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Analysis of the government bond market and monetary policy

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Institute of Business Administration (IBA), Karachi

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Analysis of the Government Bond Market and Monetary Policy

Abstract

The study explores the evolution between the yield curve and the Pakistan's economy, with a special focus on examining the effects of the monetary policy and slope of the US term structure on the emerging market yield curve and the possible feedback effect on the real sector by applying a yields-macro model. The yield curve model of this study explicitly incorporates both the yields factors (level, slope, and curvature) and the macroeconomic variables (overall economic activity, exchange rate, money supply and inflation rate). Empirical results from the yields-macro factors model show that there is a statistically significant bidirectional linkage between the macroeconomic and the yield curve factors; however, by contrast with conventional wisdom, macro variables play a less prominent role in explaining the yield factors as compared to the strength of effect from the latter to the former. Furthermore, the volatility in bond markets is found to be asymmetrically affected by positive and negative shocks and more sensitive to recent innovations rather than the lag volatility. The structural decomposition indicates that it is the entire term structure of interest rate that transmits the policy shocks to the real economy. The monetary policy signals pass through the yield curve level and the slope factors to stimulate the economic activity. Besides the slope factor, the curvature factor also reflect the cyclical fluctuations of the economy. One can infer from the overall results that the slope and the curvature factors serve as leading counter-cyclical and pro-cyclical indicators respectively. In addition, the study finds that the domestic yield curve in emerging economies has in-sample information content. The US yield curves also have in- and out-of-sample information content for future yield curve shape, inflation and growth in emerging economies. This may be due to exchange rate pegging to the US dollar in the emerging economies.

Analysis of the Government Bond Market and Monetary Policy

1. Introduction

This research study expands an earlier study, which analyses the term structure of interest rate of the Pakistani government bonds and explains the term structure theory and its application, with a particular focus on the modeling and forecasting aspects of the yield curve (Nishat and Ullah, 2015). The findings of this research indicate that forward rates are potentially useful as indicators of market expectations of future interest rates, inflation rate and exchange rate. The term structure of interest rate seems a leading indicator of the future economic activity and stock market. In the proposed study we would like to evaluate the effectiveness of the monetary policy, focusing on the expectation channel. The objective of current study is to comprehend the dynamics of the yield curve factors and macro-economy, particularly the effectiveness of monetary policy by looking at more elaborated specifications to account for time-varying volatility and a more complete set of possible macroeconomic variables and monetary policy instruments in the term structure model. This study illustrates how developments in economic theory, combined with insights learned from the Pakistan's experiences have produced the policy strategy that the other central banks may use. Moreover, we relate the factors in our model to the relevant macroeconomic variables that is helpful to interpret the factors in the macroeconomic scenario, which is one of the widely debated issues in the statistical class of term structure models. On the academic front, it arises up with a model that, besides having sound theoretical foundation and describing the market trends (optimally fit and precisely forecast), can also serve for policy analysis in order to understand important aspects of the recent intertwined financial crisis, economic recessions and policy regimes. From methodological point of view, we include the common stochastic volatility component in the term structure model that follows the EGARCH process while adopting the state-space approach. Adding a common stochastic volatility component increases the flexibility of the term structure model and enables it to fit attractively the more complex shapes of the yield curve. This issue has been addressed by a burgeoning macro-finance literature, which is described in Rudebusch and Wu (2003).

In this research study we aim to improve the understandings on the dynamic interaction between the yield curve factors and macro-economy, and the role of the term structure of interest rate in transmitting the signals of monetary policy to the real sector. The study examines the effectiveness of monetary policy tools in affecting the yield curve, the evolution between the yield curve and the economy with a special focus on examining the effects of the monetary policy on the yield curve, and the possible feedback effect on the real sector by applying a macro-finance model. The study also evaluates the potential of yield curve in predicting the future state of economy, i.e., predicting the upcoming recession and economic downturns and tests the relative potency of the expectation channel, which has attracted considerable interest after the recent ample liquidity in the Pakistani market. In particularly, we are interested to figure out the transition mechanism through which the monetary policy affects the real economy and the financial sector. Moreover, by formulating a term structure model that integrates macroeconomic factors in the state-space framework, we evaluate the policy shocks through the short, medium and long-term bond yields to the real economy. Furthermore, contrary to the results of standard term-structure models, we evaluate the expectations hypothesis of the term structure of interest rates with time-variant term premia. This study extends our earlier work (Nishat and Ullah, 2015) about the term structure of interest rate in the Pakistani bond market and provides us opportunity to relate our yield curve modeling approach to the monetary policy regimes in Pakistan. This research hence influences yield curve understanding and its manipulation across many fields such as monetary policy, deficit financing, equity market and international finance.

In the context of emerging markets, this study is the first attempt to design a macro-finance model to evaluate the transmission mechanism of monetary policy. Recent events (crisis) reveal that there is a close feedback between the real economy and financial conditions. Given that the short-term interest rate interconnects finance and the macro literature, investigation of the term structure of interest rates and inspecting the role of macroeconomic variables in the yield curve movements is a prime objective of the study. Moreover, it is helpful to pinpoint the possible role of yield curve factors for predicting the future state of economy. The analysis provides insights regarding the macroeconomic and financial stability, which is one of the key issues in the context of Pakistan economy now a day. Furthermore, until very recently, standard macroeconomic models have not incorporated long-term interest rates or the yield curve. And even when they have, most of the attention is still on the correlation between the real economy and the shortest-term interest rate in the model rather than on the whole yield curve. In this context, this study attempts to bring up a yield curve model that can predict the term structure of interest rate and macroeconomic activity on one hand, and serve for policy analysis on the other hand. The problem is appealing in the context of the current situation of Pakistani economy, where the monetary policy authorities at the SBP have lowered the policy rate by 50 basis point to stimulate the economy and facilitate the ongoing demand for investment given the current situation of economy. We are interested to see if these fluctuations in the short-term rates will translate and transmit in long end of yield curve, which in turn may cause a fluctuation in the real sector and long run dynamics of economy. It will be also of use for the financial managers to dig out the hedging opportunities and better risk management, and to help to predict the future states of economy.

The study is related to the implications of the yield curve for the macroeconomic stabilization policy by comprehending the economic and financial dynamics to financial markets. At certain point of time, the yield curve can have different shapes. These shapes, representing a time-varying relationship of the interest rate and maturity, are of great significance for various economic and financial decisions. Particularly, the inversion of yield curve in the US triggers out the signal of recession (Ang et al. 2006). The relation between the term spread and economic activity may be that the slope of yield curve reflects the stance of monetary policy variation. If the policymakers raise short-term interest rates, long-term rates are usually not increasing one-to-one with them but slightly less. Hence, the spread tightens and even might become negative. Higher interest rates slow down overall spending and, consequently, stagnates the economic growth. Therefore, a small or negative slope of the yield curve will be an indication for a slower growing economy and a decline in inflation in the future (Svensson, 2003; and Bernanke et al. 2005).

The literature places strong emphasis on the US economy and indeed, international evidence has remained scarce and limited. Furthermore, most of the evidences regarding the joint interaction between yield curve and monetary policy factors are based on pricing data obtained from the developed economies bond markets such as USA (Diebold et al. 2006) and Japan (Ullah et al. 2014), where the markets are efficient and the informational contents reflect fully and instantaneous changes in the prices. There is no study that evaluate the informational contents of the yield curve for the policy analysis in the context of emerging markets, where the market suffer from the lack of liquidity and the governments rely heavily on the bond financing to finance its deficits. Secondly, these models fail to reflect the stances of monetary policy on real economic activity through the domestic yields spread because of the high sensitivity of the yield curve shape to external shocks (fluctuation in the economic condition in the large economies, such as US and EU countries) rather than only the domestic policy shocks (Mehl, 2006). Since, the slope of yield curve in emerging markets may not be the only relevant factor to assess the impact of monetary transmission, therefore, it seems more appropriate to consider the impact on medium to long-term maturities yields rather than only the term spread; as they are the fundamental conduits for the transmission of monetary policy, include the spillovers related macro and financial factors in the model to analyze the transmission mechanism of monetary policy and the possible spillovers from the US or the Euro area yield curve to the term structure factors of emerging economies.

Objectives of the study:

This main objectives of the study is to:

- Formulate a yield curve model that integrates monetary policy as well as real economy factors in the term structure model, while simultaneously taking into account the international financial transmission of shocks from the developed markets to the emerging ones.
- Find out a more appropriate model for predicting the stances of monetary policy on the term structure of interest rate and the possible feedback on the real sector and equity market.
- Figure out the impact and transmission mechanism of external shocks on the domestic economy yield curve that accounts for time-varying asymmetric volatility in the model and

relates the model to macroeconomic scenarios.

Contribution of this study:

The study contributes in three ways.

- Firstly it is methodological. In calibration the multi-factors term structure model, we include the common stochastic volatility component that follows the EGARCH process, while adopting the state-space approach.
- Secondly, it investigates whether the slope of the yield curve in the US or the euro area helps to predict inflation and growth in emerging economies and whether the information contained in the slope of the yield curve of the emerging economies stems from the yield curve in the US.
- Thirdly, it provides a framework to assess whether the results of predicting the economic downturn and inflation with yield curve slope (presented in Ang et al. 2006; Diebold et al. 2006; Ullah et al. 2014) can be generalized to the emerging market or not?

Policy implications:

- The results of the study gives an insight and policy guidelines for the central bank in the current version of monetary policies. It will measure the effect of recent ample liquidity on the long-term interest rates, inflationary expectations and economic activity.
- On the academic front, it may be helpful to construct a statistical model that accounts for the stochastic volatility and can plot more complex shapes of the yield curve. Furthermore, important lessons from the Pakistan experience are likely to be of use to other central banks around the world contemplating similar policies.

The rest of the report is structured that section 2 discusses the term structure models and estimation method. Section 3 describes the estimation and discussion of empirical results followed by conclusion and way forward of this study.

2. Term structure model and estimation method

The macro-finance literature has convincingly advocated the case for the existence of bidirectional link between the term structure and rest of the economy. Since, we design a dynamic Nelson-Siegel (DNS) yield curve model with macroeconomic variables in the state-space framework that also allows for the time-varying stochastic volatility in yields for various maturities. In this section, we discuss the concept of time-varying factors and volatility in the DNS model. First, in subsection 2.1, we describe the model that incorporates macroeconomic variables as well as the common stochastic volatility term in state-space representation. The latent factors model is considered, because it will be a convenient way for introducing the state-space representation. Second, subsection 2.2 presents the estimation procedure of the model in the state-space framework using the Kalman filter algorithm.

2.1. Yields-macro factors model

To explore the informational contents of the yield curve in the context of emerging economies, we design a dynamic Nelson-Siegel (DNS) yield curve model with macroeconomic variables in the state-space framework that also allows for the time-varying stochastic volatility in yields for various maturities. An intuitive way to represent our model is to cast the Nelson-Siegel (1987) functional form into state-space framework, which is:

$$R_t(m) = \beta_{1t} + \beta_{2t}\left[\frac{1-\exp(-\lambda m)}{\lambda m}\right] + \beta_{3t}\left[\frac{1-\exp(-\lambda m)}{\lambda m} - \exp(-\lambda m)\right] + \varepsilon_t(m) \qquad (1)$$

where $R_t(m)$ is the zero-coupon yield for maturity m at time t, $m = 1,2,\dots,N$; $t = 1,2,\dots,T$, $\beta_t = (\beta_{1t}, \beta_{2t}, \beta_{3t})'$ is the unobservable vector of three latent factors of level, slope and curvature respectively. Finally, the parameter λ determines the maturity time at which the loading of the curvature factor β_{3t} is optimal. It also specifies the location of the hump or the U-shape on the yield curve. Since, the range of shapes the curve can take is dependent on a single parameter λ, which represents the rate at which the regressor decays to zero.

Regarding the error term, ε_t, in the Nelson-Siegel model, the earlier studies assume that $\varepsilon_t \sim N(0, \sigma^2 I_N)$. However, the interest rates are the result of trading at financial markets, therefore, the volatility in the series may have changed over time as well. That's why, we assume that:

$$\varepsilon_t = \Gamma_\varepsilon \varepsilon_t^* + \varepsilon_t^+ , \qquad \varepsilon_t^+ \sim N(0, \Omega) \qquad (2)$$

where Γ_ε and ε_t^+ are (N×1) vectors of loadings and noise component respectively, and ε_t^* is a scalar representing the common disturbance term. In this model ε_t^* and ε_t^+ are independent. The loading factor, Γ_ε, determines how sensitive the different yields are to the common shock.
The distribution of the common volatility component, ε_t^*, given the information up to time $t-1$ (denoted by ζ_{t-1}) is:

$$\varepsilon_t^* | \zeta_{t-1} \sim N(0, h_t) \qquad (3)$$

where h_t follows the EGARCH specification, which is given by:

$$\log(h_t) = \gamma_0 + \gamma_1 \frac{\varepsilon^*_{t-1}}{\sqrt{h_{t-1}}} + \gamma_2 \log(h_{t-1}) + \psi\left(\left|\frac{\varepsilon^*_{t-1}}{\sqrt{h_{t-1}}}\right| - \mathbb{E}\left[\left|\frac{\varepsilon^*_{t-1}}{\sqrt{h_{t-1}}}\right|\right]\right) \tag{4}$$

where $\mathbb{E}(|\varepsilon^*_{t-1}/\sqrt{h_{t-1}}|)$ is the expectation of the absolute value of a standard normally distributed random variable, which is equal to $\sqrt{2/\pi}$. The volatility at $t = 1$ is set equal to the unconditional expectation of the log variance, which is $\mathbb{E}[\log(h_t)] = \gamma_0(1-\gamma_2)^{-1}$. This specification for variance dynamics enable the common volatility component in the DNS model to account for asymmetric response to positive and negative shocks.

As far as the macro variables are concerned, we include six key variables: the annual growth rate in industrial production (IP_t), exchange rate (EX_t), money supply (MS_t), annual price inflation (INF_t), and slope of the US yield curve (SUS_t) in the state equation to analyse their joint dynamics with the yield curve factors. These variables represent, respectively, the level of real economic activity, foreign market competitiveness, monetary policy stances, and inflation rate, which are widely considered to be the minimum set of fundamentals needed to capture basic macroeconomic dynamics. The slope of US yield curve (SUS_t) represents the spill over effect from developed markets to the emerging economies.

We assume that the yield curve latent factors vector β_t along with the five macroeconomic factors follow a vector autoregressive process of first order, which allows us to formulate the yield curve latent factor model in the state-space form and to use the Kalman filter for obtaining maximum-likelihood estimates of the hyper-parameters and the implied estimate of β_t. In the state-space representation the model is:

$$\begin{bmatrix} R_t \\ Z_t \end{bmatrix} = \begin{bmatrix} \Lambda(\tau) & \Gamma_\varepsilon & 0 \\ 0 & 0 & I_5 \end{bmatrix} \begin{bmatrix} \beta_t \\ \varepsilon^*_t \\ \tilde{Z}_t \end{bmatrix} + \begin{bmatrix} \varepsilon^+_t \\ 0 \end{bmatrix} \tag{5}$$

$$\alpha_{t+1} = \begin{bmatrix} (I_8 - A)\mu \\ 0 \end{bmatrix} + \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \alpha_t + \begin{bmatrix} v_{t+1} \\ \varepsilon^*_{t+1} \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} \varepsilon^+_t \\ v_{t+1} \\ \varepsilon^*_{t+1} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Omega & 0 & 0 \\ 0 & \Sigma_v & 0 \\ 0 & 0 & h_{t+1} \end{bmatrix}\right) \tag{7}$$

where $\alpha_t = (\beta_{1t}, \beta_{2t}, \beta_{3t}, \widetilde{IP}_t, \widetilde{EX}_t, \widetilde{MS}_t, \widetilde{INF}_t, \widetilde{SUS}_t, \varepsilon^*_t)'$ is (9×1) latent vector, R_t is (N×1) vector of zero-coupon yields, $Z = (IP_t, EX_t, MS_t, INF_t, SUS_t)'$ is (5×1) vector of macroeconomic factors, β_t is (3×1) vector of yield curve factors, $\Lambda(\lambda)$ is (N×3) matrix of factors loadings, A is (8×8) matrix of parameters, μ is (8×1) mean vector of factors, and I_8 and I_5 are (8×8) and (5×5) identity matrices respectively and Γ_ε is (N×1) vector. Σ_v is (8×8), and Ω is (N×N), are the covariance matrices of state and measurement equations innovations. Furthermore, the variance of

ε^*_{t+1} is h_{t+1} and is modeled as EGARCH process, specified in (4).

Moreover, in (7), we assume that the innovations, ε^+_t and v_t, as well as common volatility component, ε^*_t, have Gaussian distribution. The model in equations (4 – 7) provides a flexible framework for analyzing the interaction between the yield curve and macroeconomy, while simultaneously accounts for the time-varying stochastic volatility in yields for all maturities. In addition, the proposed specification guarantees positive forward rates at all horizons and a discount factor that approaches to zero as maturity increases.

2.2. State-space estimation of the model

In this subsection, the estimation procedure based on the Kalman filter for the dynamic Nelson-Siegel model with time-varying volatility is explained. For convenience, we introduce some new notations and rewrite the signal and state equations in (5) and (6) respectively, to obtain the generalized form of DNS model with time-varying volatility in state-space form.

$$y_t = H\alpha_t + w_t \tag{8}$$

$$\alpha_{t+1} = C + K\alpha_t + Gu_{t+1} \tag{9}$$

$$\begin{bmatrix} w_t \\ u_t|\zeta_{t-1} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} R & 0 \\ 0 & Q_t \end{bmatrix}\right) \tag{10}$$

where the expressions of y_t, α_t, H, C, K, G, u_t and w_t are given in Appendix-I.

The Kalman filter algorithm consists of two steps to find a minimum mean squared error estimate of the latent vector α_t, namely the prediction and the update steps. At a given time t, we form an optimal prediction of y_t based on all information available up to time $t-1$, denoted by $y_{t|t-1}$. This prediction can be made using (8) and $\hat{\alpha}_{t|t-1}$, which can be calculated using (9) and $y_{t-1|t-1}$. After obtaining the prediction on y_t, the prediction error $\eta_{t|t-1}$ and its covariance matrix $F_{t|t-1}$ can be calculated to obtain information on α_t that is not yet contained in $y_{t|t-1}$. In the update step the estimate of α_t at time t using information up to time $t-1$, $\hat{\alpha}_{t|t-1}$ is updated by incorporating the new information from the prediction error to obtain $\hat{\alpha}_{t|t}$. The estimate $\hat{\alpha}_{t|t}$ contains information up to time t. The prediction step is summarised by the following four equations:

$$\hat{\alpha}_{t|t-1} = C + K\hat{\alpha}_{t-1|t-1} \tag{11}$$

$$P_{t|t-1} = KP_{t-1|t-1}K' + GQ_tG' \tag{12}$$

with

$$\eta_{t|t-1} = y_t - H\hat{\alpha}_{t|t-1} \tag{13}$$
$$F_{t|t-1} = HP_{t|t-1}H' + R \tag{14}$$

and the update step is described by the two equations given as follows:

$$\hat{\alpha}_{t|t} = \hat{\alpha}_{t|t-1} + P_{t|t-1}H'F_{t|t-1}^{-1}\eta_{t|t-1} \tag{15}$$
$$P_{t|t} = P_{t|t-1} - P_{t|t-1}H'F_{t|t-1}^{-1}HP_{t|t-1} \tag{16}$$

where P_t is the covariance/MSE matrix of $\hat{\alpha}_t$ in the prediction and update steps. These equations enable the Kalman filter to estimate all latent variables recursively for $t = 1,2, \ldots, T$.

Matrix Q_t contains h_{t+1} that is modeled by EGARCH process and relies on latent shocks at time t, which are unobservable. Kim and Nelson (1999) show that taking conditional expectation of the latent variables in (4) gives:

$$\log(h_t) = \gamma_0 + \gamma_1 \mathbb{E}\left(\frac{\varepsilon_{t-1}^*|\zeta_{t-1}}{\sqrt{h_{t-1}}}\right) + \gamma_2 \log(h_{t-1}) + \psi \mathbb{E}\left(\left|\frac{\varepsilon_{t-1}^*|\zeta_{t-1}}{\sqrt{h_{t-1}}}\right| - \mathbb{E}\left[\left|\frac{\varepsilon_{t-1}^*|\zeta_{t-1}}{\sqrt{h_{t-1}}}\right|\right]\right) \tag{17}$$

where the estimate of $\mathbb{E}(\varepsilon_{t-1}^*|\zeta_{t-1})$ is the last element of $\hat{\alpha}_{t-1|t-1}$ from the filtering/update step. In order to start the recursion, the initial value for α_t is set equal to the unconditional mean, $\alpha_{1|0} = \mathbb{E}(\alpha_t) = 0$, and the initial covariance matrix of the state vector, $P_{1|0}$, is:

$$P_{1|0} = \begin{bmatrix} \Upsilon & 0 \\ 0 & h_1 \end{bmatrix} \tag{18}$$

where Υ is chosen such that $\Upsilon - W\Upsilon W' = \Sigma_v$ and h_1 is the unconditional expectation of the log variance defined in section 2.1.[1] This initiation enables the Kalman filter to provide a minimum mean squared error estimate of α_t at every time $t = 1,2, \ldots, T$, given information up to time $t-1$ and given the hyper-parameters.

The Kalman filter provides estimates for the latent variables and the unknown hyper-parameters have to be estimated using maximum likelihood method. Collecting all unknown parameters of the measurement and state equations into $\theta = (\lambda, \mu, A, \Omega, \Sigma_v, \Gamma_\varepsilon, \gamma_0, \gamma_1, \gamma_2, \psi)$, and assuming that ε_t^+ and v_t are normally distributed, the distribution of y_t conditional on ζ_{t-1} is also Gaussian as $y_t|\zeta_{t-1} \sim N(y_{t|t-1}, F_{t|t-1})$; hence, the Gaussian log likelihood is given by:

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[1] We define vector ξ_t consists of the first six elements of α_t vector and model it as:

$$\xi_t = W\xi_{t-1} + v_t, \qquad v_t \sim N(0, \Sigma_v)$$

where $\xi_t = (\beta_{1t} - \mu_1, \beta_{2t} - \mu_2, \beta_{3t} - \mu_3, IP_t - \mu_4\ EX - \mu_5, MS_t - \mu_6, INF_t - \mu_7, SUS_t - \mu_8)'$, W is (8×8), and Σ_v is (8×8) covariance matrices of error term v_t. We derive the unconditional mean and covariance of ξ_t, which is summarized as $\xi_t \sim N(0, \Upsilon)$. For detail of initializing the Kalman filter, see Hamilton (1994).

$$\log L\ (\theta) = -\frac{NT}{2}\log(2\pi) - \frac{1}{2}\sum_t \log(|F_{t|t-1}(\theta)|) - \frac{1}{2}\sum_t \left(\eta'_{t|t-1}[F_{t|t-1}(\theta)]^{-1}\eta_{t|t-1}\right) \quad (19)$$

Numerical optimization of the log likelihood function (19) yields maximum likelihood estimates of the hyper-parameters. The process to find the latent factors and consistent estimates of the hyper-parameters is recursive one. The procedure is started by initiating the recursion using certain starting values for the hyper-parameters (θ^0) that enable the Kalman filter to obtain estimates of the latent factors (α_t^0), conditional on the initial choice for the parameters. Subsequently, given (α_t^0), the likelihood function (4.19) is maximized in the optimization step to obtain new estimates of the hyper-parameters, (θ^1), that yield a higher likelihood. These estimates are used in the Kalman filter again to obtain new estimates of latent factors, (α_t^1) and the corresponding likelihood value and so on. These recursive steps in the algorithm continue until the estimates of the hyper-parameters converge and we find the optimum of the likelihood function.[2]

Finally, parameter standard errors are calculated as:

$$\Sigma_\theta(\hat{\theta}) = \left(\frac{\partial^2 \log L\ (\hat{\theta})}{\partial\theta\partial\theta'}\right)^{-1}\left(\frac{\partial \log L\ (\hat{\theta})}{\partial\theta}\right)\left(\frac{\partial \log L\ (\hat{\theta})}{\partial\theta}\right)'\left(\frac{\partial^2 \log L\ (\hat{\theta})}{\partial\theta\partial\theta'}\right)^{-1}$$

3. Empirical results

We use the monthly time series panel of zero-coupon yields for Pakistani treasuries of different maturities between 2002 and 2014. We combine this panel with a data set of macroeconomic time series for the same sample period. The details of the data set are provided in section 3.1. The estimation results for the joint interaction of macro and yield curve factors along with the EGARCH results are presented in section 3.2. Section 3.3 presents the results of some formal statistical tests of contemporaneous and lagged interaction between macro and yield curve factors. Finally, in section 3.4 and 3.5, we discuss the estimation results for macroeconomic and yield curve factors impulse response functions and variance decompositions respectively.

3.1. Data description

We consider the Pakistan's government bond yields with maturities of 3, 6, 9, 12, 18, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 months. The yields are derived from bid/ask average price quotes, from August 2002 through December 2014. This is extracted from the prices of coupon bearing Pakistani government bonds using a smoothing spline technique. For the macroeconomic variables, we use data on the following three variables: the annualized growth of industrial production (IP_t), the growth rate of Pakistani Rupee to US$ exchange rate (EX_t), the growth rate of $M2$ money supply

[2] We set the convergence criterion of (10^{-6}) for the change in the norm of the parameter vector θ from one iteration to the next.

(MS_t) as an indicator of monetary policy, inflation rate (INF_t), measured as annualized monthly changes in the consumer price index, and slope of the US yield curve (SUS_t). The slope of US yield curve (SUS_t) represents the spill over effect from developed markets to the emerging economies.

As mentioned the data for the zero-coupon rates of Pakistan's government bonds is taken from the Mutual Fund Association of Pakistan (MUFAP)[3] and Pak Brunei Investment Company bonds files[4] available on their website as Pak Revaluation Rate (PKRV) data on daily basis, while for the four macroeconomic variables is obtained from the International Financial Statistics (IFS) published by International Monetary Fund (IMF). To calculate the slope of US yield curve, the data for the US government bonds zero rates id retrieved from the FED website.[5]

Summary statistics for the yields are displayed in table 1. It shows that the average yield curve is upward sloping as the mean yield is increasing with maturity. Furthermore, the short rates are more volatile than the long rates. It also seems that the skewness has the upward trend with the maturity. Moreover, kurtoses are almost similar for all rates. The yields for all maturities are also highly persistent. Figure 1 provides a three-dimensional plot of the yield curve data. It is clearly visible that the yield curves have upward slope at all points of time considered in this study. Moreover, the shape are almost stable except early 2006 and 2010. This phenomenon is also reflected in the estimated conditional volatility for the DNS-EGARCH model in figure 2. These statistics of the data provide the first evidence of a change to dynamics of the yield curve as a result of the rise in interest rates in Pakistan bond market. The descriptive statistics of the macroeconomic variables are depicted in Appendix-II.

<<Table 1>>

<<Figure 1>>

3.2. Estimation results of the model

To estimate the dynamic factors model, we use the Kalman filter algorithm suggested in Hamilton (1994). For given values of the system matrices, the Kalman filter is used to evaluate the log likelihood function via the prediction error decomposition. The maximum likelihood estimates of the unknown parameters are obtained via the numerical optimization of the Gaussian log likelihood

[3] http://www.mufap.com.pk/industry.php

[4] http://www.pakbrunei.com.pk/contact_us.html

[5] https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yieldAll

function by iterating the Marquardt algorithm, using numerical derivatives. The Kalman filter is initialized using the unconditional mean (zero) and unconditional covariance matrix of the state vector, which are derived from the Gaussian distribution for the first 6 components in state vector, given that the innovations of both signal and state equations are normally distributed.

The estimation results of the parameters of state equation are presented in the first panel of table 2. High persistency in the yield curve latent factors and macroeconomic variables can be seen from the diagonal elements of the coefficient matrix. Moreover, the lagged value of the second factor, which proxies for the yield curve slope, has a significant influence on the level factor and curvature factor along with the statistically significant lagged impact of exchange rate and inflation rate on industrial production. In addition, there is bi-directional lagged linkages between inflation rate and exchange rate. This significant relation encourages the use of a VAR model to describe the dynamics of the latent factors in the dynamic Nelson-Siegel model instead of the more parsimonious AR(1) specification. Regarding the yield macro dynamics, industrial production is positively while the growth rate of money supply and exchange rate are negatively related to the overall yield level. Furthermore, the slope factor is affected positively by the exchange rate growth rate and negatively by MS_{t-1}. This suggests that monetary policy shocks account for significant fluctuations in the yield curve shape and policy shocks to short-term interest rates are likely to affect the medium to long-term interest rates. One important channel, through which monetary policy works, is the long end of yield curve, shaping them so that, in turn, they affect the level of economic activity. This relation is consistent with the expectation hypothesis of the yield curve theory. Moreover, the industrial production and inflation rate have a positive statistically significant impact on the yield curve curvature. More importantly, the yield curve slope as well as curvature respond significantly to the US yield curve slope. The magnitude of impact is weak but statistically significant at 5% level. The result signifies the spillover effect from the US treasury market to the emerging economies markets. It suggests that slowdown in the developed economies (indicated by the fall in yield curve slope) leads to recession in the developing economies. It may due the high level of export share in the US market.

<<Table 2>>

It is interesting to observe that the impact of the yield curve factors on the macro economy is much stronger than the macro on the yield curve factors. The spread factor, often used as a predictor of economic recessions, has a negative significant effect on the level of economic activity and positive effect on inflation arte, suggesting that a decrease in the slope of yield curve (becoming flat or negatively sloped) can be considered as a signal of economic slowdown. Furthermore, the curvature factor is negatively related to the inflation rate, implying that the curvature factor contains information about the expected stance of monetary policy and could also be informative about the evolution of the economy (Monch, 2006). The results also confer that the level factor, i.e., the long term interest rate has a negative impact on the level of economic activity and inflation rate. The negative lagged impact of the level factor on inflation rate suggests that as the long end of yield curve goes down, inflationary expectations become stronger as a consequence of rise in aggregate demand. It indicates that the long end of the yield curve contains important information about the future inflation. Finally, the parameter λ is estimated at 0.1110 with a standard error of 0.0452, indicating that the estimate is highly significant. It implies that the loading on the curvature factor is maximized at a maturity of about 2 years.

As many of the coefficients (36 out of 64) in matrix A are statistically insignificant, Wald-test and LR (likelihood ratio) test for their joint significance are employed and the results are presented in the second panel of table 1. Both of the test statistics reject the null-hypothesis of the joint insignificance of the 36 individually insignificant coefficients in the state equation. This suggests that inclusion of macroeconomic factors in the Nelson-Siegel specification of yield curve improves the model's overall fit and prediction power (Ullah *et al.* 2013).

Financial market volatility in many prior studies is characterized by asymmetric volatility rather than symmetric. For example, stock market volatility tends to surge when indices are falling and revert back to normal levels only gradually when prices increase. This phenomenon is also present in interest rate markets. In order to allow for asymmetric dynamics, we estimate the EGARCH specification of the volatility process for the common component in the Nelson-Siegel model. The third panel of table 1 presents the estimates of the parameters for the EGARCH specification given by equation (4). The results support the hypothesis of asymmetric volatility dynamics in the common shock component as most of the parameters, including ψ, are statically significant, supporting the finding of Dungey *et al.* (2009).

The high and significant estimate of the γ_1 indicates that much weight is put on recent shocks. The lag volatility coefficient γ_2 in the EGARCH equation is very low and statistically insignificant. Therefore, the volatility of the common component is highly sensitive to the latest innovations; it increases quickly with large shocks and reverts back soon thereafter.

To illustrate more clearly the pattern of common volatility in bond market, in figure 2, the conditional volatility (h_t) is plotted over time.

<<Figure 2>>

Some historical events are clearly reflected in the graph, particularly the last two big jumps correspond to the monetary and fiscal policy regimes in Pakistan. It shows that the yield curve responds to the stances of monetary policy and might transmits the signals of monetary

interventions to the real sector through the alteration in the slope or/and the curvature of yield curve. Therefore, the joint interaction of yield curve factors and macro economy will be of immense interest to evaluate the impact of monetary and fiscal policies on the yield curve and the possible feedback effect on the real sector and foreign exchange market in the context of an emerging markets. However, we focus this issue in the future research.
First, the spike in late 2003 coincides with the little fall in the interest rate (the yield curve shifted down) in Pakistan bond market.[6] During this period, the Stock market remained buoyant (the KSE index reached the record level of 3100), while the bonds surged by more than 10% (excluding coupon payment), which places them amongst the best performing in emerging markets. The yields on the bonds declined by more than 300 basis points (bps).[7] The interest in Pakistani papers and bonds was mainly triggered by the revival of investors' confidence (particularly the foreign investors) in the economy, and its ability to pay back the principal in time (the government at the time was widely labeled as business friendly).

It is also obvious in the figure that the bond market suffered from a moderate level of volatility in late 2006 that continued until early 2008. In these two years, the government issued a huge amount of long-term maturity bonds. In 2006 and 2007, the country raised $500 million and $750 million respectively from 10-year maturity bonds, and another $300 million from 30-year maturity papers. All through these years, the yields on the bonds remained extremely volatile, in line with the inherent issues of political instability, fiscal imbalances, low foreign exchange reserves and terrorism (Looney R. 2008; State Bank of Pakistan, 2008).

Moreover, in the last two spikes, the first one points towards to the impact of global financial crisis of 2008. In the wake of the global financial crisis Pakistan witnessed a sharp decline in economic activity.[8] The global financial crisis accentuated the economic difficulties with widening current account and fiscal deficits, soaring inflation and weakening economic growth.[9] In response to sharply rising inflation, the Central Bank considerably tightened the monetary policy by raising the discount rate by 250 bps during 2008. The consequent rise in the rate of interest translated in the short as well as the long end of the yield curve. Hence, we observe a big spike in the estimated conditional volatility in 2008 and early 2009.[10] It is worthwhile to note that like other central banks,

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[6] However, the shift is not parallel and one can observe an increase in the slope of the yield curve, means the curve became a bit steeper.

[7] The price of the bonds maturing in 2016 came closer to their par value during this period.

[8] The economy also remained under the strain because of the macroeconomic imbalances that were building up after years of expansionary policies.

[9] Fearing an economic meltdown, Pakistan sought the support of the IMF in November 2008 to help sustain its macroeconomic recovery. Under the IMF program, Pakistan followed tight monetary and fiscal policies to restore macroeconomic stability.

[10] In late 2008 (October to December), Pakistan came near default due to these factors, which caused yields to spike to as high as 24%, that incidentally was also the period when the local equity markets had hit the pit. In order to stem the rot, corporate regulators had closed the exit door, putting the infamous floor under stocks' fall.

the State Bank of Pakistan (SBP) operates its monetary policy by directing the short-term interest rates in the interbank money market.[11] However, the SBP does not explicitly announce any desired or target level for its operational target, but uses the reverse repo rate as policy rate to give overall direction of monetary policy in the economy. Therefore, one can observe a big difference between overnight repo rate and SBP's policy rates during this period, which in turn were producing high volatility at the short end as well as the longer end of the yield curve (Mahmood, 2014).

The last big spike corresponds to the higher borrowing by the fiscal authorities from the market. Given a retirement of 198 billion (Pak rupee) in January 2013, the fiscal authorities made considerably higher borrowings from the market in the two T-bill auctions in July 2012. Consequently, the injection of liquidity is increased to 423 billion (Pak rupee) by the SBP in August 2012, in order to avoid the adverse impact of liquidity drain from the market.[12] Therefore, the overnight repo rate declined by 174 bps (State Bank of Pakistan, 2012). This relatively higher decline, compared to an earlier reduction in the policy rate, caused a considerably higher volatility in the bond market. However, the excess volatility remained limited to overnight repo rate only and other short term market interest rates largely remained unchanged. The long term interest rates, on the other hand, increased considerably as evident by the higher spread between the two ends of the yield curve during this period (figure 1). The spread between 10-year maturity bonds rate and 6-month T-bill rate increased to 121 bps (State Bank of Pakistan, 2012). In SBP's assessment, this increase was more due to a relative increase in supply of long term securities rather than expectations of rising inflation. In last half of 2012, the decline in inflation (in July 2012 to 9.6%) has had a positive impact on market expectations. As a result, there has been a decline in both short and long term market interest rates which is reflected in the downward shift of the yield curve (State Bank of Pakistan, 2012; Figure 5).

In early 2013, the trend of volatility in bond market points towards the uncertainty over the economic fundamentals. With uncertainty about politics, fiscal imbalances and dwindling foreign exchange reserves, the bonds were briskly traded, as no one was willing to hold them for long. As a result, the yields gained momentum and rose from 9% to 11.5% in March 2013. However, with improved investor perception about Pakistan's election process and the smooth transition of power from one government to the other, yields on the bonds declined by about 300bps in late 2013.[13] This phenomenon is reflected by a rise in the conditional volatility in the first half of 2013.

Overall, the estimated stochastic volatility pattern over time shows that bond market in Pakistan is highly sensitive to the policy related moves and also to the economic track and fundamentals in the

[11] According to Tylor rule (Taylor, 1993) of optimal monetary policy, the short rate, being the operational target, plays a central role in signaling the stance of monetary policy.

[12] However, it should be clear that a prudent approach in consistently implementing the requirements of the SBP Act, without adverse implications for the economy in terms of excessive borrowings from the scheduled banks, would require a consistent decline in the fiscal deficit through comprehensive fiscal reforms.

[13] The new government has struck a deal with the IMF, resolved the 503 billion (Pak rupee) circular debt issue, and approved a tax-laden federal budget 2013-14. All of this has encouraged foreign institutional investors to allocate larger sums for the equity market, as well as take a harder look at the long since laggard, Pakistani dollar bonds.

country. It confers that besides reacting to the monetary policy, the yield curve also react to the fiscal policy stances. The market is also sensitive to the external shocks that arise/happen in the world leading markets (spillover effect from rest of the world). Furthermore, the volatility is high during the periods of SBP interventions and external (global) shocks (such as evident in the case of the world financial crisis of 2008).

Furthermore, to empirically test whether the factors β_{1t}, β_{2t} and β_{3t} can be called as level, slope and curvature factors respectively, we construct a level (*L*), slope (*S*) and curvature (*C*) from the observed zero-coupon yield data and compare them with $\hat{\beta}_{1t}$, $\hat{\beta}_{2t}$ and $\hat{\beta}_{3t}$. The level of the yield curve is defined as the 20-year yield. We compute the slope as the difference between the 20-year and 3-month yield. Finally, we work out the curvature as two times the two-year yield minus the sum of the 20-year and 3-month zero coupon yields. The pairwise correlation of empirically defined level factor and $\hat{\beta}_{1t}$ (model based) is $\hat{\rho}(L_t, \hat{\beta}_{1t}) = 0.8979$, while for the slope and $\hat{\beta}_{2t}$ is $\hat{\rho}(S_t, \hat{\beta}_{2t}) = -0.9377$. The estimated correlation between curvature (C) and $\hat{\beta}_{3t}$ is $\hat{\rho}(C_t, \hat{\beta}_{3t}) = 0.6743$. The pairwise correlation along with the time series plot in figure 3 show that the estimated factors and the empirically defined factors seem to follow the same pattern, therefore, $\hat{\beta}_{1t}$, $\hat{\beta}_{2t}$ and $\hat{\beta}_{3t}$ may truly be called level, slope and curvature factors, respectively.

<<Figure 3>>

The estimate of the covariance matrix of the state innovations, as depicted by Σ_v in (7), along with the results of Wald and LR tests are shown in table 3. There are 18 out of 28 individually significant covariance terms (whereas 10 are insignificant) at the 5% level of significance. We perform the Wald and LR tests for the joint significance of the off-diagonal elements of the matrix and both of the test statistics reject the null-hypothesis of the diagonality of the Σ_v matrix with very high probability. The result is consistent with our prior expectation that the innovations of transition system are cross correlated.

<<Table 3>>

Regarding the in-sample fit of the model, in table 4 summary statistics for the fitted errors are reported. At first glance, table 4 seem to imply that model has a very good fit, both for short and long maturities. The mean absolute errors (MAE) as well as root mean squared errors (RMSE) results suggest that the long end of the yield curve has been fitted more attractively than the short end. Moreover, it is evident that the residuals autocorrelations across time for all maturities is significantly large.

<<**Table 4**>>

3.3. Formal tests for macro and yield curve factors interactions

The coefficient matrix A and the covariance matrix Σ_v, shown in table 1 and 2 respectively, are crucial for assessing the interactions between the yield curve factors and the macroeconomic variables. The (8×8) matrices A and Σ_v are partitioned into four blocks as:

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \qquad \Sigma_v = \begin{bmatrix} \Sigma_1 & \Sigma_2' \\ \Sigma_2 & \Sigma_4 \end{bmatrix} \tag{20}$$

where A_1 and A_4, being (3×3) and (5×5) matrices respectively, show the yield curve factors and the macroeconomic variables dynamics with their own lags respectively. Furthermore, A_2 and A_3, being (3×5) and (5×3) matrices respectively, show the extent of the lagged linkage from the macro-to-yields and the yields-to-macro factors respectively. Moreover, we attribute all the covariance terms given by the block Σ_2, (5×3) matrix, to the contemporaneous effect of the yield curve factors on the macro variables in accordance to the order of the yield and the macro factors employed in the state equation (6). As such, there are two links from the yields to the macroeconomy in our setup: the contemporaneous link given by Σ_2 and the effects of the lagged yields on the macroeconomy are embodied in A_3. Conversely, links from the macroeconomy to yields are symbolized in A_2.

<<**Table 5**>>

We employ the likelihood ratio (LR) and the Wald tests for the various restrictions of the yield and the macro dynamics (on the matrix A and Σ_v) and the results of both tests are reported in table 5. Both of the tests reject the no individual contemporaneous as well as the lagged interaction hypothesis (as the null hypothesis of (i) $A_2 = 0$, (ii) $A_3 = 0$, and (iii) $\Sigma_2 = 0$ are rejected). Furthermore, the null hypothesis of no interaction of the two joint restrictions and the three joint restrictions are also rejected with a very high probability (as the null hypothesis of (i) $A_2 = A_3 = 0$, (ii) $A_2 = \Sigma_2 = 0$, (iii) $A_3 = \Sigma_2 = 0$, and (iv) $A_2 = A_3 = \Sigma_2 = 0$ are rejected). The results suggest that both hypotheses, of “no macro to yields” depicted by A_2 and “no yields to macro for contemporaneous as well as lagged impact” depicted by A_3 and Σ_2 respectively, should be rejected at a very high level of significance. It confers that there is a clear statistical evidence in favor of a bi-directional link between the macroeconomy and the yield curve factors.

3.4. Macroeconomic and yield curve impulse response functions

Following Diebold *et al.* (2006) and Ullah *et al.* (2013), we consider the dynamic relationships between the macro and the yield curve factors through impulse response analysis. From an estimated VAR, we compute the variance decomposition (VDCs) and the impulse response functions (IRFs), which serve as tools for evaluating the dynamic interactions and the strength of causal relations among variables in the system. In simulating IRFs and VDCs, it should be noted that VAR innovations may be contemporaneously correlated. This means that a shock in one variable may work through the contemporaneous correlation with innovations in other variables. Therefore, the responses of a variable to shocks in another variable of interest cannot be adequately represented and isolated shocks to individual variables cannot be identified (Lutkepohl, 1991). Therefore, we use Cholesky factorization which orthogonalizes the innovations as suggested in Sims (1980) to solve this identification problem. This strategy requires a pre-specified causal ordering of the variables, because the results from VDCs and IRFs may be sensitive to the variables' ordering. The ordering of variables suggested in Sims (1980) starts with the most exogenous variable in the system and ends with the most endogenous variable.[14]

To see whether the ordering could be a problem, the contemporaneous correlations of VAR error terms are checked. The results show that there are high correlations among the three yield curve factors $(\beta_{1t}, \beta_{2t}, \beta_{3t})$ and between the yield curve factors and growth rate of money supply (MS_t). Other correlations are mostly less than 0.25. Based on the strength of the correlation, we arrange the variables according to the following order: $(SUS_t, MS_t, \beta_{3t}, \beta_{2t}, \beta_{1t}, EX_t, INF_t, IP_t)$.

There are four blocks of impulse responses, i.e., the yield curve factors responses to macro shocks, the macro variables responses to yield factors shocks, the yield-to-yield factors shocks, and the macro-to-macro variables shocks, but given the focus of this study, here we consider only the former two blocks. The results of impulse response functions of the two blocks along with 90% confidence band are presented in figure 4 and 5. Overall, the results convey an interesting message that the response of the macro variables to the yield factors is much stronger than the response of the latter to the former variables shocks.

Considering the responses of the yield curve to the macro shocks in figure 4, the level and curvature factors show a little response than the slope factors to the shocks in all the five macroeconomic variables. It attributes to the prominent role and the economic interpretation of the slope factor of the term structure. The results show that a stochastic positive shock in the industrial production immediately push down the long end with an increase in the slope factor (the yield curve become less positively sloped or more negatively sloped), suggesting that the yield curve becomes flatter in response to the supply side shocks. However, the curvature factor immediately moves to left, indicating that inflation expectation rises as a result of expansionary monetary policy in subsequent periods during recession. After 10 months, the long end goes up and the yield curve becomes

[14] To avoid the subjective criteria of pre-specified ordering of variables, we also computed the generalized impulses (GIRF) as described in Pesaran and Shin (1998). The resulting responses (not reported here to save space) are almost similar to the one obtained from Cholesky factorization.

steeper. This behavior of long rates is consistent with the inflationary expectation hypothesis of Fisher (1896). Furthermore, the behaviour of all three yield curve factors in subsequent periods is consistent with the idea that during recessions, premia on long-term bonds tend to be high and yields on short bonds tend to be low. Hence, during recessions, upward sloping yield curve does not only indicate bad times today, but also better times tomorrow.

Therefore, the rise in EX_t by analogy acts like the expansionary monetary policy. Positive shocks to money supply and exchange rate induce the long rates to rise and, hence, the slope increases (meaning β_{2t} and β_{3t} falls). The fall in the curvature factor is associated with a rise in the inflationary expectation, consistent with the expected positive impact of the expansionary monetary policy on the inflation rate. As argued by Nagayasu (2004), monetary policy mechanisms take one to two years to achieve their full effects. It seems appropriate to expect that the effectiveness of the monetary policy, if any, would result in an increase in the level factor. Therefore, the long end immediately jumps up in response to a shock in the monetary policy indicators. The rise in the level factor reflects the strengthening credibility of the SBP and, thus, the effectiveness of its policy.

Shocks in inflation rate immediately push up β_{2t} (decrease the slope) and the level factor, however, the curvature factor fall after 1 to 2 months delay. The analysis suggests that rise in short rate is much higher than the long rates and, as a result, the yield curve became flatter due to inflationary expectations. The inflation rate was very high during the last decade and the surprise to the actual inflation give a prolonged boost to long rates.

Positive shocks to money supply induce the long rates to rise and, hence, the slope increases (meaning β_{2t} falls), however, the curvature factor reacts much stronger than the former two. The fall in the curvature factor is associated with a rise in the inflationary expectation, consistent with the expected positive impact of the money supply on the inflation rate.

The figures in last row of figure 4 shows that a rise in slope of the US yield curve imply a reduction in the long rates in Pakistan and fall in the slope of yield curve (as indicated by a rise in $\beta_{2t}$and β_{3t}).

However, the slope as well as the long end of the yield curve go up with a 5 months delay, indicating that a boom like situation in the US market leads to an expansion in the economic activity. The result is consistent with the spillover effect from the US market into the emerging markets. The analysis suggests that a recession in the US economy is followed by economic slowdown in Pakistan. It may be due to higher level of inter-linkages in the financial markets and higher dependency of Pakistani economy on the exports to the US market.

<<Figure 4>>

<<Figure 5>>

Figure 5 summarizes impulse response functions of the macroeconomic variables to the unexpected increase in the yield curve factors. The level shock has a negative effect on industrial production, although its impact seems small but statistically significant. It reinforces the idea that the contribution of the macroeconomic variables to the level factor variation, if any, comes from the level of economic activity. Furthermore, a positive surprise change in the level factor indicates a sudden fall in the exchange rate but it reverts back and rises in next 2 months. The behavior of exchange rate suggests that as the long term interest rate goes up, there is inflow of foreign reserves (such as US$) in the domestic economy because of higher return on bonds in the domestic bond market and, hence, the domestic currency appreciates. However, the exchange rate rises (domestic currency depreciates) in subsequent periods indicating a fall in foreign reserves because of low exports and foreign reserves outflow. This phenomena arises, may be due to the uncertainty about the policy instability and war like situation in Pakistan during the last decade. Moreover, a positive external shock implies an increase in inflation rate. This suggests that a rise in the long term rates hurts the economic growth, which in turn leads to a higher inflation rate in the subsequent periods. The response of monetary expansion and the slope of the US yield curve to shocks in the long term rates is virtually zero.

The responses to an unexpected positive change in the slope factor are consistent with the monetary policy stances in the context of Pakistan's economy. An increase in the slope factor means a reduced spread between long-term and short-term bonds, which indicates a monetary policy tightening and, thus, economic activity declines within the upcoming 3 to 4 months.[15] The direction of reaction of the IP_t contributes to the view that the yield curve slope acts as an indicator of the future state of economy. The reaction of inflation and exchange rate look qualitatively similar to the response to the level shock. An unexpected increase of the slope factor is followed by an initial increase in inflation rate, but it is short-lived and very small. The money growth rate falls in response to the slope shock but reverts to zero immediately and then increases. It confers that the SBP implements the expansionary monetary policy, as the spread between the long and short end tightens, to avoid the upcoming recession.

Unlike Diebold *et al.* (2006), the macroeconomic variables have significant reactions to the positive change in the curvature factor. The increase in the curvature means transition from a flat yield curve to a steeper one. The economic activity expands along with an increase in inflation rate in response to an unanticipated positive shock in the curvature of the yield curve. It suggests that the curvature

[15] Normally a decrease in yield curve slope announces an economic slowdown. But, the loading of the slope factor in our model decreases with maturity and corresponds to the difference between short and long-term yields, therefore, an increase in this factor corresponds to a decrease in the term spread.

is a leading indicator/main driving force of future inflation and also reflects the cyclical fluctuations of the economy. It advocates that the curvature factor also presents the stances of monetary policy and can predict the future path of economy and inflationary expectations. The reaction of the monetary policy and US yield curve is virtually zero in response to a change in the curvature factor, consistent with the prevailing economic situation during the decade in Pakistan.

Summarizing, it turns out that the contribution of the macroeconomic variables, though small in magnitude but does not quickly shift to low levels, suggest a significant role in influencing the yield curve in Pakistan. The reaction of the macroeconomic variables in response to the shocks in the yield curve factors suggests that the monetary policy signals can be transmitted significantly and with higher probability (as all the responses are statistically significant) to the real sector through the yield curve three factors.

3.5. Macroeconomic and yield curve variance decompositions

Variance decompositions (VDCs) is an alternative method to IRFs for examining the effects of exogenous shocks on the dependent variables. It shows how much of the forecast error variance for any variable in a system is explained by innovations to each explanatory variable over a series of time horizons. Usually, own series shocks explain most of the error variance, although the shock will also affect other variables in the system. From table 6, the VDC substantiates that neither yield curve factors nor the macroeconomic factors play any significant role in explaining the variation yield curve level factor. However, the spread factor of the yield curve play a limited role at the longer horizon. It confers that the variation in the long rates are not sensitive to the macroeconomic fundamentals in Pakistan.

The variation in the slope factor mainly comes from the level factor and the slope of the US yield curve. The impact of the US yield curve spread suggests that the fluctuation in the US yield curve alter the shape of term structure in emerging markets, which is consistent with the higher dependency of developing markets on the US economy. Furthermore, the changes in the curvature factor are attributed to the shift of long end of the yield curve and the variation in the yield curve slope. However, at the longer horizon forecasts, the US yield curve slope and exchange rate play a significant role as well.

Regarding the variance decomposition of the extant of the economic activity (represented by the growth rate of the industrial production), it is apparent that the slope factor plays a crucial role at all horizons of forecasts, followed by the exchange rate and the long term interest rate in contributing the variation in IP_t. It highlights the idea that the slope of the yield curve signal out the state of economy in the near future. This indicates that the information about the slope of the yield curve might be an important signal about the future evolution of the output than the long rates and the inflationary concerns for that period.

The variation in exchange rate is explained by the slope and level factors of the yield curve to a greater extent. The behavior of EX_t, is in accordance to the economic theory and historical facts that a tightened monetary policy is implanted by most of the central banks during the inflationary pressure. However, keeping in view the prevailing economic situation during the decade in the Pakistan's economy, the reaction of money supply and exchange rate is not consistent with the expectations. The expected reaction will be zero because of the liquidity trap and stagnation like situation in Pakistan for the last 20 years.

Looking at the variance decomposition of the money supply, it shows that the curvature factor is the dominant factor, followed by the industrial production growth rate. Exchange rate shocks also contribute up to some extent in explaining the variance of monetary growth. The result is consistent with the idea that the shape and particularly the curvature of the yield curve represent the stances of the monetary policy to affect the level of the economic activity and the inflation rate in the economy.

The variation in inflation is explained by the exchange rate and money supply to a greater extent. It suggests that the demand side shocks are more influential in determining the path of inflation rather than the supply side, because the contribution of productivity shocks have negligible impact on the inflation rate. Regarding the contribution of the yield curve factors, the variation in the inflation is largely due to the level factor of the yield curve.

Finally, the US yield curve slope is explained almost by its own shocks. The yield curve factors as well as macroeconomic fundamentals do not play any significant role in explaining the variation in US yield curve as expected.

<<Table 6>>

4. Conclusion

This study explores the evolution between the yield curve and the Pakistan's economy with a special focus on examining the effects of the monetary policy and slope of the US term structure on the emerging market yield curve and the possible feed-back effect on the real sector by applying a yields-macro model. The study uses monthly Pakistani government bonds zero coupon data (yield to maturity) from August 2002 until June 2014. The yield curve model of this study explicitly incorporates both the yields factors (level, slope, and curvature) and the macroeconomic variables (overall economic activity, exchange rate, money supply and inflation rate). We also extend the model in Diebold *et al.* (2006) to include time-varying stochastic volatility in the yield model (observation equation) in the state-space framework.

Empirical results from the yields-macro factors model show that there is a statistically significant bidirectional linkage between the macroeconomic and the yield curve factors; however, by contrast with conventional wisdom, macro variables play a less prominent role in explaining the yield factors as compared to the strength of effect from the latter to the former. Furthermore, the volatility in bond markets is found to be asymmetrically affected by positive and negative shocks and more sensitive to recent innovations rather than the lag volatility. For the in-sample fit, the results show that the Nelson-Siegel model is capable to distill the term structure of interest rate quite well and describe the evolution and the trends of the market in the emerging markets as it has been evident in the context of larger and developed market. However, the magnitude of the error in emerging market is reasonably larger as compared to the developed markets. It might be due to not taking into account the arbitrage free restriction in the market or pricing error in the market (possibly because of lack of liquidity).

The structural decomposition indicates that it is the entire term structure of interest rate that transmits the policy shocks to the real economy. The monetary policy signals pass through the yield curve level and the slope factors to stimulate the economic activity. Besides the slope factor, the curvature factor also reflect the cyclical fluctuations of the economy. One can infer from the overall results that the slope and the curvature factors (in our framework) serve as leading counter-cyclical and pro-cyclical indicators respectively. In addition, the study finds that the domestic yield curve in emerging economies has in-sample information content. The US yield curves also have in- and out-of-sample information content for future yield curve shape, inflation and growth in emerging economies. This may be due to exchange rate pegging to the US dollar in the emerging economies. The US yield curve is found to be a better predictor than the domestic yield curve and to causally explain their movements.

Way forward:

The study offers many directions and opens many interesting challenges for future research. In evaluating the monetary policy shocks, it will be interesting to specify the reaction function such as a Taylor rule, of monetary authorities to macroeconomic variables. The inclusion of Taylor type rule instead of the exogenous money supply in model will be helpful to evaluate the impact of monetary policy solely on the yield curve and the feedback effect on the macroeconomy.

Though, the statistical class of models comes up with encouraging results in in-sample fit, it will be of immense interest to use more flexible form of the standard Nelson-Siegel model to fit accurately the curves with multiple maxima and/or minima. We are aware of the popular extension by Svensson (1994), but it leads to high degree of correlation between the loadings (multicolinearity) and makes it difficult to estimate the parameters precisely.

Another key aspect of the term structure is time-varying stochastic volatility. The interest rate volatility for various maturities can be modeled in many different ways. The alternative specification can be to link the volatility with macroeconomic variables as one can proceed with GARCHX specification.

Furthermore, the lag-lead (causality) analysis between the yield factors and the stock market will be of a higher significance for the efficiency analysis of both markets, the bond and stock.

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Appendix-I

Coefficients and latent variable in the general state-space form

In the statistical formulation of the models in section 2.2, the matrices and vectors for the state and observations equations should be considered as follows. The matrices and vectors in state-space system in (5-7) should be defined as:

$$y_t = \begin{bmatrix} R_t \\ Z_t \end{bmatrix} \qquad \alpha_t = [\beta_t', \widetilde{IP}_t, \widetilde{EX}_t, \widetilde{MS}_t, \widetilde{INF}_t, , \widetilde{SUS}_t, \varepsilon_t^*]'$$

$$H = \begin{bmatrix} \Lambda(\lambda) & \Gamma_\varepsilon & 0 \\ 0 & 0 & I_5 \end{bmatrix} \qquad K = \begin{bmatrix} A & 0_8 \\ 0_8' & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} (I_8 - A)\mu \\ 0 \end{bmatrix} \qquad G = I_9$$

$$w_t = \begin{bmatrix} \varepsilon_t^+ \\ 0 \end{bmatrix} \qquad u_t = \begin{bmatrix} v_{t+1} \\ \varepsilon_{t+1}^* \end{bmatrix}$$

$$R = \begin{bmatrix} \Omega & 0 \\ 0 & 0 \end{bmatrix} \qquad Q_t = \begin{bmatrix} \Sigma_v & 0 \\ 0 & h_{t+1} \end{bmatrix}$$

where $\alpha_t = (\beta_{1t}, \beta_{2t}, \beta_{3t}, \widetilde{IP}_t, \widetilde{EX}_t, \widetilde{MS}_t, \widetilde{INF}_t, , \widetilde{SUS}_t, \varepsilon_t^*)'$ is the (9×1) vector of yield curve and macroeconomic factors, $R_t(m)$ is (N×1) vector of zero-coupon yield, $Z_t = (IP_t, EX_t, MS_t, INF_t, SUS_t)'$ is the (5×1) vector of macroeconomic factors, β_t is (3×1) vector of Nelson-Siegel factors, $\Lambda(\lambda)$ is (N×3) matrix of factors loadings, A is (8×8) matrix of parameters, μ is (8×1) mean vector of factors, I_6 and I_3 are (6×6) and (3×3) identity matrices respectively and Γ_ε is (N×1) vector. Σ_v is (8×8), the covariance matrix of innovations of the transition system and Ω is the (N×N) dimension covariance matrix of the innovations to the measurement system. Furthermore, ε_t^+ is the (N×1) error vector of measurement equation and v_{t+1} is (8×1) innovation vector of first six state equations.

Appendix-II

Data description

Regarding the macroeconomic fundamental and spillover effect indicator, we consider the industrial production, real exchange rate, money supply, consumer price index and slope of the US treasuries yield curve. The data for the former four variables is obtained from the International Financial Statistics (IFS), while for US yield curve is from the FED website. The four variables are measured as the last 12 months' percentage growth rate. The IP_t is growth rate in industrial production, EX_t is the growth in real exchange rate (PK-Rupee/US-\$), MS_t is the growth rate of $M2$ monetary aggregate, INF_t is the inflation rate and is measured as 12-month percent change in the consumer price index, and SUS_t is the slope of the US yield curve, calculated as the difference between 120-month and 3-month rates. The descriptive statistics of these variables are presented in table A2.

<<Table A1>>

In table A1, the results of augmented Dickey–Fuller (ADF) unit root test suggest that all five variables are stationary at level.

Table 1. Descriptive statistics of yields data across maturities

Maturity	Mean	SD	Max	Min	Skewness	Kurtosis	$\hat{\rho}(1)$	$\hat{\rho}(6)$	$\hat{\rho}(12)$
3	8.8674	3.5770	13.4488	1.1177	-0.7999	2.5839	0.9908	0.8997	0.7221
6	8.9936	3.5782	13.7511	1.1308	-0.7864	2.6239	0.9915	0.8964	0.7128
9	9.0904	3.5419	13.9142	1.2327	-0.7881	2.6612	0.9916	0.8933	0.7048
12	9.1872	3.5068	14.1090	1.3346	-0.7893	2.6995	0.9915	0.8898	0.6961
18	9.4310	3.4044	14.5700	1.7256	-0.7880	2.6922	0.9907	0.8801	0.6881
24	9.6748	3.3156	15.0628	2.1165	-0.7731	2.6767	0.9887	0.8639	0.6747
30	9.8217	3.2683	15.2706	2.3156	-0.7614	2.6438	0.9885	0.8603	0.6703
36	9.9687	3.2271	15.4783	2.5146	-0.7446	2.6071	0.9879	0.8548	0.6639
48	10.2093	3.0917	15.8406	3.0962	-0.7098	2.6253	0.9868	0.8430	0.6539
60	10.3718	3.0144	15.8656	3.4737	-0.6855	2.5976	0.9855	0.8350	0.6420
72	10.5779	2.8548	16.0867	3.8446	-0.6711	2.7128	0.9848	0.8201	0.6191
84	10.7086	2.7689	16.1828	4.1919	-0.6393	2.6899	0.9838	0.8135	0.6147
96	10.8167	2.6802	16.2661	4.5838	-0.5846	2.5910	0.9831	0.8117	0.6172
108	10.8644	2.6650	16.3894	4.6832	-0.5957	2.6141	0.9825	0.8112	0.6200
120	10.9160	2.6792	16.5311	4.5305	-0.6162	2.7470	0.9818	0.8007	0.6019

Note: The table shows descriptive statistics for monthly yields at different maturities. The last three columns contain sample autocorrelations at displacements of 1, 6 and 12 months. The sample period is 2002:08–2014:12. The number of observations is 149.

Table 2. Latent and macro factors VAR(1) parameters estimates of the yield-macro model

Panel 1: Estimates of matrix A and vector μ

	μ	$\beta_{1,t-1}$	$\beta_{2,t-1}$	$\beta_{3,t-1}$	IP_{t-1}	EX_{t-1}	MS_{t-1}	INF_{t-1}	SUS_{t-1}
β_{1t}	**1.5214** (0.6669)	**0.8843** (0.0830)	**-0.4905** (0.1532)	0.2010 (0.1559)	**1.4385** (0.3704)	**-0.0238** (0.0110)	**-0.1908** (0.0753)	0.1864 (0.1352)	0.0273 (0.0320)
β_{2t}	**1.1200** (0.2384)	-0.0541 (0.2359)	**0.6149** (0.1017)	0.2373 (0.2404)	0.2206 (1.0618)	**0.1133** (0.0208)	**-0.3415** (0.0230)	**1.0773** (0.3550)	**0.2479** (0.0807)
β_{3t}	**1.4056** (0.0958)	0.3896 (0.1296)	**1.1194** (0.1264)	**0.7166** (0.0862)	**1.5057** (0.5895)	0.0941 (0.1053)	0.0290 (0.1262)	**1.7274** (0.1022)	**0.3626** (0.1399)
IP_t	**2.6697** (0.2825)	**-0.4786** (0.1383)	**-1.2293** (0.6003)	-0.0021 (0.1404)	**0.5392** (0.2196)	**0.1502** (0.0263)	0.2820 (0.1818)	**0.8401** (0.2168)	0.0600 (0.2080)
EX_t	**3.0916** (1.2957)	0.1679 (0.9533)	0.0414 (0.8355)	0.0333 (0.3407)	1.4048 (1.1679)	**0.6253** (0.0582)	0.5875 (0.9769)	**2.9616** (0.5965)	0.0362 (0.1757)
MS_t	**3.0606** (0.2165)	0.2741 (0.1827)	0.1772 (0.4066)	0.6931 (0.2932)	1.1176 (1.1721)	0.1691 (0.0219)	**0.7664** (0.1143)	0.9032 (0.7510)	0.0620 (0.1536)
INF_t	**1.1672** (0.6133)	**-0.4414** (0.1552)	**1.2581** (0.6790)	**-0.0397** (0.0080)	0.8271 (0.7031)	**0.1356** (0.0771)	0.1242 (0.1761)	**-0.5588** (0.1463)	-0.4374 (0.2262)
SUS_t	**1.1267** (0.3722)	0.4225 (0.6444)	0.7600 (0.6600)	0.2508 (0.1475)	0.1995 (1.7601)	**0.2215** (0.0352)	0.1054 (0.5232)	1.1300 (1.0355)	**0.6267** (0.0823)
λ_1		0.1110 (0.0452)							

Panel 2: Test for the joint-significance of individually insignificant coefficients in mean reversion matrix A

Test	Wald Test			LR Test		
	Test statistic	df	P-value	Test statistic	df	P-value
Value	163.8516	34	0.0000	128.1537	34	0.0000

Panel 3: EGARCH model parameters estimates

γ_0	γ_1	γ_2	ψ
0.0118 (0.0064)	**-0.2525** (0.0799)	0.0009 (0.1851)	**0.0804** (0.0186)

Note: The table reports the estimates for the parameters of the transition equation of yields-macro factors dynamics. Panel 1 presents the estimates for the vector μ and matrix A, while panel 2 shows the results of the Wald-test and likelihood ratio (LR) test for the joint significance of individually insignificant coefficients in matrix A. The null hypothesis is that insignificant coefficients are simultaneously equal to zero. Both the test statistics are Chi-square with their respective degrees of freedom (df). P-value is the probability value of the test statistic. Panel 3 shows the parameters' estimates of the volatility processes (EGARCH) of the common component in the yield curve model. The standard errors are in parenthesis. Bold entries denote parameter estimates are significant at the 5 percent level.

Table 3. Estimates of covariance matrix Σ_v and its diagonality test

Panel 1: Estimates of covariance matrix Σ_v

	$\Sigma_v(.,1)$	$\Sigma_v(.,2)$	$\Sigma_v(.,3)$	$\Sigma_v(.,4)$	$\Sigma_v(.,5)$	$\Sigma_v(.,6)$	$\Sigma_v(.,7)$	$\Sigma_v(.,8)$
$\Sigma_v(1,.)$	**0.5040** (0.0816)							
$\Sigma_v(2,.)$	0.2905 (0.1535)	**0.9939** (0.2450)						
$\Sigma_v(3,.)$	**0.4167** (0.0904)	**0.4435** (0.1153)	**0.4344** (0.1227)					
$\Sigma_v(4,.)$	0.8483 (0.6105)	**0.6963** (0.2702)	0.9442 (0.6198)	**3.6386** (0.9257)				
$\Sigma_v(5,.)$	**1.1413** (0.3943)	**0.6575** (0.1419)	**1.1415** (0.2885)	0.0726 (0.2275)	**2.3753** (0.1375)			
$\Sigma_v(6,.)$	0.0057 (0.1513)	-0.4209 (0.2436)	**0.3166** (0.1185)	**1.0169** (0.4087)	**2.5770** (0.1439)	**1.3690** (0.1578)		
$\Sigma_v(7,.)$	**0.5227** (0.1443)	**0.7143** (0.2928)	0.0324 (0.0581)	**0.9930** (0.3503)	0.4594 (0.2721)	**0.5161** (0.3442)	**2.9611** (0.2469)	
$\Sigma_v(8,.)$	0.2095 (0.1825)	**1.1934** (0.2525)	**0.6067** (0.0880)	**0.2412** (0.0441)	0.4832 (0.2887)	**0.9164** (0.0734)	0.3558 (0.3341)	**1.5684** (0.2803)

Panel 2: Test for diagonality of covariance matrix Σ_v

Test	Wald Test			LR Test		
	Value	df	P-value	Value	df	P-value
Test statistic	145.2894	28	0.0000	91.5428	28	0.0000

Note: The upper panel of the table reports the estimate of covariance matrix of innovations of the transition equation. The standard errors are in parenthesis. The lower panel presents the results of the Wald-test and LR-test for the null hypothesis that the covariance matrix Σ_v is diagonal. Both of the test statistics are Chi-square with their respective degrees of freedom (df). P-value is the probability value of the test statistic. Bold entries denote parameters estimates are significant at the 5 percent level.

Table 4. Tests for yields-macro factors interactions

Null Hypothesis	Number of restrictions	Wald test		LR test	
		Test statistic	P-value	Test statistic	P-value
$A_2 = 0$	15	61.2549	0.0000	81.4737	0.0000
$A_3 = 0$	15	54.6881	0.0000	90.5791	0.0000
$\Sigma_2 = 0$	15	45.7506	0.0000	22.6986	0.0000
$A_2 = A_3 = 0$	30	66.4885	0.0000	51.3379	0.0000
$A_2 = \Sigma_2 = 0$	30	75.7611	0.0000	63.2352	0.0000
$A_3 = \Sigma_2 = 0$	30	71.0593	0.0000	89.7540	0.0000
$A_2 = A_3 = \Sigma_2 = 0$	45	97.7169	0.0000	102.8498	0.0000

Note: The table presents the results of the Wald-test and LR-test for the no lagged and/or contemporaneous yields-macro factors interaction. A_2, A_3 and Σ_2 refers to the relevant blocks of A and Σ_v matrices. A_2 and A_3 show the extent of lagged linkage from macro-to-yields and yields-to-macro factors respectively, and Σ_2 refers to the contemporaneous effect of yield curve factors on the macro variables. Both of the test statistics are Chi-square with the degrees of freedom equal to the number of restrictions. P-value is the probability value of the test statistic.

Table 5. Descriptive statistics of the yield curve residuals

Maturity	Mean	SD	MAE	RMSE	$\hat{\rho}(1)$	$\hat{\rho}(6)$	$\hat{\rho}(12)$
3	0.6515	3.2653	2.4701	3.3189	0.9889	0.8903	0.7005
6	0.5666	3.4554	2.5943	3.4901	0.9912	0.8932	0.7036
9	0.4435	3.5471	2.6583	3.5629	0.9917	0.8932	0.7043
12	0.4112	3.6140	2.7204	3.6252	0.9907	0.8907	0.7024
18	0.3265	3.5218	2.6725	3.5251	0.9909	0.8868	0.7000
24	0.3166	3.3961	2.6069	3.3995	0.9889	0.8791	0.6933
30	0.3965	3.3413	2.6025	3.3536	0.9881	0.8694	0.6838
36	0.1845	3.1807	2.5061	3.1754	0.9882	0.8633	0.6758
48	0.2176	3.0557	2.4449	3.0532	0.9863	0.8455	0.6563
60	0.1560	2.9306	2.3683	2.9249	0.9856	0.8326	0.6396
72	0.2188	2.8376	2.2991	2.8365	0.9848	0.8223	0.6252
84	0.3700	2.7788	2.2506	2.7941	0.9839	0.8133	0.6132
96	0.5803	2.7540	2.2367	2.8055	0.9829	0.8053	0.6041
108	0.5088	2.7011	2.1941	2.7397	0.9822	0.8001	0.5949
120	0.5475	2.6865	2.1870	2.7328	0.9820	0.7950	0.5888

Note: The table presents summary statistic of the residuals for different maturity times of the measurement equation of the estimated yield-macro model, using monthly data 2002:08–2014:12. SD, MAE and RMSE are the standard deviation, mean absolute error and root mean squared errors, respectively. $\hat{\rho}(i)$ denotes the sample autocorrelations at displacements of 1, 6 and 12 months. The number of observations is 149.

Table 6. VDCs of yield curve factors and macroeconomic variables

Period	β_{1t}	β_{2t}	β_{3t}	IP_t	EX_t	MS_t	INF_t	SUS_t
Variance decomposition of β_{1t}								
1	96.0679	0.0758	0.0327	0.2398	0.1985	0.2141	0.0036	3.1675
12	88.8727	5.4571	0.9768	0.6278	1.1036	0.1441	0.8516	1.9663
24	82.5211	9.7763	0.8928	0.8507	2.2163	0.1272	1.1544	2.4612
40	78.6242	13.0096	0.8120	0.9242	2.9178	0.1156	1.2906	2.3060
Variance decomposition of β_{2t}								
1	47.1749	26.3322	8.5696	0.8800	13.0620	0.0287	0.3424	3.6102
12	41.8546	34.3356	4.6049	0.4389	8.9256	0.3430	0.2141	9.2832
24	38.9175	30.4435	3.2364	0.3124	6.7869	0.2852	0.2150	19.8032
40	39.9062	28.2307	2.7309	0.2632	5.9257	0.2620	0.2359	22.4456
Variance decomposition of β_{3t}								
1	7.3314	51.9263	31.7375	5.9019	0.4761	2.3931	0.2138	0.0200
12	11.5229	64.9158	7.7524	1.3985	4.7739	0.6739	1.6042	7.3583
24	28.2305	46.5924	3.7931	0.7245	4.1613	0.3535	1.1271	15.0176
40	35.4642	40.1554	2.7679	0.5642	3.8280	0.2680	1.0328	15.9196
Variance decomposition of IP_t								
1	1.2827	64.0752	0.6256	30.4539	2.8206	0.5008	0.0000	0.2411
12	2.1810	58.5305	0.8936	28.2598	3.8801	1.6721	4.1550	0.4278
24	2.2067	58.5034	0.9044	28.2468	3.8802	1.6713	4.1537	0.4335
40	2.2240	58.4855	0.9122	28.2382	3.8802	1.6708	4.1528	0.4361
Variance decomposition of EX_t								
1	2.4159	15.9820	5.8270	16.5897	52.4537	0.8306	2.9909	2.9102
12	9.0449	12.7375	4.6721	13.3268	48.8682	0.9973	7.7775	2.5757
24	10.3121	12.6150	4.5968	13.0632	47.7901	0.9768	7.6475	2.9986
40	10.6013	12.7530	4.5479	12.9382	47.3212	0.9683	7.5873	3.2826
Variance decomposition of MS_t								
1	0.1664	0.9985	25.0836	32.2644	7.6351	33.2233	0.2565	0.3722
12	0.5774	1.4248	21.4520	30.3204	8.8127	31.7964	4.4779	1.1385
24	0.6483	1.4437	21.4205	30.2788	8.8066	31.7486	4.4753	1.1781
40	0.6869	1.4701	21.4000	30.2514	8.8059	31.7178	4.4734	1.1945
Variance decomposition of INF_t								
1	0.0767	0.5291	1.5554	7.1163	11.8283	8.9362	69.0376	0.9203
12	5.0475	0.9878	2.0495	6.7018	12.2617	8.2297	63.4642	1.2578
24	6.4230	1.4481	2.0200	6.5846	12.0974	8.0328	61.9876	1.4066
40	7.1579	1.9379	1.9931	6.5045	12.0239	7.9041	61.0212	1.4576
Variance decomposition of SUS_t								
1	0.3115	0.4830	0.1535	0.1088	0.1778	0.5874	0.0395	98.1385
12	0.8173	1.3042	0.4557	1.2611	0.3576	0.4804	0.2770	95.0467
24	1.1055	1.0219	0.3422	1.3243	0.6919	0.5057	0.3657	94.6428
40	1.2235	1.6244	0.3205	1.4013	1.0556	0.5126	0.4596	93.4024

Note: The table reports the results of the variance decompositions of all the ten variables in the system. We simulate the VAR(1) model of the yield and the macro factors and compute the contribution of innovations of each explanatory variable over a series of time horizons. Each entry is the proportion of the forecast variance (at the specified forecast horizon) for a 1, 12, 24 and 40 months' time horizons that are explained by the particular factor.

Table A-1. Descriptive statistics of macroeconomic variables data

	IP_t	EX_t	MS_t	INF_t	SUS_t
Mean	0.6380	0.3541	1.1964	0.7540	2.0503
SD	8.3243	1.1499	1.7043	0.8330	1.1401
Maximum	28.3151	6.2548	5.9316	3.2824	3.7900
Minimum	-22.0582	-3.4900	-3.1454	-1.3317	-0.6000
Skewness	0.5364	1.6890	0.1152	0.3148	-0.7268
Kurtosis	4.1098	10.1636	3.0545	3.2048	2.5585
$\hat{\rho}$ (1)	-0.0088	0.3809	-0.2887	0.2312	0.9641
$\hat{\rho}$ (6)	-0.1397	0.0682	0.7157	0.1086	0.7947
$\hat{\rho}$ (12)	0.5456	0.0618	0.7112	0.2152	0.4923
ADF-statistic	-4.3439	-5.1081	-3.1965	-9.5570	-3.0198
P-value (ADF-stat)	0.0006	0.0000	0.0895	0.0000	0.0920

Note: The table presents summary statistics for macroeconomic variables data 2002:08–2014:12. The four macroeconomic variables are measured as the last 12 months percentage growth rate. The IP_t is annual growth rate in industrial production, EX_t is the growth of real exchange rate, MS_t is the growth of $M2$ money supply, INF_t is the 12-month percent change in the consumer price index and SUS_t is the spread between the 10-year and 3-month maturity bond yields of the US treasuries. $\hat{\rho}$ (i) denotes the sample autocorrelations at displacements of 1, 6 and 12 months. The last two rows contain augmented Dickey–Fuller (ADF) unit root test-statistic and its p-value.

<<Figures>>

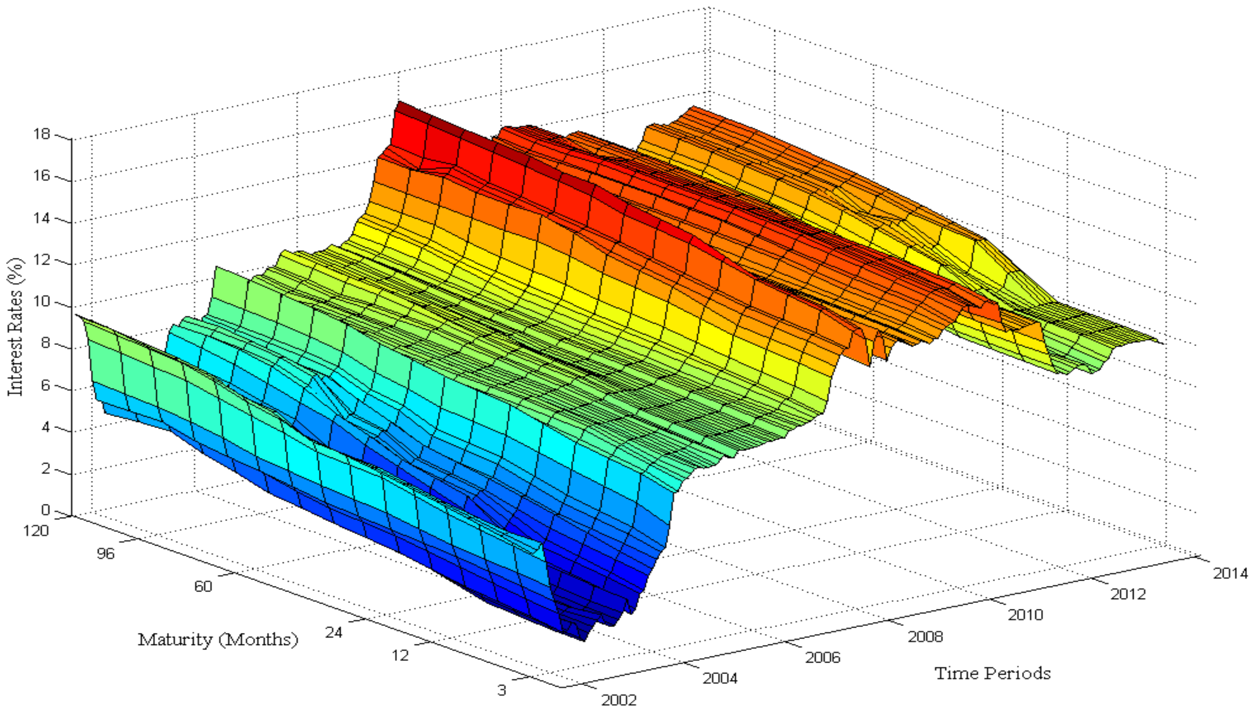

Figure 1. The figure shows the yield curves, 2002:08–20143:12. The sample consists of monthly yield data from January 1996 to December 2013 (216 months) for maturities of 3, 6, 9, 12, 18, 24, 30, 36, 48, 60, 72, 84, 96, 108, and

120 months (15 maturities).

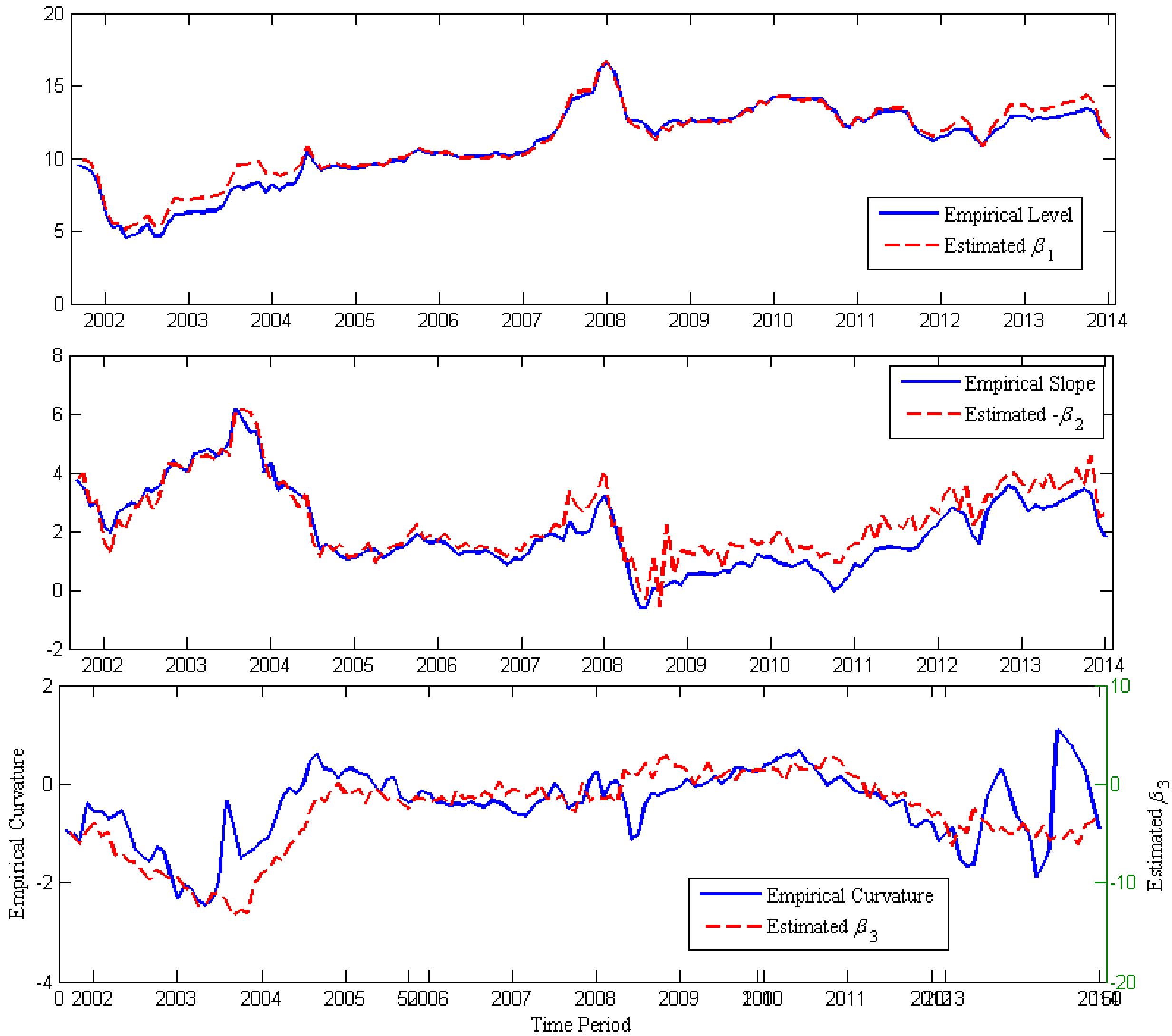

Figure 2. Time series plot of Nelson-Siegel estimated factors and empirical level, slope and curvature**.** Model-based level, slope and curvature (i.e., estimated factors vector $\hat{\beta}_t$) vs. data-based level, slope and curvature are plotted, where level is defined as the 25-year yield, slope as the difference between the 25-year and 3-month yields and curvature as two times the 2-year yield minus the sum of the 25-years and 3- month zero-coupon yields. Rescaling of estimated factors is based on Diebold and Li (2006).

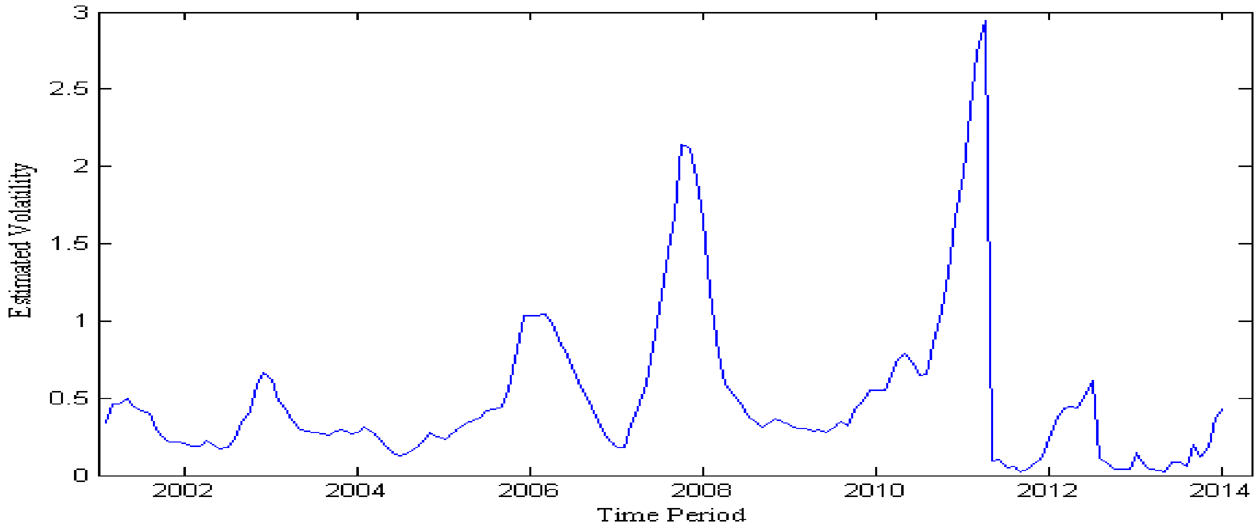

Figure 3. Common Volatility (h_t). The figure shows the plot of the volatility (h_t) of the common shock component (ε_t^*), which is modelled as EGARCH process, over time for the dynamic Nelson-Siegel EGARCH model.

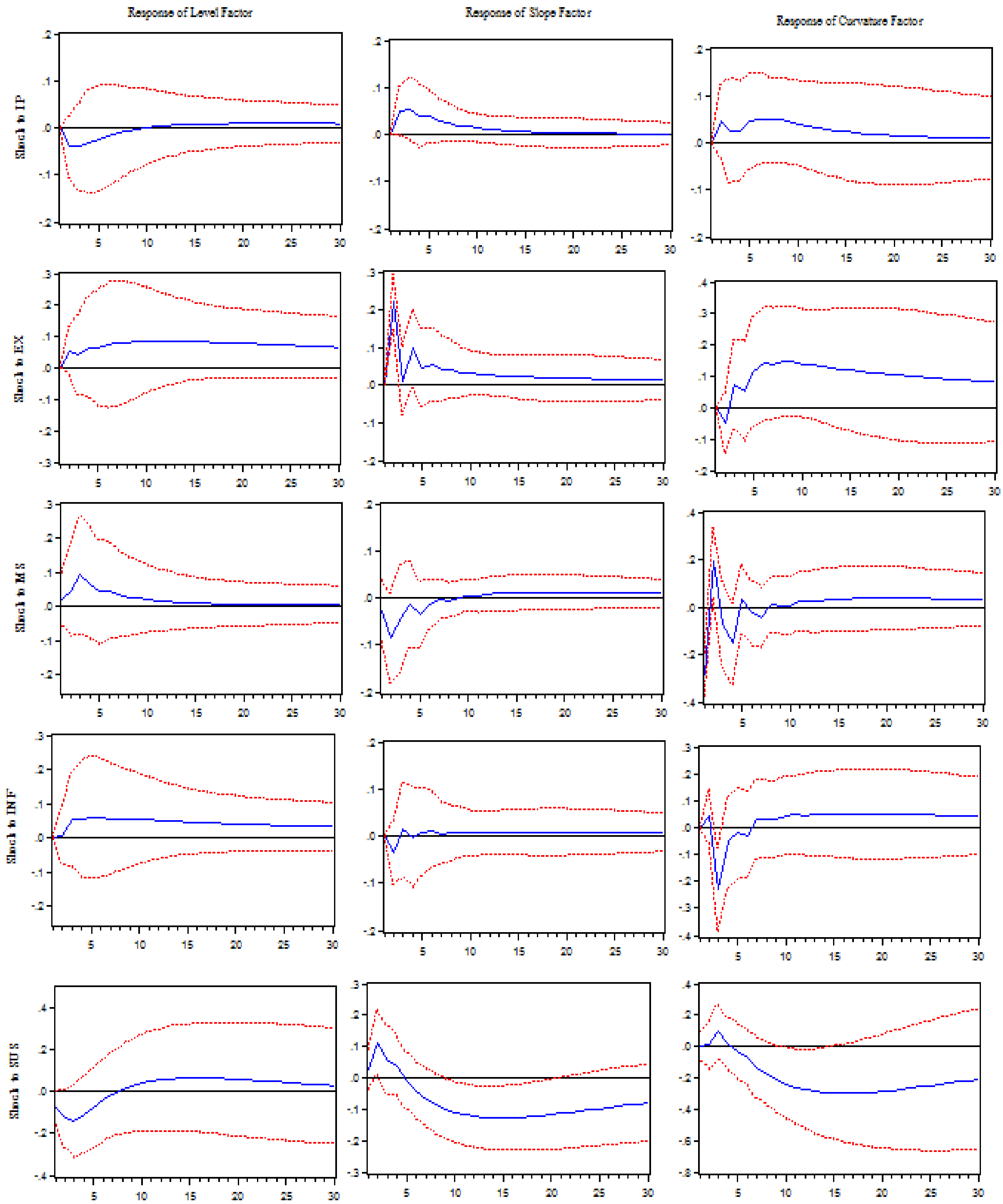

Figure 4. The figure shows the reactions of three yield curve factors (i.e., $\beta_{1t}, \beta_{2t}, \beta_{3t}$, denoted by level, slopes and curvatures factors respectively) to a shock in the set of five macroeconomic variables (i.e., $IP_t, EX_t, MS_t, INF_t, SUS_t$) to a shock in each exogenous variables in the VAR(2) model over 30 months. We simulate the VAR(2) model of yield and macro factors and compute response of each factor to Cholesky one unit innovation. The solid blue line denotes the estimated response, while the red dashed line shows $\pm 2(SE)$ (plus-minus two standard error) confidence band.

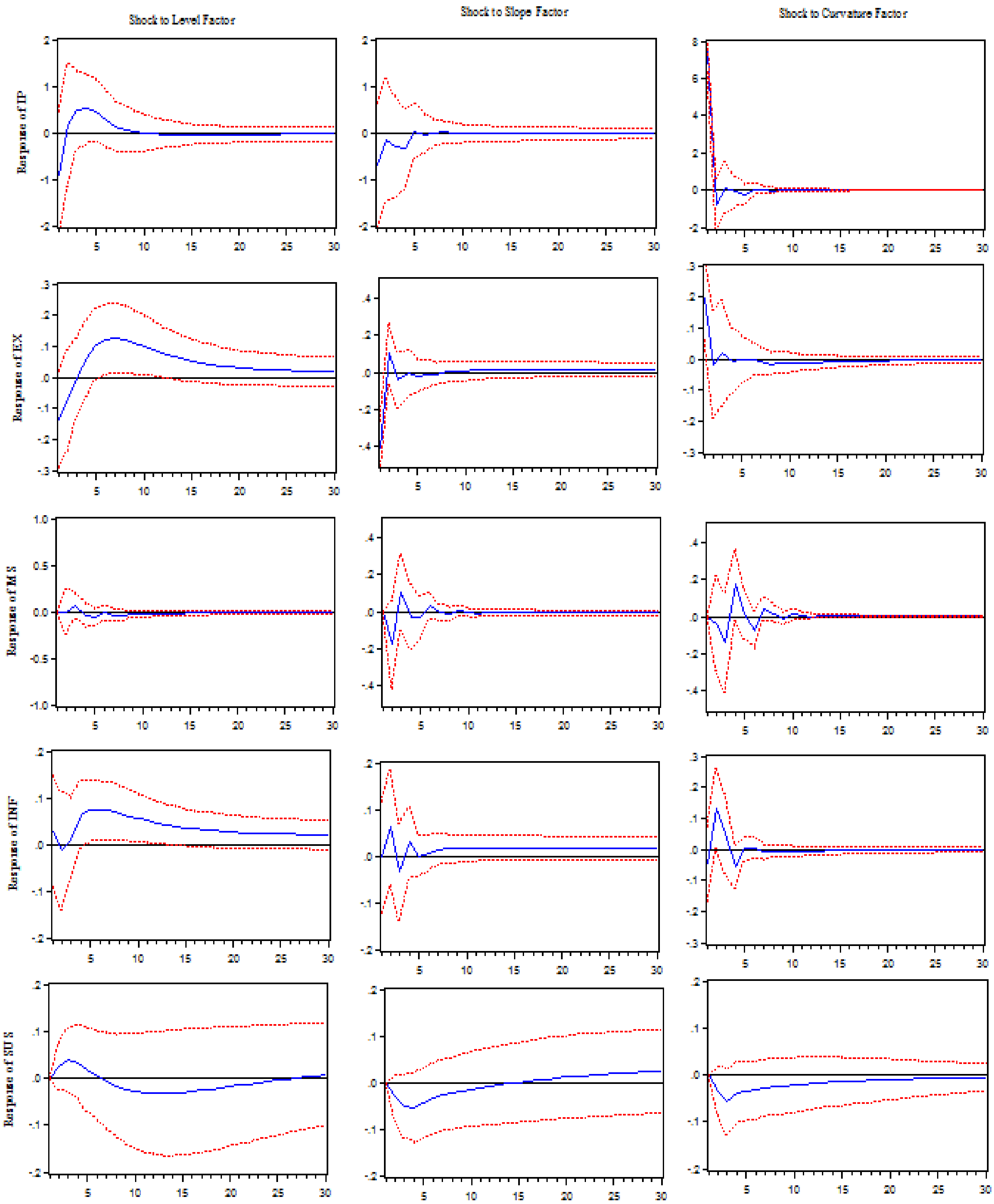

Figure 5. The figure shows the reactions of five macroeconomic factors five macroeconomic variables (i.e., $IP_t, EX_t, MS_t, INF_t, SUS_t$) to a shock in the three yield curve factors (i.e., $\beta_{1t}, \beta_{2t}, \beta_{3t}$, denoted by level, slopes and curvatures factors respectively) in the VAR(2) model over 30 months. We simulate the VAR(2) model of yield and macro factors and compute response of each factor to Cholesky one unit innovation. The solid blue line denotes the estimated response, while the red dashed line shows $\pm 2(SE)$ (plus-minus two standard error) confidence band.

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