Quantitative Urban Models

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Motivation

- Complexity of modelling spatial interactions implies that theoretical urban literature has traditionally focused on stylized settings
  - Monocentric city
  - Symmetric locations
  - Hard to connect directly with observed data

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  - Capture first-order features of the data, such as many locations, heterogeneous productivity and amenities, and transport networks
  - Small number of structural parameters to estimate
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- Focus of this lecture is on quantitative urban models
  - Focus on internal city structure (network within a single city)
  - Contrast with quantitative spatial models concerned with systems of cities or regions (network between cities of regions)
  - Main new margins are separation of residence from workplace and locations of consumption

Outline

1. Traditional Theoretical Literature
2. Baseline Quantitative Urban Model
3. Applications
4. Conclusions

Traditional Theoretical Literature

- Alonso-Muth-Mills monocentric city model is the traditional theoretical model of internal city structure
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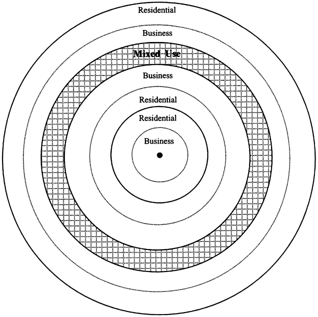
- Path-breaking theoretical models of non-monocentric cities
  - Fujita and Ogawa (1982) (linear city)
  - Lucas and Rossi-Hansberg (2002) (symmetric circular city)



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Quantitative Urban Model

- Model of the equilibrium distribution of residents, workers and land rents across locations within a city following Ahlfeldt et al. (2015)
- Rationalize observed data on thousands of city blocks
  - Employment by workplace and by residence
  - Or data on bilateral commuting flows
  - Land rents
  - Bilateral transport network and travel times
- Capture empirically relevant differences across locations in
  - Productivity
  - Amenities
  - Supply of floor space
  - Transportation infrastructure
- Endogenous agglomeration and dispersion forces
  - Production externalities
  - Residential externalities
  - Supply of floor space
  - Commuting costs

Preferences

- Utility for worker ψ from occupation o in residence n and workplace i

$$U_{ni}(\omega) = \frac{B_n b_{ni}(\omega) w_i}{\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}}, \qquad 0 < \alpha < 1$$

- with idiosyncratic preferences $b_{ni}(\omega)$, amenities B_n, wage w_i, goods price P_n, price of residential floor space Q_n, and commuting costs κ_{ni}

$$\kappa_{ni} = e^{\kappa \tau_{ni}} \in [1, \infty)$$

- Idiosyncratic preferences

$$G(z) = e^{-b^{-\epsilon}}, \qquad \epsilon > 1$$

- Residential amenities (B_n) depend on residential fundamentals ($\overline{B}_n$) and residential externalities ($\mathbb{B}_n$)

$$B_n = \overline{B}_n \mathbb{B}_n^{\eta^B}, \qquad \mathbb{B}_n \equiv \sum_{i \in \mathbb{N}} e^{-\delta^B \tau_{ni}} R_i$$

Residence-Workplace Choices

- Probabilistic sorting across residence-workplace pairs

$$\lambda_{ni} = \frac{L_{ni}}{L_{\mathbb{N}}} = \frac{(B_n w_i)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum\limits_{k \in \mathbb{N}} \sum\limits_{\ell \in \mathbb{N}} (B_k w_{\ell t})^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}}$$

- Residents (R_n) and employment (L_i)

$$\lambda_n^R = \frac{R_n}{L_{\mathbb{N}}} = \sum_{i \in \mathbb{N}} \lambda_{ni}, \qquad \lambda_i^L = \frac{L_i}{L_{\mathbb{N}}} = \sum_{n \in \mathbb{N}} \lambda_{ni}$$

- Uutility equalized across residence-workplace pairs

$$U = \vartheta \left[\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} (B_k w_{\ell})^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon} \right]^{\frac{1}{\epsilon}}, \quad \vartheta \equiv \Gamma\left(\frac{\epsilon - 1}{\epsilon}\right)$$

- Commuter market clearing condition

$$L_i = \sum_{n \in \mathbb{N}} \lambda_{ni|n}^R R_n = \sum_{n \in \mathbb{N}} \frac{(w_i / \kappa_{ni})^{\epsilon}}{\sum_{\ell \in \mathbb{N}} (w_{\ell} / \kappa_{n\ell})^{\epsilon}} R_n$$

Production

- Single final good produced using labor and floor space under conditions of perfect competition and costless trade

$$1 = \frac{1}{A_i} w_i^{\beta} q_i^{1-\beta}$$

- where q_i is the price of commercial floor space
- Productivity (A_i) depends on production fundamentals ($\overline{A}_n$) and production externalities ($\mathbb{A}_n$)

$$A_n = \overline{A}_n \mathbb{A}_n^{\eta^A} \qquad\qquad \mathbb{A}_n \equiv \sum_{i \in \mathbb{N}} e^{-\delta^A \tau_{ni}} L_i$$

Floor Space Clearing

- Given supplies of residential and commercial floor space (H_n^R, H_n^L), prices of floor space (Q_n, q_n) floor space are determined as:

$$Q_n = \frac{(1-\alpha)\, v_n R_n}{H_n^R}$$

$$q_n = \frac{1-\beta}{\beta} \frac{w_n L_n}{H_n^L}$$

- where

$$v_n = \sum_{i \in \mathbb{N}} \lambda_{ni|n}^R w_i = \sum_{i \in \mathbb{N}} \frac{(w_i / \kappa_{ni})^{\epsilon}}{\sum_{\ell \in \mathbb{N}} (w_\ell / \kappa_{n\ell})^{\epsilon}} w_i$$

Uniqueness

- The general equilibrium spatial distribution of economic activity within the city is determined by the model parameters (α, β, κ, ϵ, η^B, δ^B, η^A, δ^A) and the following exogenous location characteristics: residential fundamentals ($\overline{B}_n$), production fundamentals ($\overline{A}_n$), the supplies of residential and commercial floor space (H_n^R, H_n^L), and the transport network (τ_{ni})
- Given these parameters and exogenous location characteristics, the closed-city general equilibrium of the model is referenced by residents (R_n), employment (L_n), wages (w_n), average residential income (v_n), the prices of residential and commercial floor space (Q_r, q_r), and expected utility (U), given exogenous total city population ($L_{\mathbb{N}}$)
- Conditions for general equilibrium can be written in the form required to apply Theorem 1 from Allen et al. (2024) for uniqueness:

$$x_{nh} = f_{nih}(x_i) = \sum_{i \in \mathbb{N}} \mathcal{K}_{nih} \prod_{h' \in \mathbb{H}} x_{ih'}^{\gamma_{nhh'}}$$

- Spectral radius of a matrix of parameters less than or equal to one

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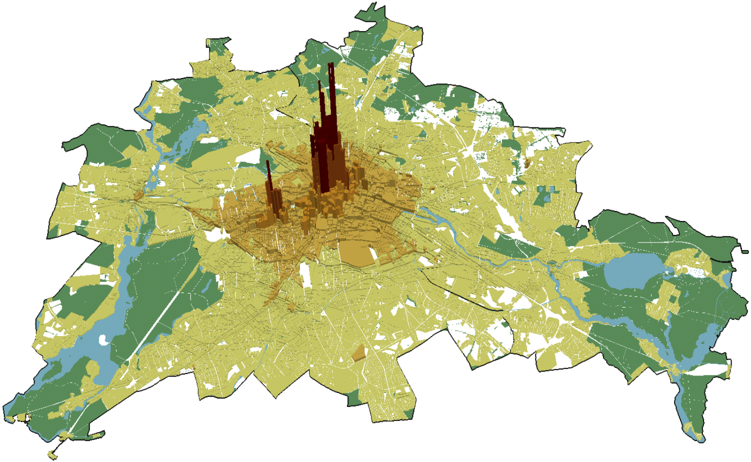
Applications

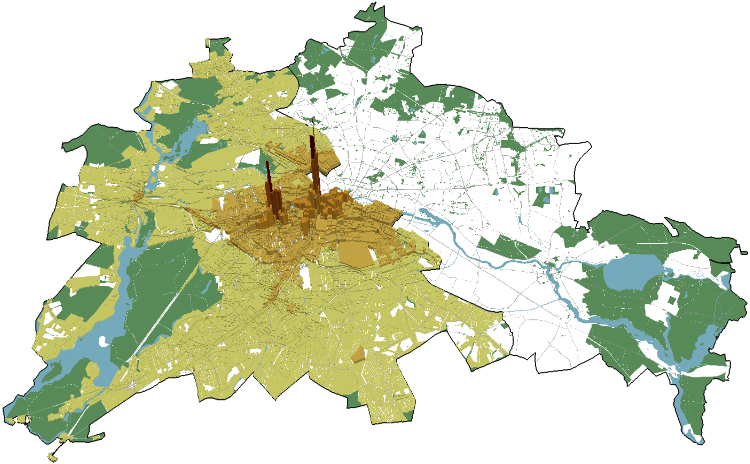
- Consider two different applications of quantitative urban models
- Structural estimation of agglomeration and dispersion forces
  - Ahlfeldt, Gabriel, Stephen J. Redding, Daniel M. Sturm and Nikolaus Wolf (2015) "The Economics of Density: Evidence from the Berlin Wall," *Econometrica*, 83(6), 2015, 2127-2189.
- Quantify the impact on transport infrastructure on the spatial distribution of economic activity within cities
  - Heblich, Stephan, Stephen J. Redding and Daniel M. Sturm (2020) "The Making of the Modern Metropolis: Evidence from London," *Quarterly Journal of Economics*, 135(4), 2020, 2059-2133.

The Economics of Density: Evidence from the Berlin Wall

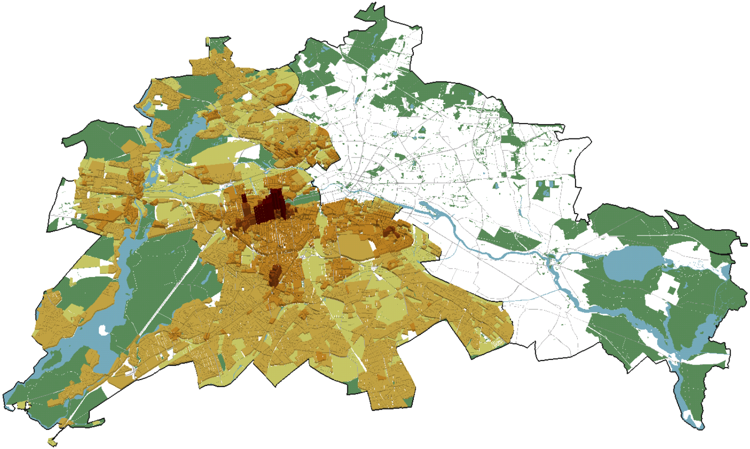
Berlin 1936



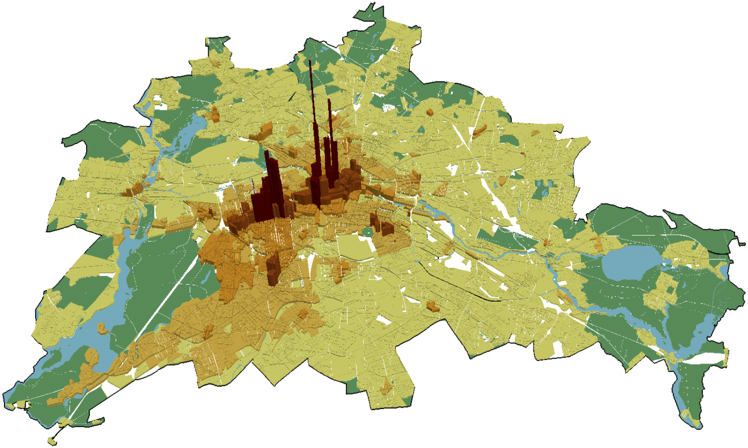
West Berlin 1936



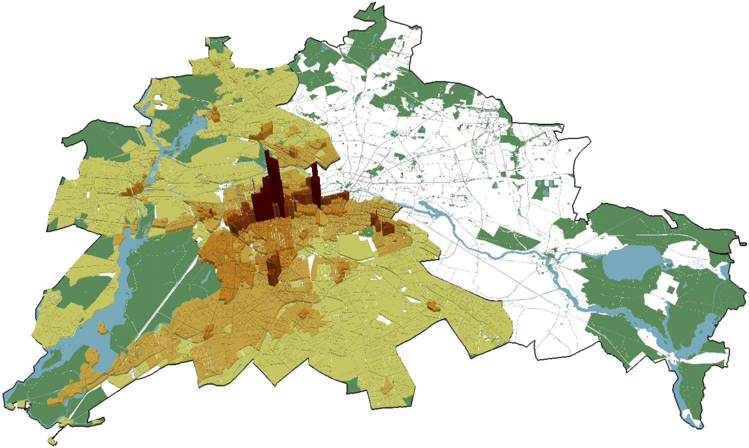
West Berlin 1986



Berlin 2006



West Berlin 2006



Parameters

Assumed Parameter		Source	Value
Residential land	$1-\beta$	Morris-Davis (2008)	0.25
Commercial land	$1-\alpha$	Valentinyi-Herrendorf (2008)	0.20
Fréchet Scale	T	(normalization)	1
Expected Utility	$\bar{u}$	(normalization)	1000

Estimated Parameter	
Production externalities elasticity	λ
Production externalities decay	δ
Residential externalities elasticity	η
Residential externalities decay	ρ
Commuting semi-elasticity	$\nu = \epsilon\kappa$
Commuting heterogeneity	ϵ

Estimated Parameters

	(1) Division Efficient GMM	(2) Reunification Efficient GMM	(3) Division and Reunification Efficient GMM
Commuting Travel Time Elasticity ($\kappa\varepsilon$)	0.0951***	0.1011***	0.0987***
	(0.0016)	(0.0016)	(0.0016)
Commuting Heterogeneity (ε)	7.6278***	7.7926***	7.7143***
	(0.1085)	(0.1152)	(0.1049)
Productivity Elasticity (λ)	0.0738***	0.0449***	0.0657***
	(0.0056)	(0.0071)	(0.0048)
Productivity Decay (δ)	0.3576***	0.8896***	0.3594***
	(0.0945)	(0.3339)	(0.0724)
Residential Elasticity (η)	0.1441***	0.0740***	0.1444***
	(0.0080)	(0.0287)	(0.0073)
Residential Decay (ρ)	0.8872***	0.5532	0.7376***
	(0.2774)	(0.3699)	(0.1622)

Note: Generalized Method of Moments (GMM) estimates. Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors in parentheses (Conley 1999). * significant at 10%; ** significant at 5%; *** significant at 1%.

Localized Externalities

	(1) Production Externalities ($1 \times e^{-\delta\tau}$)	(2) Residential Externalities ($1 \times e^{-\rho\tau}$)	(3) Utility after Commuting ($1 \times e^{-\kappa\tau}$)
0 minutes	1.000	1.000	1.000
1 minute	0.698	0.478	0.987
2 minutes	0.487	0.229	0.975
3 minutes	0.340	0.109	0.962
5 minutes	0.166	0.025	0.938
7 minutes	0.081	0.006	0.914
10 minutes	0.027	0.001	0.880
15 minutes	0.005	0.000	0.825
20 minutes	0.001	0.000	0.774
30 minutes	0.000	0.000	0.681

Note: Proportional reduction in production and residential externalities with travel time and proportional reduction in utility from commuting with travel time. Travel time is measured in minutes. Results are based on the pooled efficient GMM parameter estimates: δ=0.3594, ρ=0.7376, κ=0.0128.

- Matching the estimated impact of division and reunification requires strong and localized agglomeration forces

The Making of the Modern Metropolis: Evidence from London

Empirical Setting

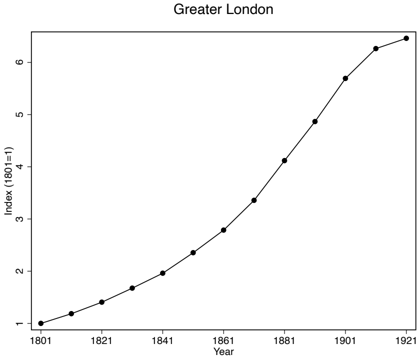
- 19th-century London is the poster child for large metropolitan areas
  - In 1801, around 1 million people, and a walkable city 5 miles E-W
  - By 1901, over 6.5 million people, 17 miles from E-W, and the metropolis that we would recognize today

- Major change in transport technology during the 19th century
  - First steam railways haul freight at mines (Stockton-Darlington 1825)
  - First dedicated passenger steam railway (London and Greenwich 1836)

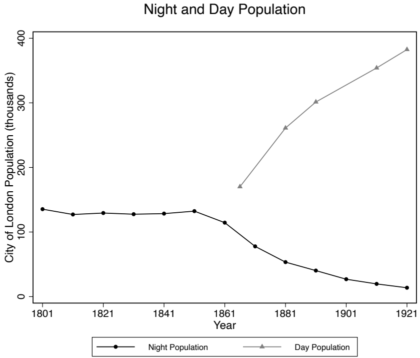
- Estimation methodology uses bilateral commuting data at the end of our sample and undertakes comparative statics back in time
  - Observe historical data on employment by residence and land values
  - Recover missing data on employment by workplace using the model

- Quantitative analysis has a recursive structure
  - In initial steps, predictions for employment by workplace use only gravity and commuter and land market clearing
  - In later steps, use more of the model's structure to recover productivity, amenities and floor space and undertake counterfactuals

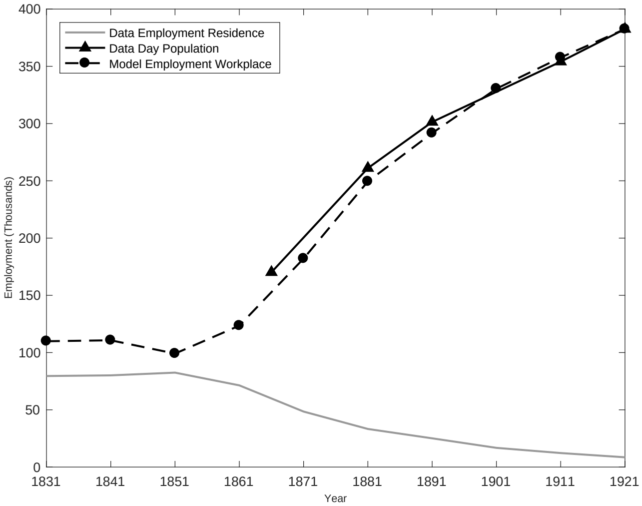
Residential (Night) Population



Day and Night Population



Workplace Employment



- Model captures sharp concentration of workplace employment in the City of London from 1860s onwards

Looking Ahead

- Many exciting areas of theoretical research
  - Dynamics of the spatial distribution of economic activity
  - Heterogeneous agent models
  - Optimal transport infrastructure investments
  - Political economy infrastructure investments
  - Microfoundations for endogenous amenities
  - Microfoundations of agglomeration forces
  - Reorganization of cities in the wake of COVID-19, the shift to remote/hybrid working, and the growth of online retail
- Wealth of Geographical Position System (GPS) data
  - Ride-hailing data (e.g. Uber and Lyft)
  - Smartphone data
  - Firm-to-firm data from sales (VAT) tax records
  - Credit card data with consumer and firm location
  - Barcode scanner data with consumer and firm location
  - Public transportation commuting data (e.g. Oyster card)
  - Satellite imaging data

Thank You