- This theory of business cycles is one reason that the 2004 Nobel Prize was awarded to Finn E. Kydland and Edward C. Prescott.

Sources of fluctuations

- Similarly, unexpected policy shocks can contribute to boom and bust cycles of real GDP.
- Classical view:
  - Monetary policy is ineffective
  - Fiscal policy through government spending and changes in tax rates affects labor supply and thus business cycles.
  - Government deficits increase production but make people worse off by crowding out investment and consumption.
  - Since wealth effect on labor supply is small, increase in government spending is partially offset by decrease in investment and consumption.
- Keynesian view:
  - Main assumption: prices do not adjust in short-run
  - Business cycles can be caused by monetary shocks
  - Government spending shocks have a potentially much larger impact

Sources of fluctuations

1. 1929 Great Depression: Bad policy and stock market crash
2. 1974 Recession: OPEC–quadrupling of oil prices
3. 1980 Recession: Iranian Revolution–sharp increase in oil price
4. 1982 Recession: Good Fed Policy! (Volker's disinflation)
5. 1990 Recession: Consumer pessimism
6. 1990's Japan: End to speculative stock market and housing bubbles, bad policy, accumulation of debt and liquidity traps
7. 2001 Recession: Consumer confidence loss due to burst of dotcom bubble
8. 2009 Recession: Collapse of housing prices and credit crunch

Neoclassical growth model: Household

Allow for population growth, $L_t = (1+n)L_{t-1}$ and technology growth, $z_t = (1+g)z_{t-1}$, but technology is stochastic.

The problem facing the household is

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1-l_t) L_t$$

subject to

$$C_t + A_{t+1} = L_t \hat{w}_t l_t + A_t(1+\hat{r}_t) - T_t,$$

where T_t are lump-sum taxes levied by the government.
In per-capita terms

$$\max E_0 \sum_{t=0}^{\infty} (1+n)^t \beta^t u(c_t, 1-l_t)$$

$$s.t. \quad c_t + (1+n)a_{t+1} = \hat{w}_t l_t + a_t(1+\hat{r}_t) - \tau_t,$$

where $\hat{r}_t = r_t - \delta$ and $\hat{w}_t$ are the after-tax return on capital and wage rate received by the household.

Neoclassical growth model: Firm

The problem facing the firm is

$$\max Y_t - w_t N_t - r_t K_t,$$

where

$$Y_t = K_t^{\alpha}(z_t N_t)^{1-\alpha}, \quad 0 < \alpha < 1.$$

Production function in per-capita terms is

$$\begin{aligned} Y_t &= K_t^{\alpha}(z_t N_t)^{1-\alpha} \div L_t \\ \frac{Y_t}{L_t} &= \left(\frac{K_t}{L_t}\right)^{\alpha}\left(z_t \frac{N_t}{L_t}\right)^{1-\alpha} \\ y_t &= k_t^{\alpha}(z_t l_t)^{1-\alpha} \end{aligned}$$

Neoclassical growth model

Market clearing conditions are:

1. Labor market: $N_t = L_t l_t$
2. Capital/asset market: $K_t = A_t$
3. Goods market:

$$X_t = Y_t - C_t - T_t + T_t - G_t$$

where $X_t = K_{t+1} - (1-\delta)K_t$ is investment.

In per capita terms:

$$(1+n)k_{t+1} - (1-\delta)k_t = y_t - c_t - G_t/L_t$$

Sources of uncertainty in the model

So, we assume that technology and government purchases are stochastic. We need to specify their evolution.

Technology:

$$\begin{aligned} \ln z_t &= \ln z_0 + gt + \tilde{z}_t \\ \tilde{z}_t &= \rho_z \tilde{z}_{t-1} + \varepsilon_{zt}, \quad |\rho_z| < 1 \end{aligned}$$

Government purchases:

$$\begin{aligned} \ln G_t &= \ln G_0 + (g+n)t + \tilde{G}_t \\ \tilde{G}_t &= \rho_g \tilde{G}_{t-1} + \varepsilon_{gt}, \quad |\rho_g| < 1 \end{aligned}$$

Notice that these processes, when re-written in levels, have a more familiar form:

$$\begin{aligned} z_t &= z_0 (1+g)^t \exp(\tilde{z}_t) \\ G_t &= G_0 \left((1+g)(1+n)\right)^t \exp(\tilde{G}_t) \end{aligned}$$

Solving the model

First-order conditions for the household are:

$$\begin{aligned} c_t &: \quad [\beta(1+n)]^t \, u'_{ct} = \lambda_t \\ l_t &: \quad -[\beta(1+n)]^t \, u'_{lt} = \hat{w}_t \lambda_t \\ a_{t+1} &: \quad \lambda_t (1+n) = E_t \lambda_{t+1} (1 + \hat{r}_{t+1}) \end{aligned}$$

which after combining them give:

$$\begin{aligned} u'_{ct} &= \beta E_t u'_{ct+1} (1 + \hat{r}_{t+1}) \\ -u'_{lt} &= \hat{w}_t u'_{ct} \end{aligned}$$

Firm's first-order conditions are

$$\begin{aligned} k_t &: \quad \alpha \frac{y_t}{k_t} = r_t \\ l_t &: \quad (1-\alpha) \frac{y_t}{l_t} = w_t \end{aligned}$$

We also know:

$$\begin{aligned} \hat{r}_t &= r_t - \delta \\ \hat{w}_t &= w_t \end{aligned}$$

How to solve a general model?

The algorithm for solving dynamic stochastic general equilibrium (DSGE) models generally consists of the following steps:

Step 1. Derive the first-order conditions of the model.

Step 2. Find the steady state.

Step 3. Linearize the system around the steady state.

Step 4. Solve the linearized system of equations (i.e. decision rules for jump variables and laws of motion for state variables).

We must do Step 1. DYNARE can implement Steps 2-4 for us.

Step 1. First-order conditions

Assume

- Log utility: $u_t = \ln c_t + b\ln(1 - l_t)$
- $g = n = 0$ and $z_0 = 1$ which give:

$$z_t = exp(\tilde{z}_t), G_t = G_0 exp(\tilde{G}_t)$$

- with

$$\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \varepsilon_{zt}$$
$$\tilde{G}_t = \rho_g \tilde{G}_{t-1} + \varepsilon_{gt}$$

First-order conditions for the household are:

$$\begin{aligned} 1/c_t &= \beta E_t(1 + r_{t+1} - \delta)/c_{t+1} \\ b/(1 - l_t) &= w_t/c_t \end{aligned}$$

Step 1. First-order conditions

Firm's first-order conditions are:

$$\alpha y_t / k_t = r_t$$
$$(1-\alpha) y_t / l_t = w_t$$

Market clearing:

$$c_t + x_t + G_0 exp(\tilde{G}_t) = y_t$$

Investment, x_t:

$$k_{t+1} - (1-\delta) k_t = x_t$$

Output:

$$y_t = exp(\tilde{z}_t)^{1-\alpha} k_t^{\alpha} l_t^{1-\alpha}$$

Step 1: Summary

Unknowns: 9

- Output y
- Consumption c
- Labor input l
- Investment x
- Capital input k
- Prices w, r
- Productivity z
- Government consumption G

Equations: 9

The number of unknowns and equations must be the same.

Step 2: Calibration

We need to assign values to the parameters used in the model. These are usually chosen based on the accepted values in the literature or to match some data moments. Let's start with the following values:

- capital income share: $\alpha = 0.33$
- depreciation $\delta = 0.025$
- preferences: $\beta = 0.99$, $b = 2$
- productivity: persistence $\rho_z = 0.95$; variance $\sigma_z^2 = 0.0001$
- government consumption: persistence $\rho_g = 0.95$; variance $\sigma_g^2 = 0.0001$

Step 2: Steady state

1. Consumption-leisure choice:

$$\frac{b}{1-l} = \frac{w}{c}$$

2. Capital accumulation:

$$1 = \beta(r + 1 - \delta)$$

3-4. Shocks

$$\tilde{z} = 0$$
$$\tilde{G} = 0$$

5. Production function

$$Y = exp(\tilde{z})^{1-\alpha} k^{\alpha} l^{1-\alpha}$$

Step 2: Steady state

6-7. Factor prices

$$\begin{aligned} r &= \alpha y/k \\ w &= (1-\alpha)y/l \end{aligned}$$

8. Investment

$$x = \delta k$$

9. Market clearing

$$c + x + G_0 exp(\tilde{G}) = y$$

These 9 equations can be solved for 9 unknown steady state values of our variables.

Step 3: DYNARE

The next step is to linearize the system of equations and solve the resulting system of difference equations. DYNARE can do that.

- open matlab
- add path to dynare by typing in the matlab command window: addpath c:\dynare\4.4.2\matlab
- guide matlab to the folder with the .mod file and add path
- to run the program type in matlab command window: dynare *program-name*.mod

Step 4: Analyzing the model

We can study the model by looking at several characteristics:

- Unconditional moments: variances, covariances, persistence, etc.
- Variance decomposition: the share of variation in a given endogenous variable explained by a particular shock
- Conditional moments: impulse responses of various variables to shocks

Results: Unconditional moments

VARIABLE	MEAN (LOG)	MEAN (LEVEL)	STD. DEV.	VARIANCE
y	0.027	1.027	0.044	0.002
c	-0.613	0.542	0.051	0.003
n	-0.946	0.388	0.014	0.000
k	2.001	7.399	0.050	0.003
w	0.572	1.772	0.047	0.002
r	-3.083	0.046	0.033	0.001
x	-1.688	0.185	0.120	0.014
$\tilde{z}$	0.000	1.000	0.032	0.001
$\tilde{G}$	0.000	1.000	0.032	0.001
c/y		0.528		
x/y		0.180		
G/y		0.292		

Results: Variance decomposition, in percent

	ε_z	ε_g
y	99	1
c	97.65	2.35
n	71.49	28.51
k	99.77	0.23
w	99.57	0.43
r	96.34	3.66
x	99.77	0.23
$\tilde{z}$	100	0
$\tilde{G}$	0	100

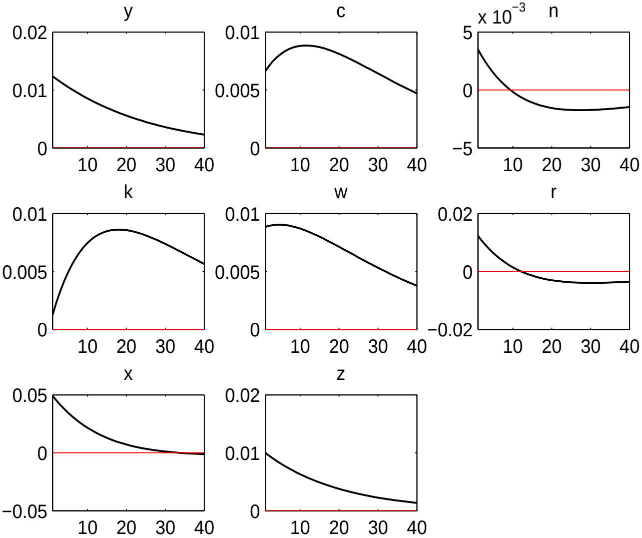
Results: Correlations

Variables	y	c	n	k	w	r	x	$\tilde{z}$	$\tilde{G}$
y	1.00	0.89	-0.07	0.80	0.96	0.17	0.88	0.99	0.10
c	0.89	1.00	-0.50	0.96	0.99	-0.25	0.64	0.88	-0.15
n	-0.07	-0.50	1.00	-0.61	-0.36	0.89	0.26	-0.05	0.53
k	0.80	0.96	-0.61	1.00	0.93	-0.46	0.44	0.75	-0.04
w	0.96	0.99	-0.36	0.93	1.00	-0.10	0.74	0.94	-0.06
r	0.17	-0.25	0.89	-0.46	-0.10	1.00	0.58	0.24	0.19
x	0.88	0.64	0.26	0.44	0.74	0.58	1.00	0.92	-0.04
$\tilde{z}$	0.99	0.88	-0.05	0.75	0.94	0.24	0.92	1.00	0.00
$\tilde{G}$	0.10	-0.15	0.53	-0.04	-0.06	0.19	-0.04	0.00	1.00

Results: Autocorrelations

Order	1	2	3	4	5
y	0.96	0.92	0.88	0.84	0.81
c	0.99	0.98	0.97	0.95	0.93
n	0.95	0.90	0.86	0.82	0.78
k	1.00	0.99	0.99	0.98	0.97
w	0.98	0.96	0.94	0.92	0.90
r	0.92	0.85	0.78	0.71	0.65
x	0.91	0.83	0.75	0.68	0.62
$\tilde{z}$	0.95	0.90	0.86	0.81	0.77
$\tilde{G}$	0.95	0.90	0.86	0.81	0.77

Impulse responses: Positive shock to productivity



Positive productivity shock: Consumption

- Given the process for z, a positive shock z lasts for a long time: i.e. z remains above its initial level for a while. This means that economy will remain more productive than usual for quite some time.
- An increase in z immediately raises output, which in turn increases household's lifetime wealth. This wealth effect acts to increase their consumption. The increase in consumption is smaller than the rise in output since the shock is temporary.

Positive productivity shock: Interest rate

- Higher z also implied higher return to capital, which raises interest rate in the economy. As usual, this has two effects: income and substitution effect.
  - On the one hand, higher r makes saving and investment more attractive: substitution effect.
  - On the other hand, higher r makes household richer and reduces the incentive to save: income effect.
  - Since the shock is long-lasting but temporary, the substitution effect dominates. Thus, k_t is gradually rising.

Positive productivity shock: Investment

- Since the increase in z dies out slowly, $r = \alpha y/k - \delta$ remains high since k can not rise abruptly. So, both saving and investment remain above their initial levels for a while.
- As technology returns to its initial level, the slow adjustment in k eventually causes y/k to fall below its initial level and thus causes r to fall below its initial level.

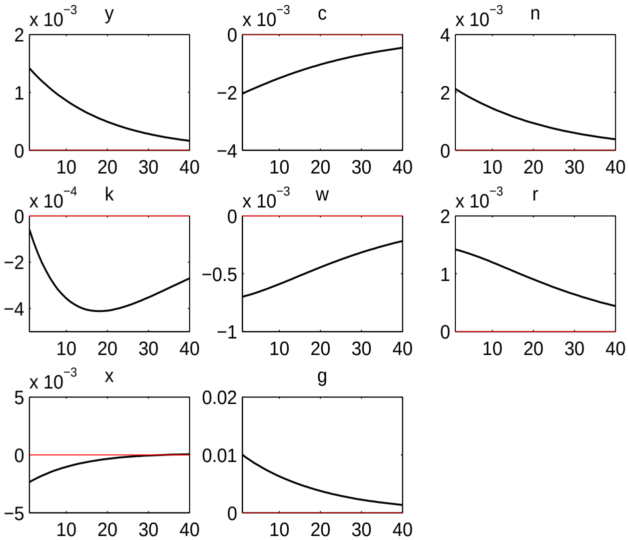
Positive productivity shock: Labor

- Higher z also implied higher wages. Again, income and substitution effects compete:
  - Higher w leads to higher labor input: substitution effect
  - Higher w also makes household richer for a given l, thus lowering her labor input: income effect
  - Productivity being higher but dissipating over time, makes it especially appealing time to work more, so l rises.

Positive productivity shock: Capital

- Since capital adjusts only slowly, adjustments in labor supply balance out these slow adjustments in k: households build up k during the early phase partly by increasing labor supply; and bring it back to the initial level in the later phase by decreasing labor supply.
- Persistence of the shock to z matters for these dynamics by affecting the strength of the income effects in the model.

Impulse responses: Positive shock to government spending



Positive government spending shock: Summary

- Given the process for G, a positive shock G lasts for a long time.
- An increase in G causes a decline in lifetime household's income, leading to a fall in consumption and an increase labor supply.
- Since consumption decline is smaller than the decline in household's disposable income, saving fall as well. Investment declines.
- Capital stock thus gradually declines.
- Because technology, z, is unchanged after the shock to G, this decline is k is very small. Output $y_t = k_t^{\alpha}(z_t l_t)^{1-\alpha}$ thus tracks employment quite closely.
- A fall in k and an increase in l lead to a rise in the return to capital r and a fall in the wage rate w.
- Persistence of the shock to G matters for these dynamics by affecting the strength of the income effects in the model.