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Using quantitative spatial-equilibrium models to inform urban policymaking

This report uses the City of Cape Town as a case study to illustrate the power of quantitative spatial equilibrium models and explores the impact of four urban policy interventions. The IGC Cities Spatial Model was produced through a collaboration with researchers from the World Bank and policymakers from the City of Cape Town.

Maria Del Mar Gómez
Román David Zárate

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Hugh Cole
Paul Court
Alfred Moyo
Nick Tsivanidis
Victoria Delbridge

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Using Quantitative Spatial-Equilibrium models to inform urban policy making*

Maria Del Mar Gómez† Román D. Zárate‡

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Abstract

This report explores a classic Quantitative Spatial Equilibrium model to assess the impacts of urban policy interventions. Quantitative Spatial Equilibrium models are a recent development in the Urban Economics literature providing a theoretical framework to capture the forces that shape the distribution of economic activity and allowing to recover some of the fundamental variables that shape the distribution of economic activity over space, such as productivity, the density of land development, and amenities. The advantages of these models include their ability to capture realistic features of economies and perform counterfactual analyses with basic data requirements. However, their technical complexity and computational requirements have hindered a broader adoption among less academic audiences. The IGC's Cities Spatial Model toolkit aims to bridge this gap by explaining the components of a seminal Quantitative Spatial Equilibrium model and creating a user-friendly R package to facilitate its practical implementation. This report uses the City of Cape Town as a case study to illustrate the power of Quantitative Spatial Equilibrium models and explores the impact of four urban policy interventions: the construction of new BRT lines, the complete failure of the railways' network, an enhancement of residential amenities, and an intervention to improve firms' productivity.

Keywords: Quantitative Spatial Equilibrium models; Distribution of economic activity; Urban policy; Trasnport; Welfare.

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†World Bank, email: mgomezortiz@worldbank.org
‡World Bank, email: rzaratevasquez@worldbank.org

1 Introduction

Economic activity is unevenly distributed across space. Even areas within the same city have enormous differences in jobs, schooling opportunities, access to transport, salaries, and prices. What determines the differences in the density of economic activity across geographic space is the interaction of two opposite forces: agglomeration and congestion. On the one side, agglomeration forces incentive agents to gather by conveying benefits from locating in a specific area. Examples include local institutions, natural resources, facilities, and infrastructure. Conversely, congestion forces spread people around the space because they encompass the disadvantages of locating close to each other. For example, traffic, high land prices, pollution, and other urban issues make people desire to move further from each other. The interplay of these opposite forces determines the geographical location of agents, the remuneration of factors and workers and the productivity of the overall economy. More importantly, it determines the effectiveness of public policies in different locations and their ability to foster economic growth.

Recent advancements in the field of Urban Economics have introduced Quantitative Spatial Equilibrium models (QSE) as a framework to conceptualize the agglomeration and dispersion forces and capture their effect over the distribution of economic activity across geographic space. In doing so, they allow to gain a deeper understanding of the impact that certain policies would have. QSE models assume that people assess the benefits of living and residing in different locations and that they choose freely the location that provides the greater benefit. Then, due to these choices, housing prices and wages adjust to balance the existing supply of residential units and employment.

QSE models prove that there exists a strong link between the variables that emerge after individuals choose their preferred locations, which are easily observable in the data, such as employment and wages, or residents and housing prices, and other unobserved variables, like productivity and amenities. This connection is strong enough to allow for the estimation of the unobserved variables based on the observable ones. This feature enables QSE models to provide valuable insights into the potential effects of various policies because variables like productivity and amenities are often challenging to measure directly, yet they are vital components of the economy that determine its overall performance.

QSE models have two additional advantages: first, they can accommodate realistic features of economies, such as heterogeneous locations and commuting costs. Second, they are tractable and have very basic data requirements that can easily be met by researchers and policymakers. These features allow QSE models to perform counterfactual exercises that assess the impact of different urban interventions and estimate their impact on aggregate welfare. Not surprisingly, the economic literature has continuously expanded the use of QSE models in a broad range of fields, including trade, development, transportation, and environmental economics.

It is clear that QSE models offer valuable insights for policymakers seeking to assess the impacts of hypothetical urban interventions. However, their widespread practical adoption faces two significant barriers. First, their inherent technical complexity demands specialized knowledge, making it challenging for policymakers without a quantitative modeling background to effectively employ these models. Second, implementing QSE models usually necessitates access to high-performance computing resources and licensed software, which can be costly and have access barriers.

To address these challenges, the Cities Spatial Model toolkit aims to bridge the gap between policymakers and cutting-edge advancements in the Urban Economics literature by fostering a better understanding of QSE models and facilitating their practical utilization. The toolkit is structured in two components: a theoretical and a practical part. This report corresponds to the first component of the toolkit and seeks to translate the seminal work from Ahlfeldt et al. (2015) into a broader audience by explaining the intuition behind the model, the equations, and the interpretation of the results. It does so by using the City of Cape Town as a case use that illustrates how QSE models can assess the impact of several urban interventions. The second component of the toolkit is an open-source R package called IGC.CSM, available for download on the CRAN repository that develops the necessary functions to solve the model with very basic programming knowledge.

The remainder of this report is organized as follows: the following section develops a conceptual framework to describe QSE models and its capacity to offer insights into the effects of urban interventions, section 2 outlines the main components of the model created by Ahlfeldt et al. (2015), section 4 provides an overview of the model's underlying assumptions and a discussion of the limitations it has on the analysis of public policies. Following the theoretical explanation of the model, section 5 and 6 describe the use case of the model in the City of Cape Town and present the results from four counterfactual exercises. Finally, in section 7, we conclude by summarizing the model's potential in guiding urban policy decision-making.

2 Conceptual framework

The variation in the density of economic activity across geographic regions is shaped by the interplay of two opposite forces: agglomeration and congestion. Agglomeration forces encourage employment to cluster together by offering advantages for locating in specific areas. Conversely, congestion forces disperse people across the space due to the disadvantages of being in close proximity. When the benefits of living or working in a particular area balance with the disadvantages, a spatial economic equilibrium is reached. In simpler words, a spatial economic equilibrium is a state in which individuals are happy with their choices of residential and work locations. In the equilibrium state, wages equalize the supply and demand of workers and floor space prices equalize its supply and demand.

QSE models obtain a diagnosis of the baseline equilibrium in the status quo of the economy and simulate the equilibrium that would result from counterfactual scenarios. They can incorporate changes generated by policy interventions and analyze their impacts on welfare. Moreover, they do so using a general equilibrium framework that allows them to capture the impacts of policies that go beyond their beyond the partial local effects and to understand how they affect the economy as a whole. Moreover, QSE models are sufficiently complex to include relevant characteristics of the data, such as multiple locations with diverse geographical, productivity, or quality-of-life-related factors, and to account for critical interconnections between the locations, such as the exchange of goods, migration, and commuting costs (Redding and Rossi-Hansberg, 2017).

The QSE model that this report explores was developed by Ahlfeldt et al. (2015) as a seminal work in the Urban Economics literature to explain the changes in the internal structure of Berlin due to its division and reunification. The main contribution of this model is that it can accommodate a set of heterogeneous locations that differ in terms of their natural advantages for production, residential amenities, land supply, and transport infrastructure and that are also subject to agglomeration and congestion forces. This work develops a theoretical framework that accounts for the observed changes in the distribution of economic activity within the city while remaining tractable and amenable to empirical analysis.

3 Model

The model operates in the following manner. Initially, it uses observed data regarding the number of workers and residents, the available amount and prices of floor space, and the commuting time across locations to establish a connection between the observed values and the values predicted by a set of equations. Specifically, the model ensures that the predicted labor supply in a particular location corresponds to the number of workers observed in the data, and similarly for the population. Subsequently, the model generates three variables that define the fundamentals of the economy and characterize the baseline scenario upon which we can compare counterfactual simulations. In this section we describe the components of the model and the main equations that define the economic equilibrium [1].

- The first component of the model is **a discrete set of heterogeneous locations** within a city[2] that are indexed by $i = 1, .., N$, each one with an effective supply of land (K_i) with associated floorspace prices (Q_i) and a number of residents (L_{Ri}) and workers (L_{Mi}). The travel time to commute between locations (τ_{ij}) is given by the modes of transport and the existing road infrastructure.

- The **floor space** is supplied by a competitive construction sector that uses land (K_i) and capital (M_i) an inputs

  $$F_i = M_i^{\mu} K_i^{(1-\mu)}$$

  The term M_i^{μ} is equivalent to the density of land development φ_i. It determines the relationship between floor space and land area

  $$\varphi_i = \frac{F_i}{K_i^{1-\mu}}$$

  The total floor space F_i is endogenously allocated into commercial (F_{Mi}) and residential (F_{Ri}) use.

- There is **a mass of workers** that are perfectly mobile within the city. They choose the pair of residence and employment locations within the city that maximize their utility $U_{ij\omega}$. The model can assume that workers are fixed inside the city or that they are mobile between the city and the larger economy to account for the presence of migration. The utility that a worker ω obtains from living in location i and working in location j depends on: the wage paid at this workplace w_j, the time it takes to commute across te locations τ_{ij}, the floor space price at the residence location Q_i, the amenities at the residence location B_i and an idiosyncratic shock $\epsilon_{ij\omega}$

  $$U_{ij\omega} = \frac{B_i w_j \epsilon_{ij\omega}}{\exp(\epsilon\ \tau_{ij}) Q_i^{1-\alpha}},$$

- **Firms** produce a single final good Y_j, which is traded within the city and the larger economy. The firms use a Cobb-Douglas production function that employs commercial floor space F_{Mj} and workers L_{Mj} taking as given the productivity A_j. Firms choose their location of production and inputs of workers and commercial floor space to maximize profits.

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[1] Section C from the appendix provides a more detailed technical explanation of the model.

[2] For example, we can define the locations using the census tracts or the districts of a city.

$$Y_j = A_j L_{Mj}^{\beta} F_{Mj}^{1-\beta}$$

From the previous equations, the model can generate a theoretical prediction for the number of workers choosing to work in a specific location (L_{Mj}^{model}). Subsequently, by considering the observed workplace choices (L_{Mj}), the model iteratively determines an estimated vector of wages, aligning the observed employment level with the employment level projected by the theoretical prediction. From this point, the model can build three fundamental variables that define the equilibrium state of the economy[3]:

- **Residential amenities** (b_i): captures nonmonetary characteristics that make a place more attractive to live. These are all the variables that make a location better to live in and are unrelated to prices -for example, parks, clean air, security, access to restaurants, etc.
- **Productivity** (a_i): captures how good firms are in a particular location and corresponds to the comparative advantage of a place to produce. In other words, this variable captures how efficiently a place transforms the different inputs into output.
- **The density of land development** (φ_i): measures how much land is developed in a particular location based on the total floor space. A location with a higher density of development implies that the location has more buildings than other areas in the city because the available land area has been transformed into floor space.

It is important to highlight that when the model estimates the residential amenities and productivity in a certain location it also accounts for a range of externalities from neighboring locations, with the extent of this influence varying based on the distance.

After recovering the fundamental variables of the economy, the model can run counterfactual exercises that capture different urban policy interventions such as infrastructure investments that reduce travel times t_{ij}, revitalization efforts that improve the amenities b_i or innovations that improve productivity a_i and estimate a new equilibrium[4]. The new variables capture how the the economy would be if the counterfactual policies were implemented.

The model can also construct a measure of aggregate welfare that captures all the previous variables and summarizes how they affect the well-being of agents. Essentially,

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[3] The `IGC.CSM` package operationalizes this procedure using the function `InvertModel`.

[4] The `IGC.CSM` package operationalizes this procedure using the function `SolveModel`.

the aggregate welfare measure collects the expected value of the utility from all the locations. This implies that the aggregate welfare is negative correlated with floor space prices, and travel times and that it is positively correlated with amenities. Specifically, in the model aggregate welfare is determined by the following equation:

$$\bar{U} = \gamma \left(\sum_i \sum_j \frac{B_i w_j}{Q_i^{(1-\alpha)} exp(\epsilon \tau_{ij})} \right)$$

where γ is a constant term and all the other terms are the variables that we previously defined

4 Assumptions and Limitations

The model's ability to be analyzed easily and estimated with minimal data requirements is achieved by making some specific assumptions, which include:

- The model assumes that all workers exhibit risk neutrality and possess perfect information.
- It considers a scenario where there is a single tradable good, excluding the consumption of non-tradable items like services.
- The production of both floor space and final goods occurs within a framework of perfect competition.
- Labor stands as the exclusive source of income within the city, with the model omitting considerations of floor space rent.
- Full employment is a given, ensuring all workers can secure employment in their preferred locations.
- The model assumes uniformity among workers, lacking the inclusion of worker types that differ in terms of their skill level, sector or gender.

While essential to maintain the tractability of the model, these assumptions lead to a simplification of reality and the omission of crucial economic factors that undeniably shape the influence of public policies. Another enormous simplification in the model is its static design. This implies that the model exclusively examines two states: the existing economic equilibrium and the subsequent equilibrium following policy implementation. In other words, the model does not consider the dynamic aspects of policies and cannot provide insights into the transitional process as policies are implemented and economic variables are adapting in response to them.

One additional limitation of these models is that they rely on a calibration process. Calibrating a model means determining values for a range of parameters, such as the elasticity of substitution between inputs or the influence of agglomeration and congestion forces, that shape the degree of interaction between different variables[5]. Several methods exist for parameter calibration, including the use of historical data or linear regressions. While some of these methods may produce parameter values that closely describe the economic system and, consequently, lead to highly accurate model predictions that align with real-world measurements, it is important to note that the outcomes of the model remain sensitive to the selection of these parameters.

In this context, it is important to understand the model as a tool that augments the information available for policymaking rather than one that furnishes a comprehensive estimation of their effects. For instance, the model can be utilized to rank policies based on their impact on overall welfare, but it is not well-suited for determining how a policy affects the distribution of welfare among different segments of the population.

Recent developments in the academic literature have successfully addressed these constraints by extending the foundational work of Ahfeldt et al. (2015) to incorporate richer features of the economy. For example, Monte et al. (2018) develop a model in which locations are connected in both goods markets, through trade, and factor markets, through migration and commuting, Tsivanidis (2019) include multiple groups and transportation modes, and non-homothetic preferences for housing implying that relative price changes affect people's decisions differently, Zárate (2022) distinguishes between formal and informal workers, and Redding (2016) models the ownership of land to determine the equilibrium allocation of the model[6].

5 Data

One of the main goals of the Cities Spatial Model toolkit was to create a practical demonstration of the QSE model exposed in the previous section that could showcase the significant potential of the model for policymakers and highlight its diverse range of applications, despite the minimal data requirements it relies upon.

In recent years, the city of Cape Town has undergone a transformative journey that

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[5] Table 7 lists the set of parameters from the model along with its description and value. We take the values from the calibration made in Ahfeldt et al. (2015)

[6] See Redding and Rossi-Hansberg (2017) for a comprehensive review of the different building blocks and variations of the QSE models.

has placed a strong emphasis on evidence-based decision-making and the utilization of data as a valuable economic asset (Carol et al., 2022). This shift has resulted in a greater focus on harnessing the power of data and research to guide the decision-making process. The city's approach provided an ideal setting for applying the Cities Spatial Model toolkit, allowing us to evaluate various hypothetical urban policy interventions and demonstrate the crucial role of data in informing and shaping effective decision-making when combined with a structural approach.

First, we employed the Cities Spatial Model toolkit to build a baseline equilibrium scenario and estimate the values of the fundamental variables of the economy. Once we had built this benchmark, we analyzed the resulting equilibrium after implementing four policy scenarios: the construction of phase 2A of the MyCiTi BRT system, the complete failure of the railway system, an increase of 10% in the amenities of the Southeast region of the city and an increase of 5% in the top decile of the most productive locations.

Baseline equilibrium

To define the locations of our model, we employed the subplace division, which represents the second most detailed geographic unit defined in the National Census of South Africa. We defined 921 distinct locations within the city, each spanning an average area of 8 square kilometers. As previously mentioned in Section 3, for each of these locations, we required data on factors such as the population of workers and residents, the availability and pricing of floor space, and the travel times to and from all other locations. The rest of this section outlines the methodology for creating each of these variables, utilizing the data sources accessible in Cape Town.

1. **Available floorspace (K_i):** First, we determined the total area in square kilometers of each spatial unit using the geoprocessing tools from ArcGIS. However, we needed to consider that in South Africa the land use regulation established protected areas as zones reserved for the preservation of nature and biodiversity and establish prohibitions or restrictions on human activity. To account for the impact of these regulations on economic activity, we only measure the total area of each spatial unit that is not covered by protected areas. Additionally, we excluded from our estimation the spatial units whose area covered by protected areas is greater than 85%. Therefore, we estimated our model only over 896 of the initial 921 units. The protected areas are indicated in panel a of Figure 1.
2. **Floorspace prices (Q_i):** in order to recover the average floorspace prices, we used data from the valuation roll that has the price per square meter for over 850,000 residential and commercial units in Cape Town and its coordinates. We

georeferenced each of these observations to locate them inside one of our locations and then computed an average measure of floor space prices for each unit. Panel b from figure 1 plots all the observations classified according to their floor space price inside the spatial units.

3. **Number of residents (L_{Ri})** To determine the resident count (L_{Ri}), we rely on population estimates made by the South African Department of Statistics and derived from the Census from 2011. The map shown in panel c from Figure 1 plots the population distribution across the locations in Cape Town showing that the center of the city has a higher density of residents.

4. **Number of workers (L_{Mi}).** In order to acquire data regarding the population's work locations, we faced the challenge that the census solely provides information about their residential addresses. To address this limitation, we utilized data sourced from a mobile phone company located in Cape Town.

   We count the number of active cell phones within a specific polygon during working hours as a means to estimate the workforce distribution across the City. More specifically, we define the number of workers in each location as the average number of unique cell phones active between 10:00 am and 3:00 pm of workdays between March 1 and March 15 of 2020.

   We choose this time window based on Kreindler and Miyauchi (2021) who have used call detail record data from large cellphone operators in Sri Lanka and Bangladesh to compute detailed commuting matrices. They construct commuting trips by assigning "home" and "work" locations for each user of the cellphone network and identify that work locations can be identified as the most frequent towers with a transaction between 10:00 am to 3:00 pm during weekdays, while home locations can be identified as the most frequent towers with transactions between 9pm and 5am.

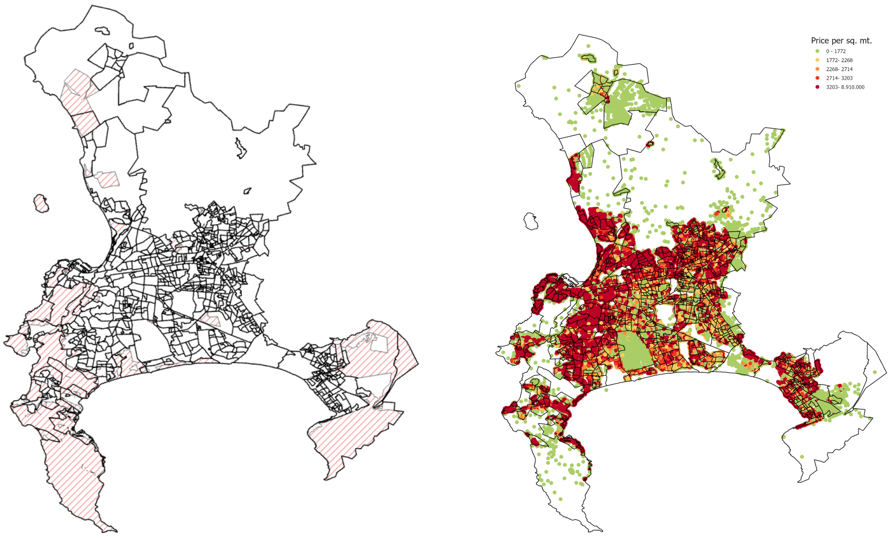
   Although there might be some concerns about the representativeness of commuters in cell phone data, Kreindler and Miyauchi (2021) have proven the robustness of this approach by showing that the commuting flows constructed with cellphone data correlate strongly with commuting flows from a transportation survey.

   The map shown in panel d from Figure 1 plots the employment distribution across the locations. There are two main nodes of employment: the center-west and the southeast parts of the city.

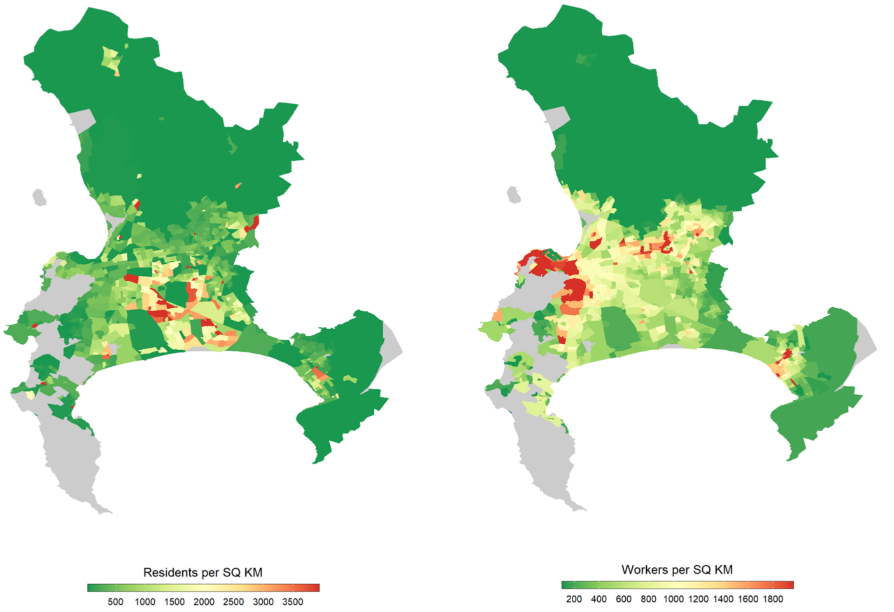
5. **Travel Times (t_{ij}).** We use the existing road network, the lines of the Golden Arrow Bus, and the railway network to calculate travel times across different locations in the City using the network analyst toolkit from ArcGIS. Based on Hitge

and Vanderschuren (2015), who make a comparison of travel times for different modes of transport in Cape Town, we assign a speed of 8km/hour to the roads which reflects an average of the walking and car speed, of 16 km/hour to the golden bus lines and of 30km/hour to the lines of the railway system. Figure 2 shows the transport network we consider.

Figure 1: Characteristics of the locations in Cape Town

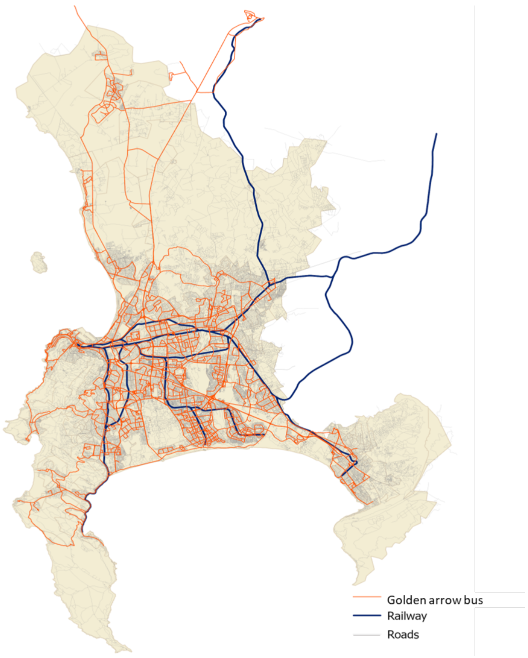

(a) Available floorspace

(b) Prices per sq mt


(c) Population

(d) Employment

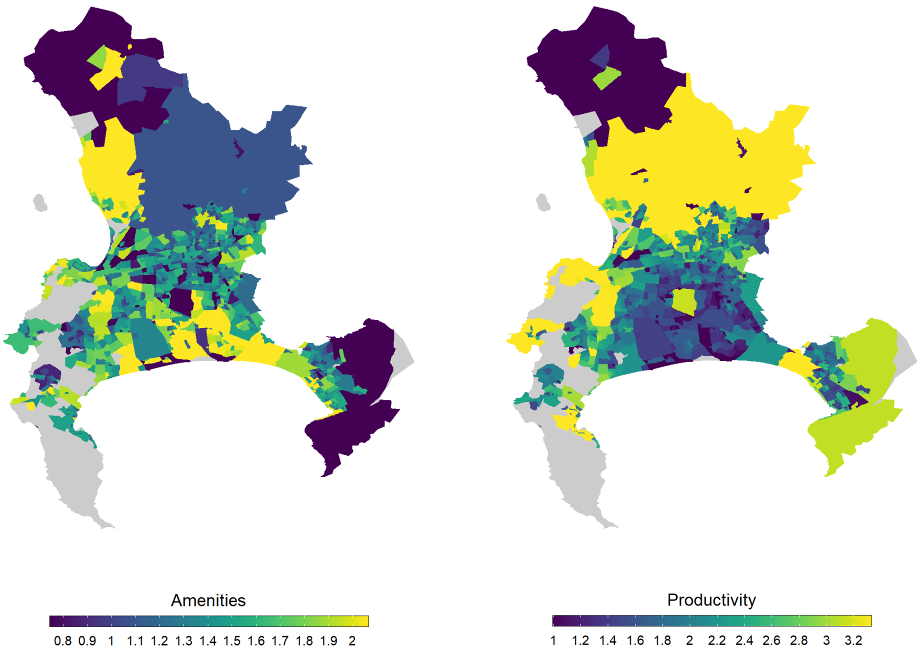
Figure 2: Baseline transport network



With this data, we can proceed to invert the model and recover the three fundamentals of the economy (amenities, productivity, and density of land development) that define the baseline equilibrium. The inversion of the model shows the following findings:

- Amenities are concentrated in the South-Center and North-West parts of the city. 8 shows that there is a strong spatial correlation between the levels of amenities predicted by the model and observed amenities such as parks, libraries and hospitals. These areas also observe a higher concentration of residents according to our population data.
- The productivity levels are higher in the North-East and Center-West of the city. In particular, we observe that the level of productivity is higher in the surroundings of the CBD district. These areas also observe a higher concentration of employment according to our data.
- The density of land development is higher in the central part of the City. This coincides with areas that have had recent expansion and increases and that have a concentration of low-income housing. On the opposite side, the density of land development is lower in the North part of the city, where the main use of land is agriculture.

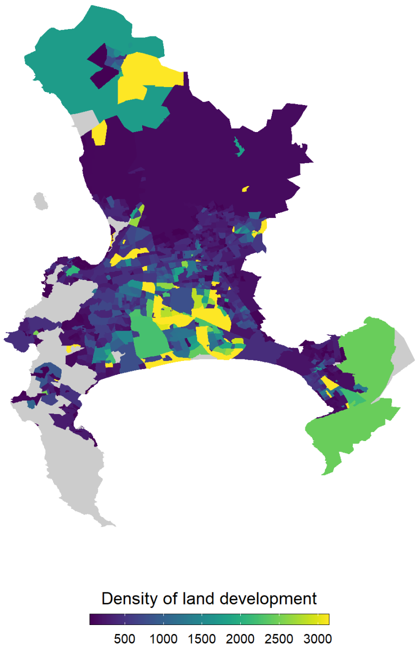
Figure 3: Fundamentals of the economy:



(a) Amenities

(b) Productivity

(c) Density of land development



Counterfactuals

By establishing a benchmark equilibrium through the inversion of the model, we create a reference point that can be compared with the results from hypothetical scenarios resulting from the implementation of public policies. Most frequently explored counterfactuals involve a transit improvement, which reduces commuting times between various locations. Nevertheless, multiple other simulations can be conducted using this model. For example, the model can assess the impacts of housing subsidies, adjustments in amenities, production subsidies, changes in productivity, or policies targeting the growth of specific areas through the expansion of available land. In this work we employed the IGC.CSM package, to simulate the following four hypothetical scenarios within the City of Cape Town:

1. **Implementation of the phase 2A from the MyCiTi transport plan:** The phase 2A of the MyCiTi transport plan is the construction of a public transport corridor served by a Bus Rapid Transit (BRT) system that comprises trunk routes and direct routes, facilitating the movement of people in Cape Town (Swart and Sasman, 2023). BRT systems typically demonstrate superior efficiency and reliability compared to traditional bus services due to their utilization of exclusive road lanes or more advanced vehicles.

   We assign the speed for these new BRT lines based on the records for Cape Town from the Global BRT database (BRT+ Centre of Excellence and EMBARQ, 2023), which compiles the average speed of the BRT systems around the world. We set a speed of 30 km/hour for the trunks and 26 km/hour for the feeders. Figure 4 illustrates the spatial distribution of lines from the MyCiTi phase 2A BRT lines within the city, showing in green the trunk lines and in purple the feeder lines. The main difference between the trunk and feeders or direct services lies in the fact that trunk lines operate on dedicated BRT corridors, transporting passengers to and from designated stations, while the latter operated on shared used lanes.

2. **Complete failure of the railway system:** The Cape Town railway system is currently grappling with significant issues related to its reliability, connectivity, and overcrowding. In our baseline equilibrium analysis, we assume account that the railway system operates at a speed of 30 kilometers per hour, a speed below what is typically expected for efficient railway systems. To explore a hypothetical scenario, we consider a situation in which the entire existing railway network becomes completely non-functional. Consequently, in this counterfactual scenario, we assume that the railway lines have a speed of 0 kilometers per hour.

   Figure 5 provides a visual representation of the location of these railway lines within the city.

3. **Improvements in amenities:** In our third hypothetical scenario, we model the impact of a 5% enhancement in the level of amenities over the locations in the South East of the city. Figure 6 maps these particular locations.

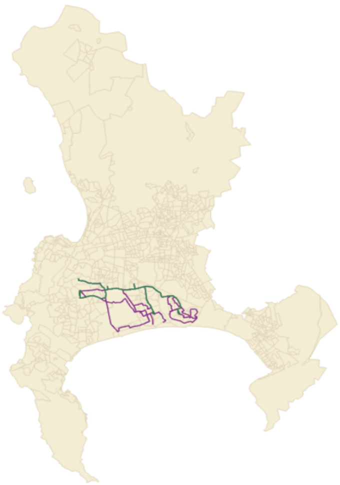
   Amenities encompass the non-monetary factors that enhance the attractiveness of a location, including aspects such as the quality of public services (e.g., schools, healthcare facilities), access to recreational spaces, cultural offerings (e.g., museums, theaters), and environmental conditions (e.g., air quality, green spaces). In that sense, urban policies that can improve amenities in a city encompass a range of initiatives, from investing in essential public services like education and healthcare to enhancing green spaces and parks for recreation.

   Amenities play a crucial role in individuals' location choices and significantly influence patterns of economic activity across space. Even though the counterfactual scenario being considered does not correspond to a specific policy proposed by the government and is purely speculative, it provides insights into how various policy interventions across a wide spectrum would function and their potential effects on the economy.

4. **Increase in productivity:** In our fourth hypothetical scenario, we investigate the consequences of a 10% increase in productivity for locations positioned in the top decile of the productivity distribution. Figure 7 depicts the geographical distribution of these specific locations.

   Although this simulation is speculative and does not reflect a particular policy, it offers an opportunity to assess the potential ramifications of interventions aimed at augmenting the production capabilities of firms, which may include technological advancements, improved resource access, and enhanced human capital.

Figure 4: BRT lines



Note: This figure plots the BRT lines from the phase 2A of the MyCiTi transport plan

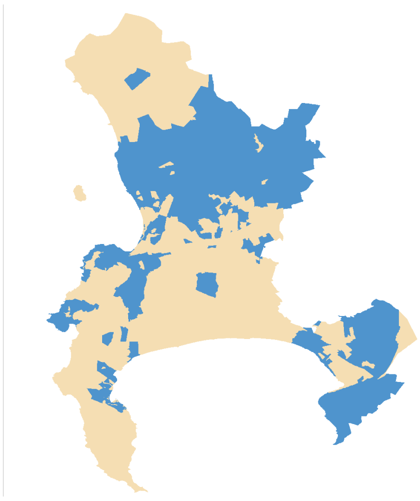
Figure 5: Railway lines

Note: This figure plots the existing railway lines in Cape Town

Figure 6: Increase in amenities

Note: This figure plots in blue the locations in Cape Town where we assume an increase in the amenities level of 5%

Figure 7: Increase in productivity



Note: This figure plots in blue the locations in Cape Town where we assume an increase in the productivity level of 10%

6 Results

We characterize the results of our counterfactual exercises using a comparative statics analysis over the results about the equilibrium states produced by the model. This approach has three main advantages. First, it allows us to analyze the changes resulting from policy interventions, such as the ones described in the previous section, in terms of effects relative to the baseline equilibrium. Second, it allows exploring the effects of variations in certain factors while holding other relevant variables constant. Such differentiation is often hard to achieve in empiric analyses where there can be multiple confounding factors. Finally, as the model operates within a framework of general equilibrium, the results take into account the response of all agents throughout the city and of both labor and floor space markets. Moreover, the results incorporate the externalities and spillovers across locations.

The first part of this section focuses on the effects of the four public policies outlined in the preceding section on six economic variables: the count of residents and workers, wage levels, floor space prices, and market access for both residents and firms. The market access of residents is captured by the Commuter Market Access (CMA) variable, which measures workers' access to employment via the commuting network, and the market access of firms is captured by the Firm Market Access (FMA) variable that measures the firms' accessibility to residents.

The second part of this section concentrates on providing an estimate of the welfare impact of each of these counterfactual interventions. For each of the hypothetical scenarios, we conduct simulations in two settings: a closed economy scenario, in which workers can only move within the city, and an open economy scenario, where workers have the flexibility to move both within and beyond the city's boundaries. The rest of this section explains our findings for each of the counterfactuals. Tables 1 to 4 show the average effect in terms of the six mentioned variables under the open and closed scenarios, and figures 10 to 16 in the appendix use maps to show the disaggregation of these results across all the locations.

Overall, according to the model's intuition, we expect to find that locations boasting higher productivity attract more workers due to their inherent comparative advantage, while areas offering superior amenities draw residents, leading to a specialization in housing. We also expect wages, the labor market price, to be positively correlated with demand for workers and negatively correlated with the supply of workers. Similarly, as residents' preferences influence floor space prices, increased desirability for specific locations is expected to drive up floor space prices. However, before delving into the interpretation of the results, it is important to reiterate the model's limitations previously discussed in Section 4, and to approach the results with a degree of caution.

MyCiTi BRT phase 2A

Under the closed city scenario, residents are drawn to areas near the BRT construction due to their preference for well-connected locations, driving up property prices in these sought-after areas while the rest of the city sees modest decreases. Proximity to the BRT leads to increased employment opportunities, often at the expense of less productive regions experiencing a workforce decline as they become more residential in nature. Property prices rise in areas near the BRT, including those in close proximity but not directly connected to the new lines, benefiting from enhanced commuting infrastructure. Wages in BRT-adjacent areas experience slight decreases as their improved connectivity results in a surplus of available workers. The CMA increases across most of the city, but locations near the BRT construction show a more pronounced increase in this measure, reflecting the impact of the city's commuting infrastructure on these better-connected areas. The FMA also increases in the locations in the proximity of the BRT lines, but it decreases across the rest of the city.

In an open city scenario, when we assume that there is migration between the city and the rest of the economy, average property prices increase citywide, with a more significant effect seen in areas directly affected by the new BRT construction, as they are the most desired. Meanwhile, the influx of workers attracted by the infrastructure exerts downward pressure on salaries.

Table 1: Average effect counterfactual MyCiTi BRT phase 2A

Variable	**Effect: closed city (%)**	**Effect: open city (%)**
Number of residents	-4.16	-.73
Number of workers	-.99	2.56
Wages	.32	-.7
Floor space prices	-.7	1.81
FMA	-.33	.32
CMA	.74	-.29

Failure of railways

Assuming that there is no migration out of or into the city, the railway's failure prompts a dispersal of residents across the city's periphery, particularly in areas in the North and South East that were not part of the railway's network. This shift is driven by these locations losing their comparative advantage for attracting workers and witnessing growth in their residential population. Similarly, central city areas that previously exhibited higher employment concentrations and transport connectivity experience a decline in the workforce, while areas in the center and South, which relied on alternative modes of transport, encounter milder effects or even an increase in workers. As workers disperse across the periphery, salaries decrease in most areas, but places that become disconnected due to railway failure see a labor supply reduction and wage increases. Conversely, locations that become less connected due to railway failure experience significant declines in property prices, while areas attracting more residents see price increases. The removal of railways results in a drop in FMA in the city center as connectivity weakens and in citywide CMA decreases, with the most pronounced losses in areas that were directly served by the railways.

In an open city scenario the number of residents also decreases in areas previously connected by railways. However, this reduction is more pronounced than in the case of the closed city. Given the reduced overall attractiveness of the city, and the ability of residents to migrate, floor space prices also decrease in all the city. As workers seek locations out of the city in order to maximize their utility, there is an overall reduction in the labor force supply that leads to salary increases.

Table 2: Average effect counterfactualrailway system

Variable	**Effect: closed city**	**Effect: open city**
Number of residents	17.51	-8.12
Number of workers	3.18	-19.33
Wages	-1.04	6.28
Floor space prices	1.85	-14.48
FMA	-3.74	-8.01
CMA	-8.27	-1.48

Amenities improvement

As expected, areas in the city's South East, where we simulated enhanced amenities, witness a rise in resident numbers, while unaffected areas lose their housing attractiveness and experience minor resident decreases. These locations also receive a higher supply of workers, which generates a reduction in the wages. This amenity-driven residency focus causes a decrease in the CMA measure for the locations that underwent the amenity enhancement. Additionally, areas becoming more appealing for residents due to improved amenities observe higher property prices. The FMA increases in locations with greater resident and employment presence. One important feature of the amenity improvements is that they have spatially concentrated effects, leading to minimal changes elsewhere in the city.

When we factor in the potential for migration in and out of the city, the simulation results remain quite similar but exhibit a more strong impact due to the influx of workers from outside the city seeking to benefit from the improved amenities. Specifically, areas in the southeast experience a rise in both resident and worker numbers, leading to an increase in real estate prices and a decrease in wages. Much like the scenario in an open city, the concentration of residents in these areas contributes to an increase in the FMA and a decrease in the CMA measure.

Table 3: Counterfactual failure of railways

Variable	**Effect: closed city**	**Effect: open city**
Number of residents	1.07	1.92
Number of workers	.24	1.08
Wages	-.06	-.31
Floor space prices	.17	.76
FMA	.13	.28
CMA	-.08	-.32

Increase in productivity

The productivity shock prompts areas to specialize according to their comparative advantage, with those impacted by the increase in productivity decreasing their resident populations, shifting their focus towards employment, while unaffected areas concentrate more on residents. Likewise, areas experiencing heightened productivity observe an influx of workers, leveraging their production advantage and attracting more employees. Notably, the productivity shock is the only scenario in which all locations experience wage increases, with the most significant gains in directly affected areas due to the positive correlation between wages and firm output. While all city locations see an increase in their CMA, this effect is more pronounced in directly affected areas. In contrast, the FMA decreases throughout the city, with a more noticeable decline in areas directly impacted by the shock.

In the context of the open city scenario, the affected areas also witness a decline in their resident population, although the impact is less pronounced. Unlike the closed city scenario, this reduction in residents is counterbalanced by the arrival of new workers, and only the areas directly impacted by the productivity shock can sustain the productivity increases. Due to the less pronounced reduction in the number of residents, there is a homogeneous increase of floor space prices across all the city. Finally, only the locations directly affected by the productivity shock will see an increase in their CMA measure and a decrease in the FMA.

Table 4: Counterfactual failure of railways

Variable	**Effect: closed city**	**Effect: open city**
Number of residents	.72	13.61
Number of workers	-6.92	4.99
Wages	4.5	.91
Floor space prices	-3.92	4.66
FMA	-.57	1.66
CMA	4.5	.91

6.1 Welfare

As indicated in section 3, the QSE model utilized in our analysis explicates welfare as an outcome shaped by the interplay of decisions made by people coupled with the dynamics within the residential and labor markets. In this subsection, we delve deeper into comprehending the changes in welfare from the four distinct counterfactual scenarios by translating its effects into a measure of compensated variation. In that sense, table 5 outlines the effects that each of these policies would have on aggregate welfare.

Under the open city scenario, the failure of railways would have the largest negative effects, decreasing the total welfare by 10%. The magnitude of the effects from the BRT is much smaller, generating only a welfare gain of 1.52%. This difference in magnitude was expected, as the magnitude of the BRT lines is much smaller than that one of the railways. The improvement in amenities has a smaller effect, increasing aggregate welfare by 0.35%. Finally, the shock to productivity has the larger positive effect of all the interventions. It could generate gains of nearly 5.33%.

One important aspect of these results is that we assumed a speed of 30 km/hour for the railways -a speed below the international standards for the railways- to account for the already present failures in the system. However, if we considered an appropriate functioning of the railways as the baseline scenario, the losses obtained in this counterfactual would be even larger.

To provide a clearer understanding of the extent of the welfare changes resulting from each of the hypothetical policies, we calculate what's known as the compensated variation (CV). The compensated variation represents the additional yearly income that individuals would need to receive to keep their well-being at the same level it was before the hypothetical change took place. For example, if floor space prices were to rise, CV helps us determine the extra money individuals would need to counterbalance the negative impact of the price increase and maintain the same level of well-being. We calculate the CV using as point of reference the average annual salary in Cape Town, which according to the Household Survey from 2021, was equivalent to 123,408 ZAR. Column 2 in Table 5 displays the CV values in terms of the South African rand for each counterfactual scenario.

Table 5: Effects on aggregate welfare (open city)

Scenario	**% change**	**CV in SA rand**
MyCiTi BRT phase 2A	1.53	1888.16
Failure of railways	-10.09	-12,451.98
Amenities improvement	0.35	431.93
Increase in productivity	5.33	6577.70

The open version of the model assumes that the welfare of the city remains unchanged and agents will move into the city whenever they can find a location that maximizes their utility. The welfare increases are absorbed by the new agents, until there is a net zero effect on aggregate welfare. However, we can calculate the change in output that would result from the updated number of workers in the city. Similar to the previous result, we also calculate the CV from each of these counterfactuals. These values are shown in column 2 of Table 6.

Table 6: Effects on output (closed city)

Scenario	**% change**	**CV in SA rand**
MyCiTi BRT phase 2A	2.4	2,961.72
Failure of railways	-16.17	-19,995
Amenities improvement	0.63	777.47
Increase in productivity	14.26	17,597

7 Conclusion

In conclusion, the rapid urbanization worldwide presents significant challenges for urban populations, demanding that policymakers reach a balance between achieving economic growth and ensuring citizens' quality of life. The IGC's Cities Spatial Model toolkit seeks to contribute to this endeavor by exploring a QSE model as a tool for comprehensively assessing the economy-wide impacts of urban policies. In particular, this toolkit bridges the gap between policymakers and the latest urban economics advancements by promoting a deeper understanding of these models and facilitating their practical application.

In this report, we described the main components of the QSE model developed by Ahfeldt et al. (2015) and demonstrated its utilization through a case study in the City of Cape Town that simulates the effect of four urban policy interventions. Our analysis involved examining labor and employer distribution, wage dynamics, housing prices, and market accessibility for both firms and commuters. The results highlight the influence of transport infrastructure on wages, floor space prices, and the geographical distribution of the population -factors that ultimately determine welfare.

Despite the potential of this toolkit to estimate the impact of a vast range of policies, it is essential to recognize some of its limitations. QSE models are based on abstractions, are grounded in theoretical assumptions, and remain highly sensitive to a set of parameters. Its use should be viewed as a tool to assist policymakers in decision-making rather than an absolute truth. Taking this approach can help assess urban interventions while considering broader fundamental aspects of the economic activity distribution.

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A Additional figures

Figure 8: Observed amenities

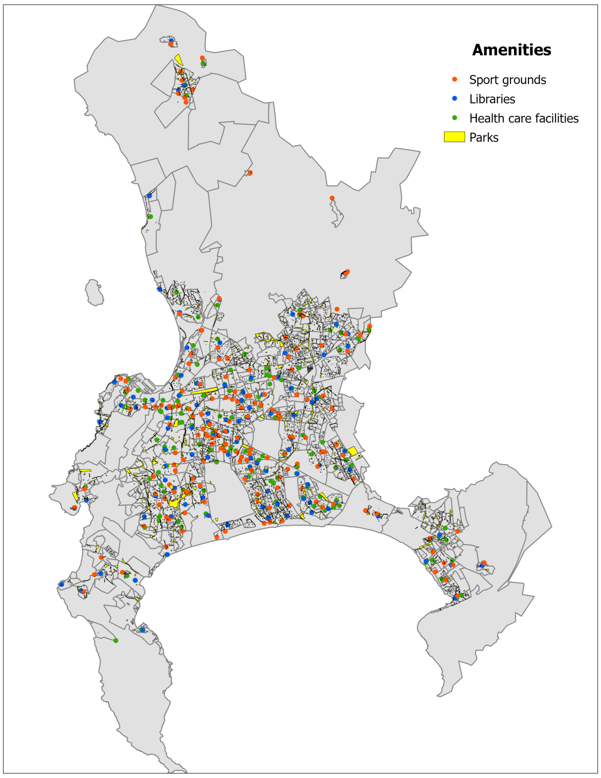


Figure 9: Changes in travel times across locations

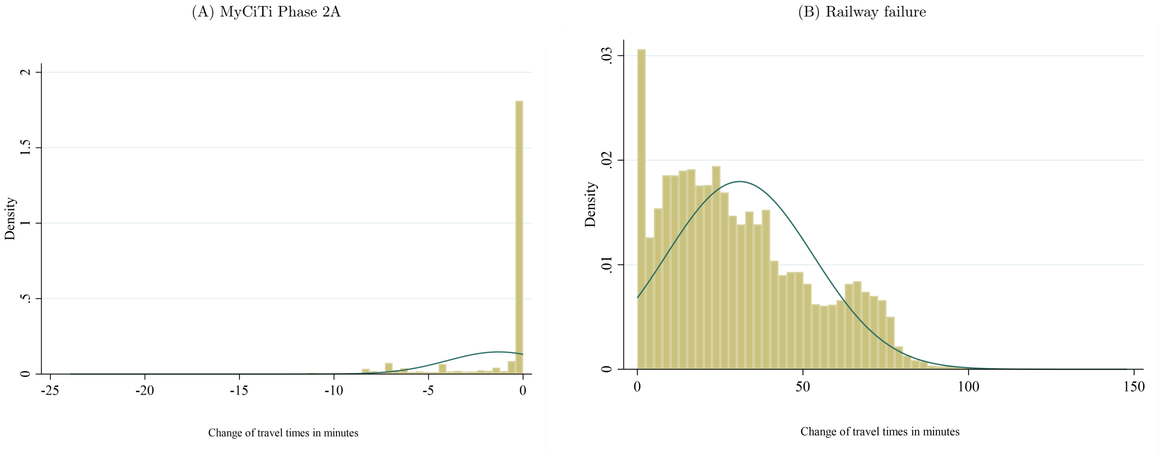
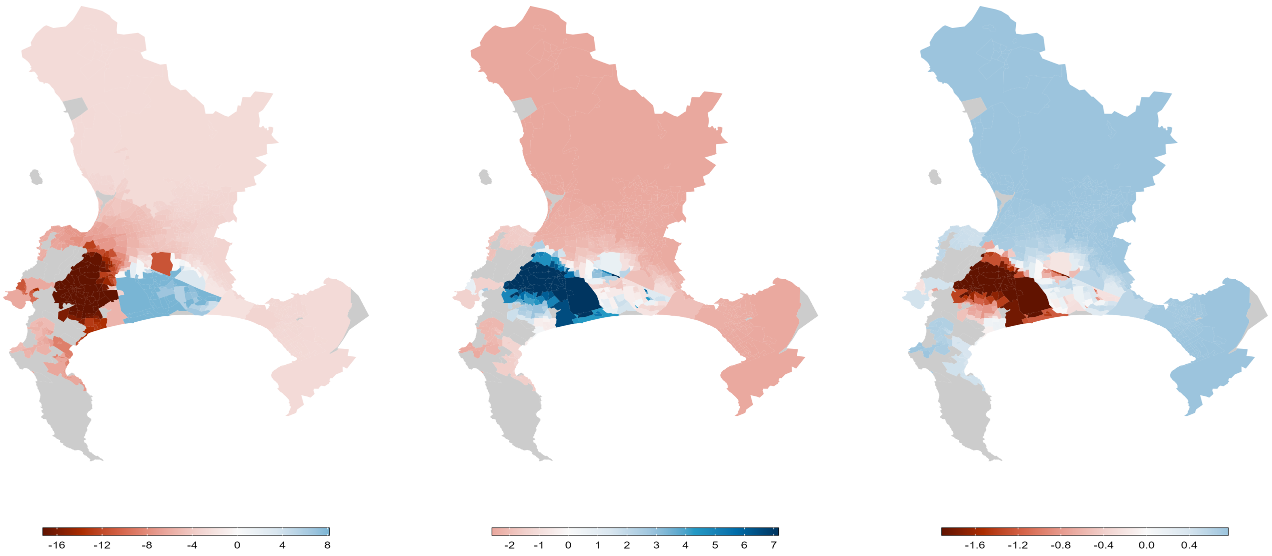


Figure 10: Results MyCiTi BRT Phase 2A - Closed City

(a) Residents

(b) Workers

(c) Wages



(d) Floor space prices

(e) FMA

(f) CMA

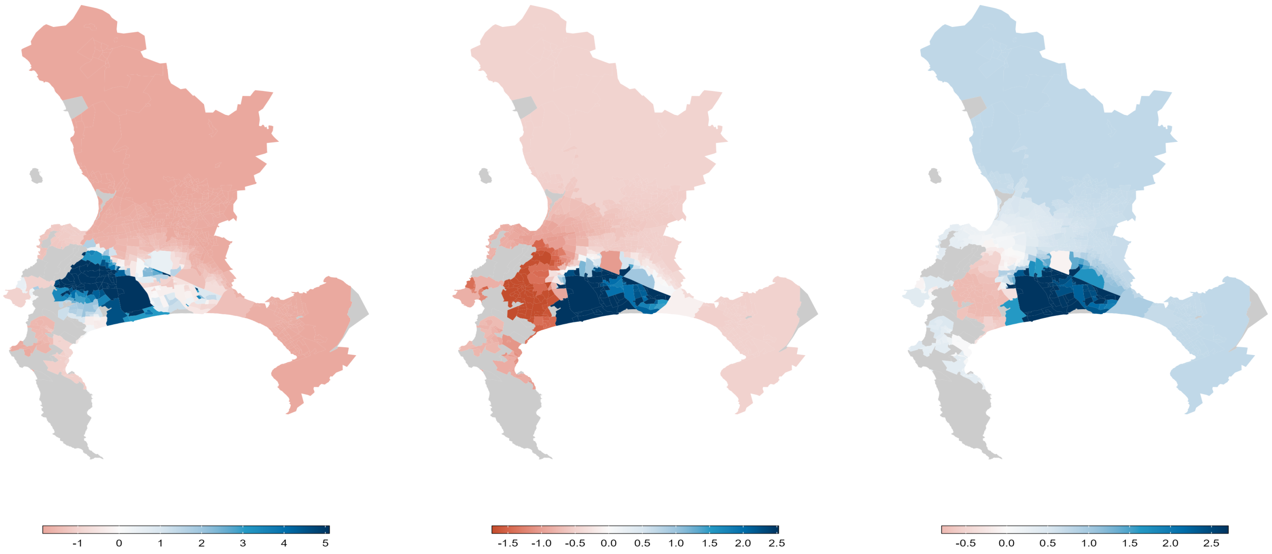
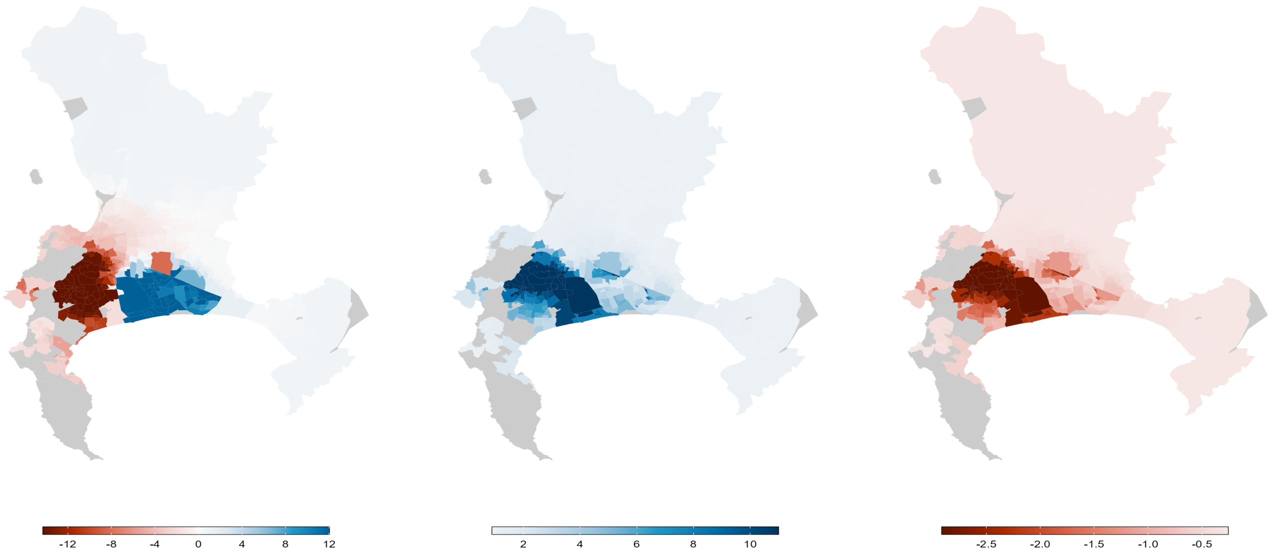
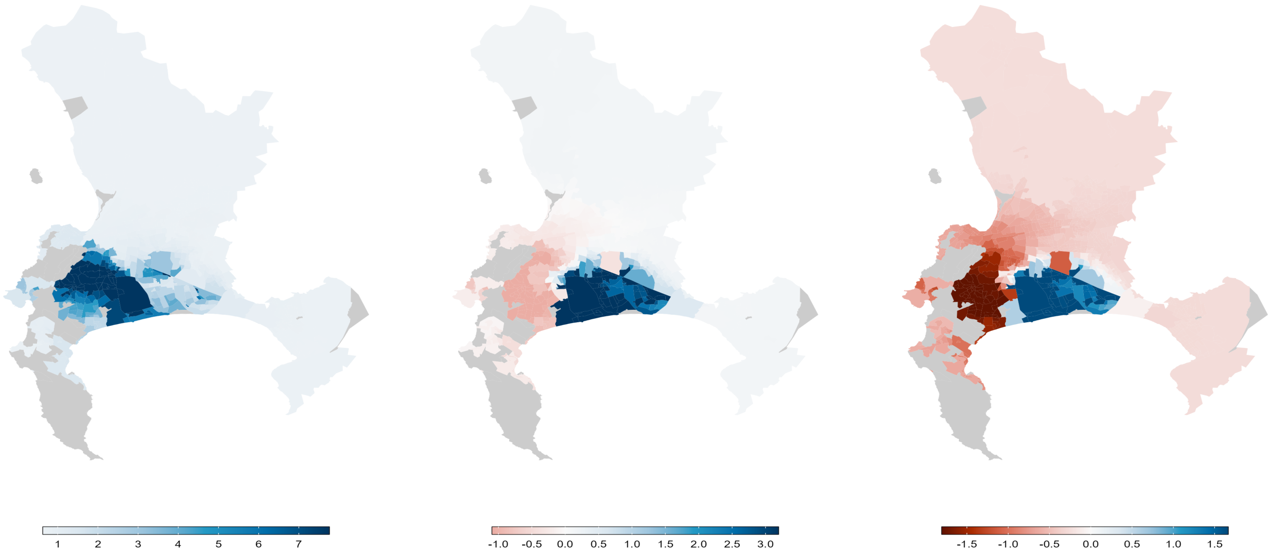


Figure 11: Results MyCiTi BRT Phase 2A - Open City

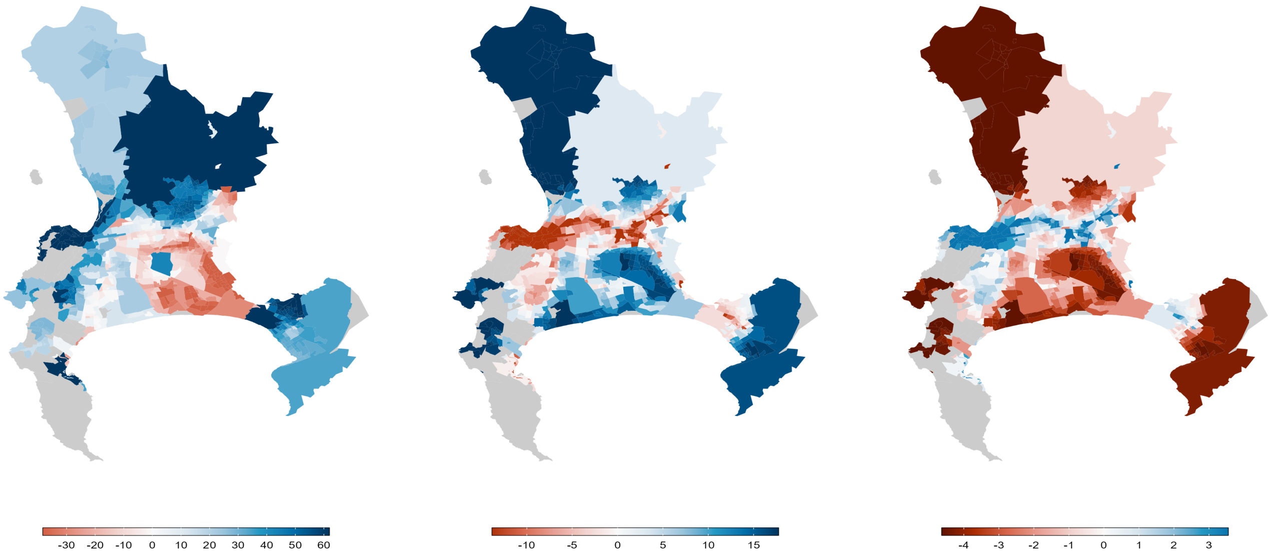


(a) Residents (b) Workers (c) Wages



(d) Floor space prices (e) FMA (f) CMA

Figure 12: Results Failure of Railways - Closed City


(a) Residents (b) Workers (c) Wages

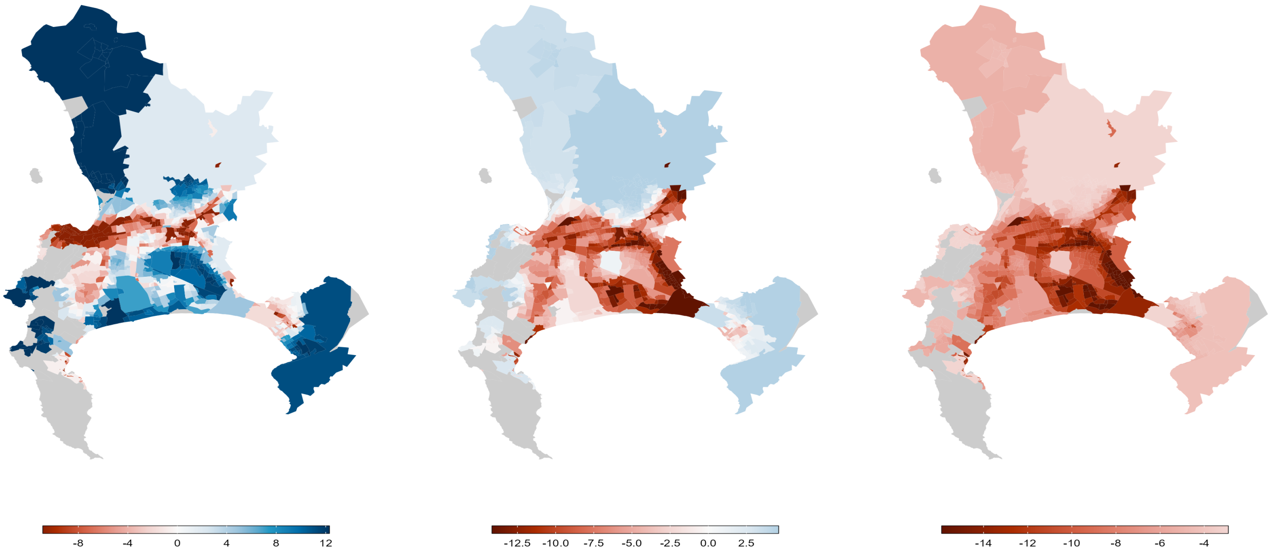
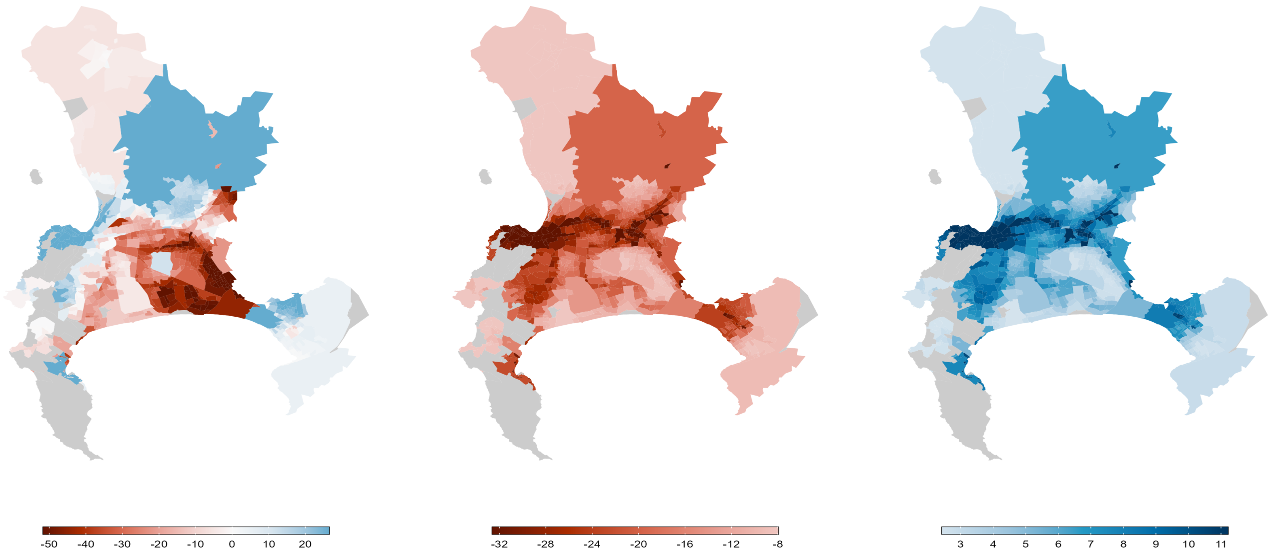

(d) Floor space prices (e) FMA (f) CMA

Figure 13: Results Failure of Railways - Open City

(a) Residents (b) Workers (c) Wages



(d) Floor space prices (e) FMA (f) CMA

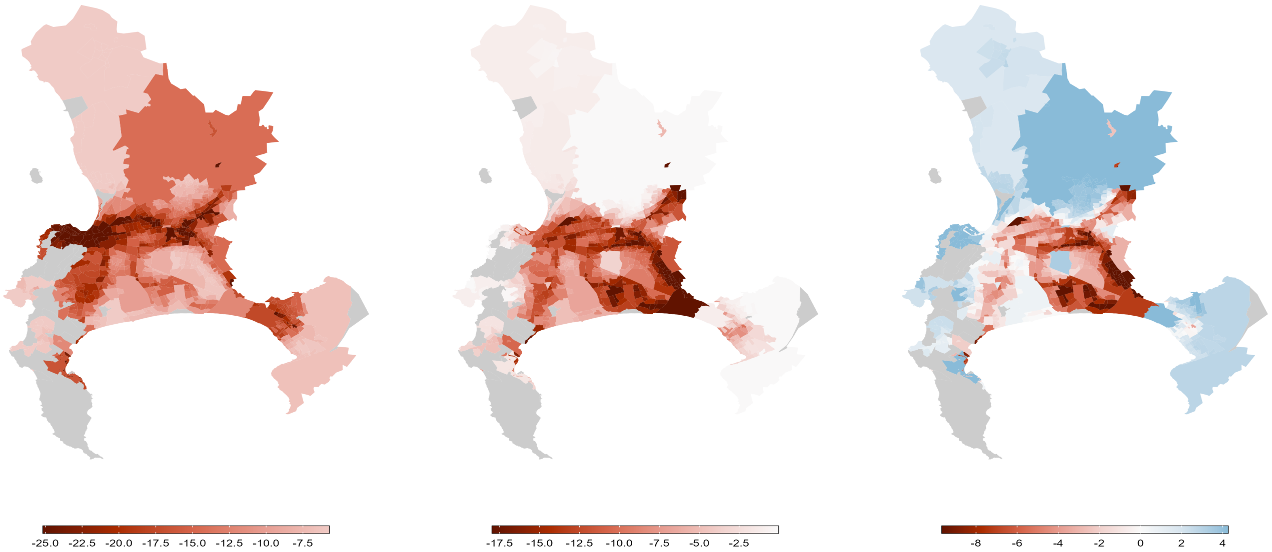
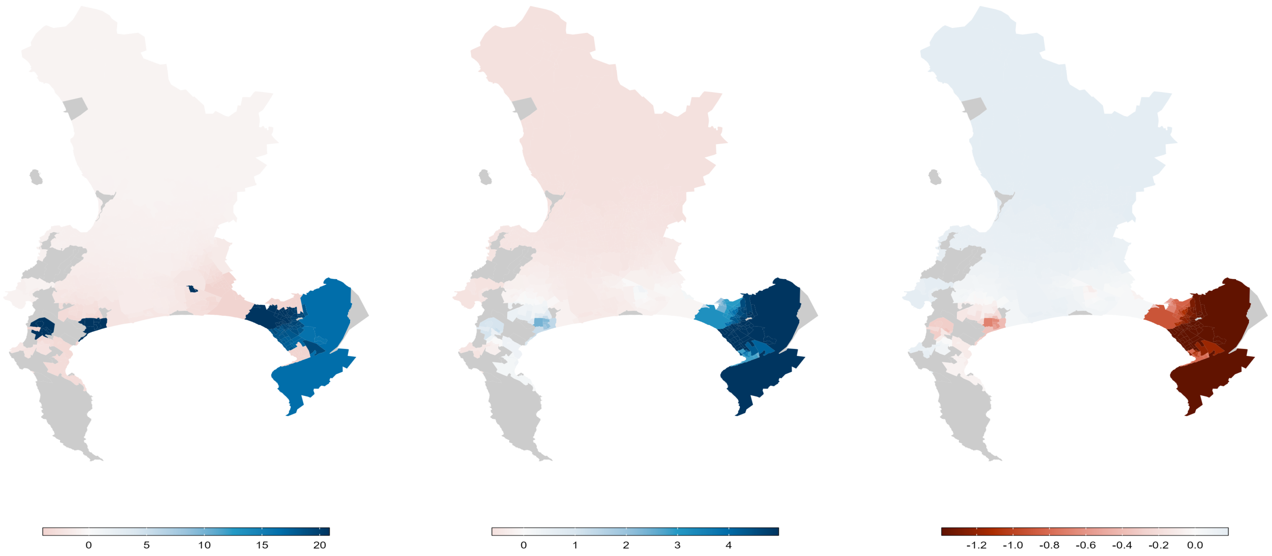


Figure 14: Amenities improvement - Closed City


(a) Residents (b) Workers (c) Wages

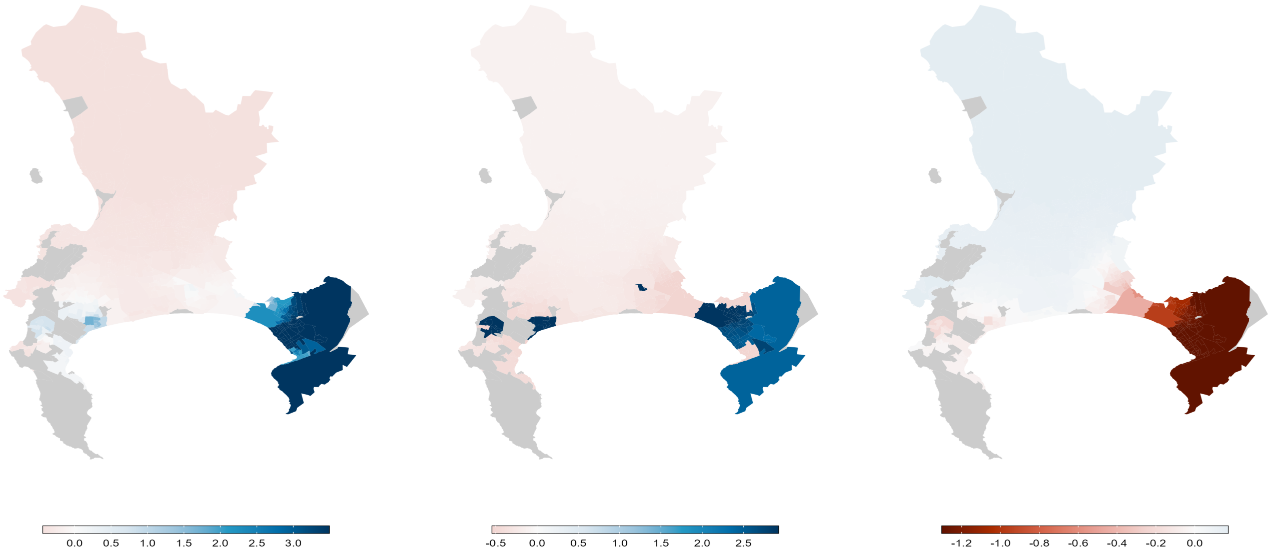
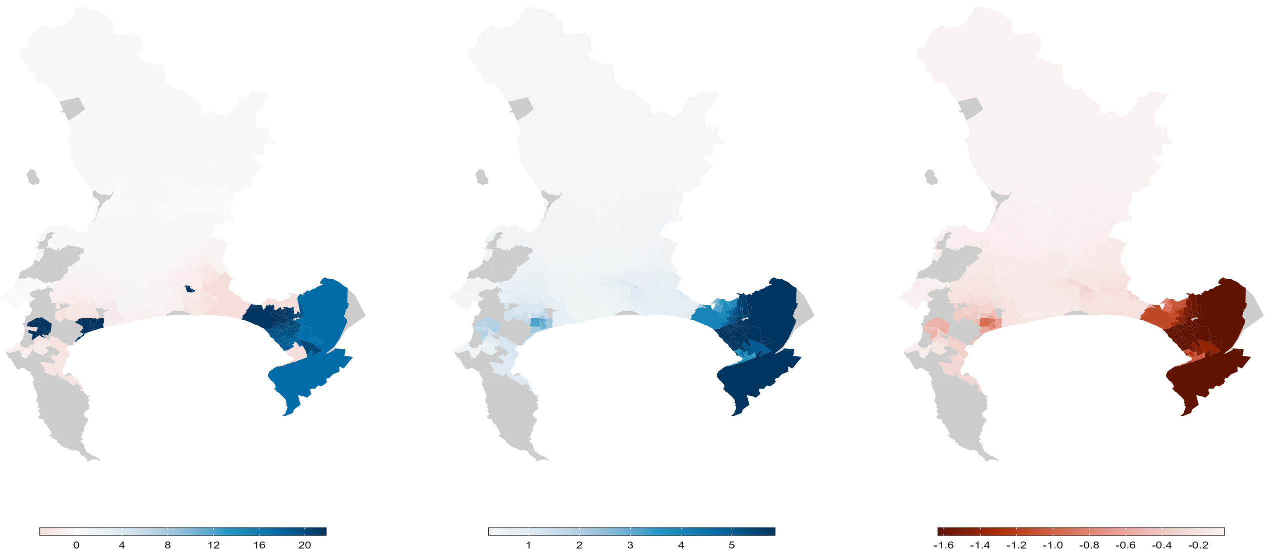

(d) Floor space prices (e) FMA (f) CMA

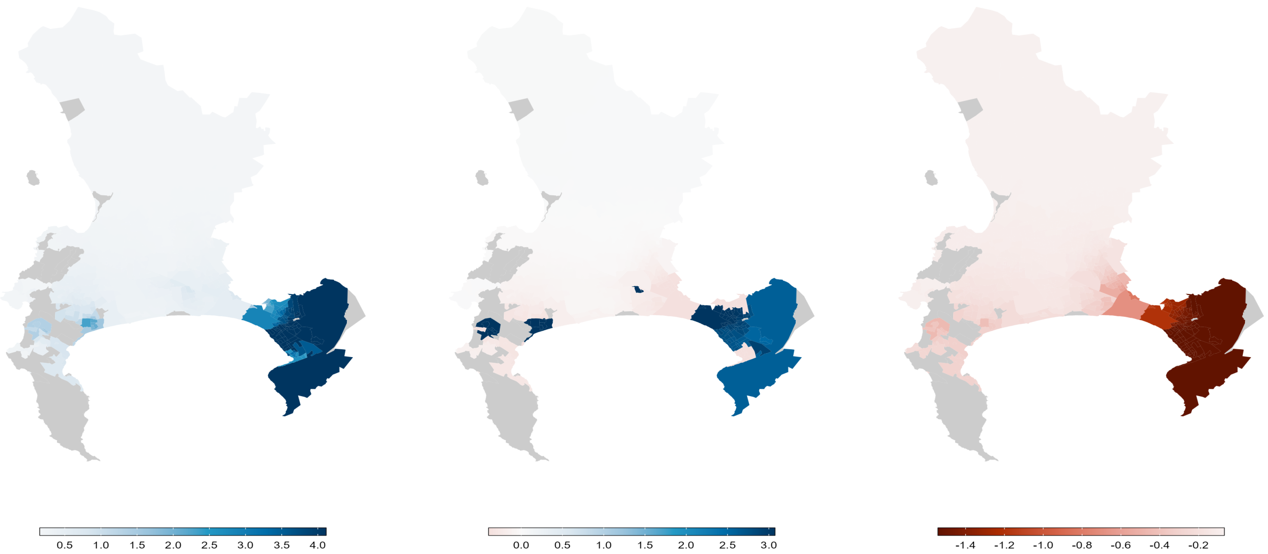
Figure 15: Amenities improvement - Open City



(a) Residents

(b) Workers

(c) Wages

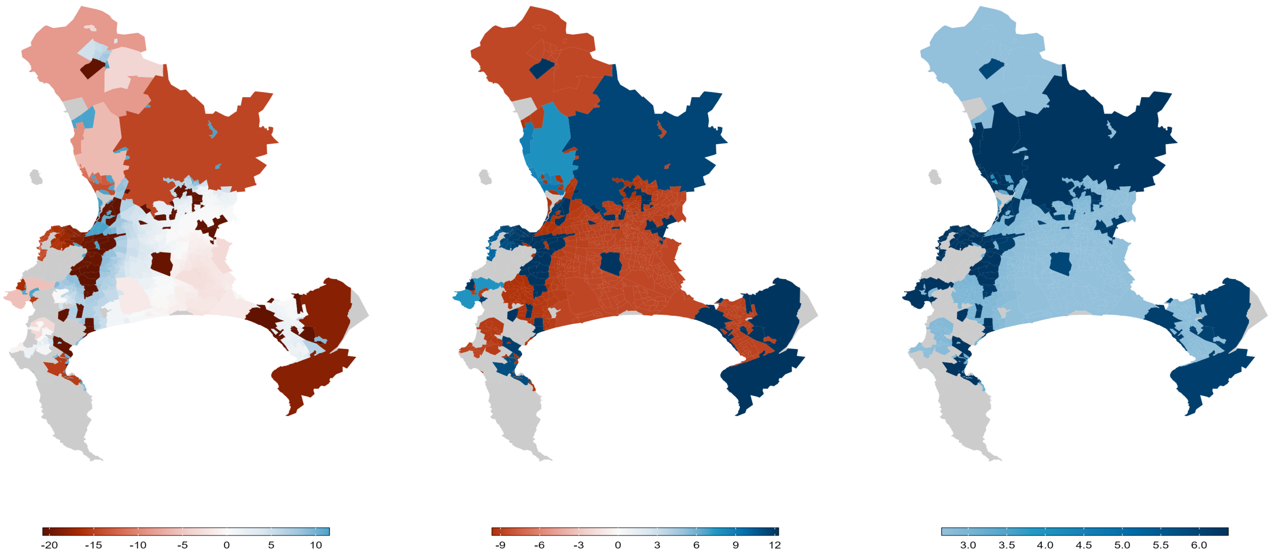


(d) Floor space prices

(e) FMA

(f) CMA

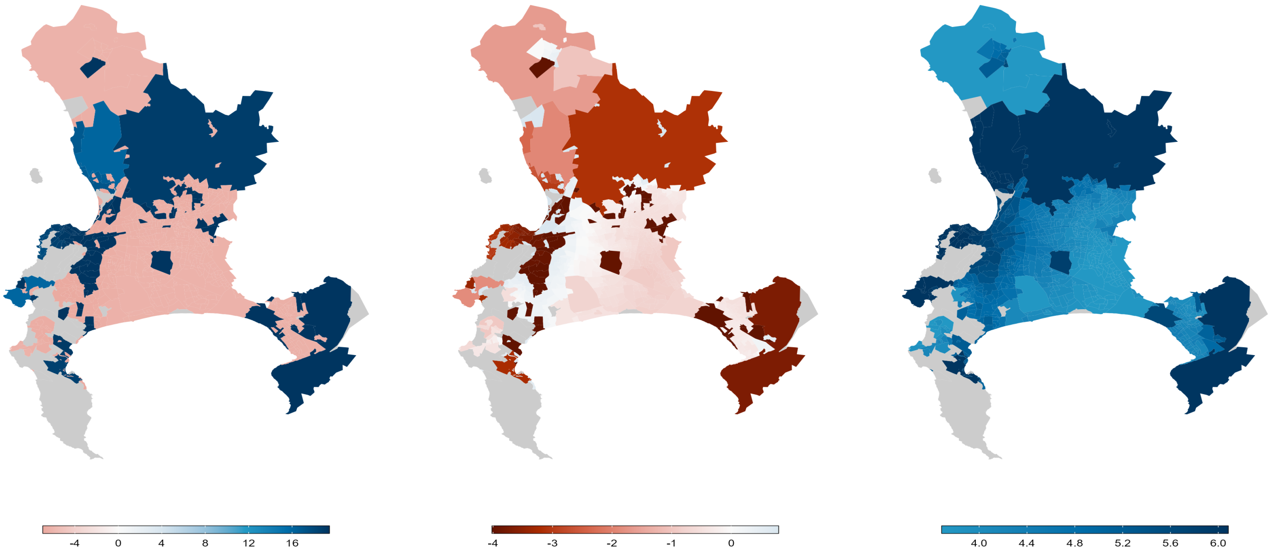
Figure 16: Increase in productivity - Closed City



(a) Residents

(b) Workers

(c) Wages



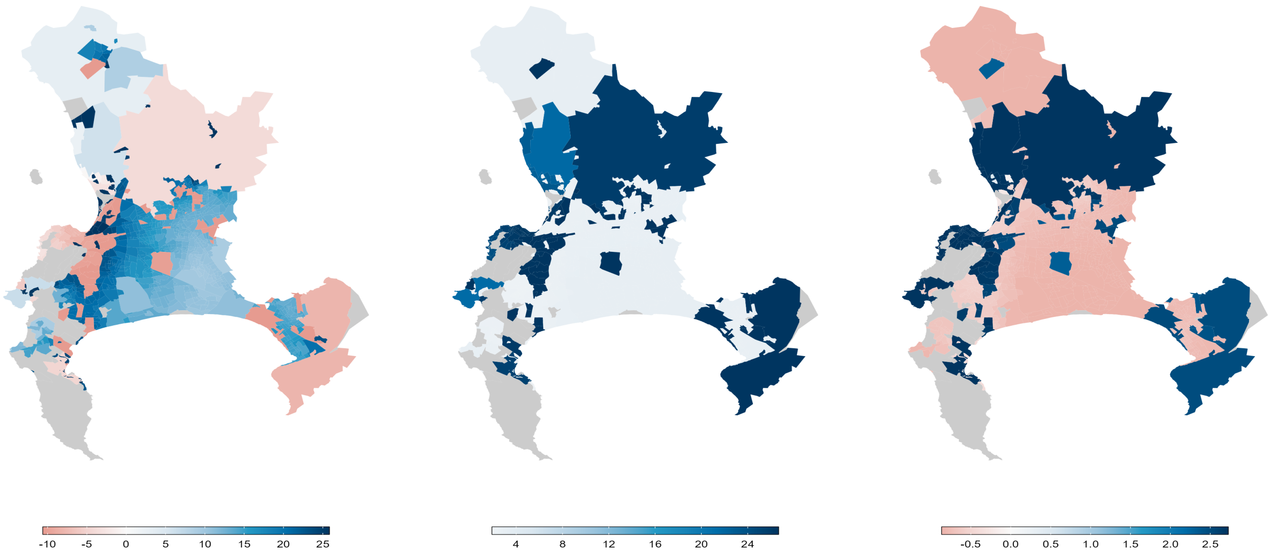
(d) Floor space prices

(e) FMA

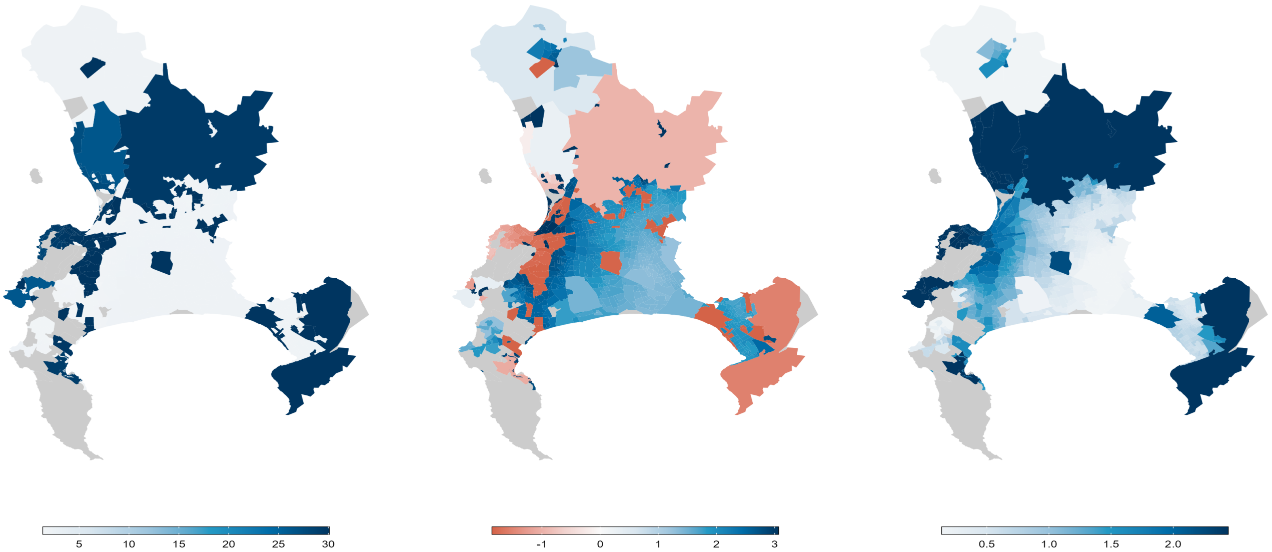
(f) CMA

Figure 17: Increase in productivity - Open City

(a) Residents (b) Workers (c) Wages



(d) Floor space prices (e) FMA (f) CMA



B Tables

Table 7: Paramters of the model

Parameter	**Description**	**Value**
α	Exp. share in the consumption good (the price is normalized to 1)	0.8
$1-\alpha$	Exp. share in housing	0.2
θ	Commuting and migration elasticity	7
β	Output elasticity w.r.t labor	0.7
$1-\beta$	Output elasticity w.r.t land	0.3
μ	Prod. function of floorspace, output elasticity w.r.t. capital	0.3
$1-\mu$	Prod. function of floorspace, output elasticity w.r.t. land	0.7
δ	Decay parameter agglomeration forces	0.3585
λ	Agglomeration externality	0.01
ρ	Decay parameter amenities	0.9094
η	Congestion force	0.02
ν	Convergence parameter of the contraction mapping	0.05

C Tecnhical description of the model

C.1 Model setup

- The travel times between two locations are transformed into commuting costs using the following equation:

$$d_{ij} = \exp(\epsilon\, \tau_{ij})$$

- All workers are ex-ante the same, but they differ in random utility draws that are drawn from an extreme value type distribution. The utility levels also depend on location amenities, wages, commuting costs, and housing prices. Therefore, the indirect utility of worker ω that lives in i and works in j is:

$$U_{ij\omega} = \frac{B_i w_j \epsilon_{ij\omega}}{d_{ij} Q_i^{1-\alpha}},$$

- By the properties of extreme value type shocks, the commuting share from location i to location j is:

$$\lambda_{ij|i} = \frac{w_j^{\theta} d_{ij}^{-\theta}}{\sum_r w_r^{\theta} d_{ir}^{-\theta}}$$

- The average income in location i is:

$$\bar{y}_i = \sum_j \lambda_{ij|i} w_j$$

- The share of workers that decide to live in location i is:

$$\lambda_i = \frac{B_i^{\theta} Q_i^{-\theta(1-\beta)} \sum_j w_j^{\theta} d_{ij}^{-\theta}}{\sum_l B_l^{\theta} Q_l^{-\theta(1-\beta)} \sum_r w_r^{\theta} d_{lr}^{-\theta}}$$

- All firms produce a homogeneous good whose price is normalized to 1. The production function in location j is:

$$Y_j = A_j L_{Mj}^{\beta} F_{Mj}^{1-\beta}$$

- The total floor space supplied in location i is:

$$F_i = \varphi_i K_i^{1-\mu}$$

- The floor space used for commercial purposes is:

$$F_{Mi} = \frac{(1-\beta)Y_i}{Q_i} \text{ if } A_i > 0$$

- The floor space used for resdiential purposes is:

$$F_{Ri} = \frac{(1-\alpha)\bar{y}_i L_{Ri}}{Q_i} \text{ if } B_i > 0$$

- The land market clearing condition implies

$$F_{Mi} + F_{Ri} = F_i$$

- The labor demand is:

$$L_{Mj} = \frac{\beta Y_j}{w_j}$$

- The labor market clearing condition implies

$$\frac{\beta Y_j}{w_j} = \sum_i L\lambda_i \lambda_{ij|i}$$

- There is an agglomeration force that affects total factor productivity in each location i. In particular:

$$A_i = a_i \Upsilon_i^{\lambda}, \quad \Upsilon_i = \sum_l \exp(-\delta t_{il}) \left(\frac{L_{Ml}}{K_l}\right)$$

- There is a congestion externality force that affects the amenity value in each location i:

$$B_i = b_i \Omega_i^{\eta}, \quad \Omega_i = \sum_l \exp(-\rho t_{il}) \left(\frac{L_{Rl}}{K_l}\right)$$

- Average welfare is normalized to a constant term:

$$\bar{U} = \gamma \left(\sum_i \sum_j B_i^{\theta} Q_i^{-\theta(1-\beta)} w_j^{\theta} d_{ij}^{-\theta}\right)^{\frac{1}{\theta}},$$

  where γ is a constant term.

Based on the previous set of equations the model can perform the inversion of the baseline equilibrium and the solving of other counterfactual equilibria.

C.2 Model Inversion

Wages

First, based on the number of residents L_{Ri} and workers L_{Mj}, commuting costs d_{ij}, and a set of exogenous parameters ϵ and θ the model makes a theoretical prediction of the number of workers by solving the following equation:

$$L_{Mj}^{\text{model}} = \sum_i L_{Ri} \frac{w_j^\theta d_{ij}^{-\theta}}{\sum_l w_l^\theta d_{il}^{-\theta}}$$

With this predicted number of workers, the model can compute the worker commuting probabilities that can be then used to solve for unique adjusted wages consistent with commuting market clearing. This is the algorithm used to recover the vector of wages (w_i) that matches the predicted (L_{Mj}^{model}) and the observed number of workers (L_{Mj})

1. Start with an initial vector of wages $\mathbf{w}_0$.
2. Construct the shares
$$\lambda_{ij|i}^1 = \sum_i L_{Ri} \frac{w_{j,0}^\theta d_{ij}^{-\theta}}{\sum_l w_{l,0}^\theta d_{il}^{-\theta}}$$
3. Construct the total employment implied by the model:
$$L_{Mj}^{\text{model},1} = \sum_i L_{Ri} \lambda_{ij|i}^1$$
4. New vector of wages:
$$w_{j,1} = \left(\frac{L_{Mj}}{L_{Mj}^{\text{model},1} / w_{j,0}} \right)^{\frac{1}{\theta}}$$
5. Update the new vector of wages using a contraction mapping:
$$w_{j,2} = \nu w_{j,1} + (1-\nu) w_{j,0}$$
6. Normalize the vector of wages such that the geometric mean is equal to 1.
$$\exp\left(\frac{1}{N} \sum_{i=1}^N \ln w_j \right) = 1$$
7. Repeat the algorithm until the difference $|\mathbf{w}_{r+1} - \mathbf{w}_r| < tol$, where tol is some tolerance factor.

Density of land development

The second step of the inversion process recovers the development density φ_i. Given data on: floorspace prices Q_i, wages w_i, number of workers L_{Mi}, number of residents L_{Ri}, and a set of exogenous parameters β, α, μ the model recovers the density development φ_i and the total amount of floorspace supplied by performing the following steps:

1. Normalize the housing prices:

$$\bar{Q} = \exp\left(\frac{1}{N}\sum_i \log Q_i\right)$$

$$\tilde{Q}_i = \frac{Q_i}{\bar{Q}}$$

2. Compute the residential and commercial floorspace:

$$F_{Mi} = \left(\frac{1-\beta}{\beta}\right)\left(\frac{w_i L_{Mi}}{\tilde{Q}_i}\right)$$

$$F_{Ri} = (1-\alpha)\frac{\bar{y}_i L_i}{\tilde{Q}_i}$$

3. Compute the total demand for floorspace and the share used commercially:

$$F_i = F_{Mi} + F_{Ri}$$

$$\tilde{\theta} = \frac{F_{Mi}}{F_i}$$

4. Recover the development density:

$$\varphi_i = \frac{K_i^{1-\mu}}{F_i}$$

5. Find the total amount of floorspace supplied:

$$F_i^{\text{supply}} = \varphi_i K_i^{1-\mu}$$

Productivity

The third step consists of recovering the productivity vector a_i. Given data on housing prices $\tilde{Q}_i$, wages w_i, workers L_{Mi}, available floorspace K_i, commuting costs d_{ij} and the set of parameters λ, δ, the model can recover the vector of adjusted productivity a_i by following these steps:

1. Recover the productivity vector A_i based on the FOC of the cost minimization problem of firms:

$$A_i = \beta^{-\beta}(1-\beta)^{-(1-\beta)}\tilde{Q}_i^{1-\beta}w_i^{\beta}$$

2. Construct the level of externalities:

$$\Upsilon_i = \sum_l \exp(-\delta t_{il})\left(\frac{L_{Ml}}{K_l}\right)$$

3. Recover the productivity term a_i

$$a_i = \frac{A_i}{\Upsilon_i}$$

4. Transform the productivity term to zero if there are no workers in location i

$$a_i = \frac{A_i}{\Upsilon_i} \times \mathbf{1}\{L_{Mi} > 0\}$$

Amenities

The fourth step of the model inversion consists of recovering the amenity distribution. Given data on the number of residents L_{Ri}, wages w_i, housing prices $\tilde{Q}_i$, and commuting costs d_{ij}, and a set of exogenous parameters θ, α, ρ, and η, the model can recover the vector of adjusted amenities b_i by performing the following steps:

1. Normalize the total number of residents

$$\bar{L} = \exp\left(\frac{1}{N}\sum_i \log L_i\right)$$

$$\tilde{L}_{Ri} = \frac{L_i}{\bar{L}}$$

2. Normalize the market access measure

$$W_i = \left(\sum_j w_j^{\theta} d_{ij}^{-\theta}\right)^{\frac{1}{\theta}}$$

$$\bar{W} = \exp\left(\frac{1}{N}\sum_i \log W_i\right)$$

$$\tilde{W}_i = \frac{W_i}{\bar{W}}$$

3. Recover the amenity vector B_i:

$$B_i = \left(\frac{\tilde{L}_{Ri}\tilde{Q}_{Ri}^{1-\alpha}}{\tilde{W}_i}\right)^{\frac{1}{\theta}}$$

4. Compute the congestion externalities:

$$\Omega = \sum_l \exp(-\rho t_{il})\left(\frac{L_{Rl}}{K_l}\right)$$

5. Recover the term b_i

$$b_i = \frac{B_i}{\Omega^{-\eta}}$$

6. Transform the amenity term to zero if there are no residents in location i:

$$b_i = \frac{B_i}{\Omega^{-\eta}} \times \mathbf{1}\{L_{Ri} > 0\}$$

C.3 Counterfactuals

After recovering the fundamentals of the economy (productivity, amenities, density of land development), the model can run counterfactual exercises that capture different urban policy interventions such as infrastructure investments that reduce travel times t_{ij}, revitalization efforts that improve the amenities b_i or innovations that improve productivity a_i.

For that purpose, what the model does is that given the set of exogenous fundamentals (a_i, b_i, t_{ij}, K_i) recovered in the inversion, it estimates the new levels of the endogenous variables $(\tilde{\theta}_i, w_i, Q_i, L_{Ri}, L_{Mi})$ by solving the following equations:

- Travel times to commuting costs:

$$d_{ij} = \exp(\epsilon\, t_{ij})$$

- The endogenous amenities:

$$\Omega_{i,1} = \sum_l \exp(-\rho t_{il}) \left(\frac{L_{Rl,0}}{K_l} \right)$$

$$B_i = b_i \Omega^{-\eta}$$

- The residential and employment shares:

$$\lambda_{ij,1} = \frac{B_i^\theta w_{j,0}^\theta Q_{i,0}^{-\theta(1-\alpha)} d_{ij}^{-\theta}}{\sum_l \sum_r B_l^\theta w_{r,0}^\theta Q_{l,0}^{-\theta(1-\alpha)} d_{lr}^{-\theta}}$$

- The conditional commuting shares:

$$\lambda_{ij|i,1} = \frac{w_{j,0}^\theta d_{ij}^{-\theta}}{\sum_r w_{r,0}^\theta d_{ir}^{-\theta}}$$

- The total number of workers:

$$L_{Mj,1} = \sum_i \lambda_{ij|i,1} L_i = \sum_i \lambda_{ij,1} \bar{L}$$

- Productivity and agglomeration externalities:

$$\Upsilon_{i,1} = \sum_l \exp(-\delta t_{il}) \left(\frac{L_{Ml,0}}{K_l} \right)$$

$$A_{i,1} = a_i \Upsilon_{i,1}^\lambda$$

- Total output in each location:

$$Y_{i,1} = A_{i,1} (L_{Mi,1})^\beta (\tilde{\theta}_{i,0} \varphi_i K_i^{1-\mu})^{1-\alpha}$$

- Average income and commuter market access:

$$\bar{y}_{i,1} = \sum_j \lambda_{ij|i,1} w_{j,0}$$

$$W_{i,1} = \left(\sum_j w_{j,0}^\theta d_{ij}^{-\theta} \right)^{\frac{1}{\theta}}$$

Finally, from these latter variables, the model can re-estimate the endogenous variables:

- Estimate number of residents:

$$L_{Ri,1} = \sum_j \lambda_{ij} \bar{L}$$

- Estimate wages:

$$w_{i,1} = \frac{\beta Y_{i,1}}{L_{Mi,1}}$$

- Estimate housing prices:

$$Q_{i,1} = \frac{(1-\beta)Y_{i,1}}{\tilde{\theta}_{i,0}\varphi_i K_i^{1-\mu}} \text{ if } a_i > 0, b_i = 0 \text{ or } a_i > 0, b_i > 0$$

$$Q_{i,1} = \frac{(1-\alpha)\bar{y}_{i,1}L_{i,1}}{(1-\tilde{\theta}_{i,0})\varphi K_i^{1-\mu}} \text{ if } a_i = 0, b_i > 0 \text{ or } a_i > 0, b_i > 0$$

- Estimate the share of floorspace used commercially:

$$\tilde{\theta}_{i,1} = 1 \quad \text{if} \quad a_i > 0, b_i = 0$$

$$\tilde{\theta}_{i,1} = 0 \quad \text{if} \quad a_i = 0, b_i > 0$$

$$\tilde{\theta}_{i,1} = \frac{(1-\beta)Y_{i,1}}{Q_{i,1}\varphi_i K_i^{1-\mu}} \quad \text{if} \quad a_i > 0, b_i > 0$$

The model iteratively repeats this updating process until each of the endogenous variables converges across the iterations, which means that the economy reaches a new equilibrium state. To do so, the model performs the following algorithm:

1. Start with an initial vector of endogenous variables $\{\tilde{\theta}_{i,0}, w_{i,0}, Q_{i,0}, L_{Ri,0}\}$
2. Compute the new vector of endogenous variables $\{\tilde{\theta}_{i,1}, w_{i,1}, Q_{i,1}, L_{Ri,1}\}$
3. Update the variables

$$L_{Ri,2} = \nu L_{Ri,0} + (1-\nu)L_{Ri,1}$$

$$w_{i,2} = \nu w_{i,0} + (1-\nu)w_{i,1}$$

$$Q_{i,2} = \nu Q_{i,0} + (1-\nu)Q_{i,1}$$

$$\tilde{\theta}_{i,2} = \nu\tilde{\theta}_{i,0} + (1-\nu)\tilde{\theta}_{i,1}$$

4. Compute the difference between the endogenous variables for the simulations:

$$z_{i,x} = |x_{i,1} - x_{i,0}|,$$

   where x corresponds to $L_{Ri}, w_i, Q_i, \tilde{\theta}_i$.

5. Repeat the algorithm until:

$$\max(\mathbf{z}_L, \mathbf{z}_w, , \mathbf{z}_Q, , \mathbf{z}_{\tilde{\theta}}) < tol$$

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