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Macroeconomics Track

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Balance of Payments Crises

1 Introduction

Open goods markets come with many positives including access to cheaper goods, better technology, better management methods, etc.. Open capital accounts though are more of a mixed bag. One the one hand, capital mobility allows countries to access cheaper sources of credit or find more profitable avenues for investment. In as much as FDI brings in new investment, capital mobility also engenders productivity gains. However, capital mobility also brings with it the potential downside instability. In particular, capital that is invested in financial assets is often prone to sudden turnarounds. This volatility of capital flows often cause currency crises, banking instability as well as debt crises. In these lectures we will analyze these crisis episodes focusing on their origins, their consequences and the possible policy responses.

Currency crises typically refer to episodes where the exchange rate depreciates very sharpy in a very short period of time. These episodes typically occur in countries which have fixed exchange rates or heavily manage their exchange rates. There is a voluminous literature on this topic. Work on these episodes are broadly classified into first-generation models, second-generation models and third-generation models. We shall analyze these, in turn.

2 First-generation models

First-generation currency crisis models explain these episodes as the outcome of fiscal policies that are incompatible with the exchange rate policy. In particular, these models posit that a fiscal deficit induces the monetary authority to finance the gap through money printing. This strategy runs into trouble due to the phenomenon called the *trilemma*: open economies can simultaneously choose only two out of the three policy targets of control over the exchange rate, monetary policy autonomy and free capital mobility. In an environment with frictionless capital flows across borders, trying to expand money supply to support a fiscal deficit causes the fixed exchange rate regime to become unsustainable and ends eventually in a currency crisis.

The first-generation models that explain currency crises were inspired by the work of Krugman (1979) who himself had built on the work of Salant and Henderson (1978). In this lecture we study a version of this model originally due to Calvo (1987).

Consider the standard small open economy model that we have been analyzing. The infinitely lived representative agent maximizes lifetime welfare given by

$$V = \int_{t=0}^{\infty} e^{-\rho t} u(c) dt$$

subject to the flow constraint given by

$$\dot{b} = rb + y + g - c - \dot{m} - \pi m - s(m)$$

where we have suppressed time subscripts to economize on notation. $s(m)$ denotes transactions costs faced by households. Assume that $s' < 0, s'' > 0$. Hence, transactions costs are strictly decreasing and strictly convex in m. Using the definition $a = b + m$ we can rewrite the flow constraint as:

$$\dot{a} = ra + y + g - c - s(m) - im \tag{2.1}$$

The first-order conditions are

$$\begin{aligned} u'(c) &= \lambda \\ \dot{\lambda} &= (\rho - r)\lambda \\ -s'(m) &= i \end{aligned}$$

We continue to assume that $\rho = r$ throughout. Hence, λ is constant over time which implies that c is constant over time. Note that along all paths with a constant i, real money balances must be constant as well.

The government prints money, holds reserves and makes lump-sum transfers. We assume that the government has a fixed level of domestic spending $\bar{g}$ which is invariant over time. Hence, the government's flow constraint is given by

$$\dot{R} = rR + \dot{m} + \pi m - \bar{g}.$$

We also assume that the central bank announces a path for the nominal exchange rate. In particular, it picks a fixed exchange rate for the domestic currency:

$$E_t = \bar{E} \qquad \text{for all } t.$$

As we noted earlier, a fixed exchange rate regime entails that the central bank stands ready to buy and sell foreign currency in exchange for domestic currency at the pre-announced exchange rate. But this requires the central bank to have adequate foreign reserves at all times to defend the announced fixed exchange rate regime.

The law of one price combined with the fixed exchange rate implies that

$$\pi_t = 0 \qquad \text{for all } t.$$

Hence, the flow constraint for the government reduces to

$$\dot{R} = rR + \dot{m} - \bar{g}.$$

Since $\dot{R} = \dot{m} - \dot{d}$ from the central bank balance sheet, it follows that

$$\dot{d} = \bar{g} - rR.$$

Note that one can integrate this flow constraint for the government to get the lifetime budget constraint

$$\bar{g} = rR_0 - rm_0 + r\int_{t=0}^{\infty} e^{-rt} i_t m_t dt \tag{2.2}$$

In order to generate a currency crisis in these models, one needs two key assumptions. First, assume that

$$R_0 < \frac{\bar{g}}{r} \equiv \bar{R}.$$

This assumption implies that at date $t = 0$ the government has a budget deficit. Second, assume that the fixed exchange rate regime will stay in place as long as foreign exchange reserves with the central bank are positive. Everyone expects the government to abandon the fixed exchange rate regime and switch to a flexible exchange rate regime as soon as foreign reserves hit zero. Moreover, the flexible exchange rate regime will be characterized by the central bank choosing to expand nominal domestic credit at the constant rate μ_T.[1]

[1] This lower level of zero reserves is a specific statement about a more general constraint that is being imposed here. In particular, the crucial restriction for all these currency crises results to go through is that there exists some finite lower level for reserves below which the central bank will not allow reserves to fall.

The first assumption implies that $\dot{d} > 0$ as long as the fixed exchange rate regime is in place. This has two implications. First, m is constant as $i = r$ under a fixed exchange rate regime. $\dot{d} > 0$ along with $\dot{m} = 0$ implies that $\dot{R} < 0$ from the central bank balance sheet. Hence, reserves must be falling secularly during the fixed exchange rate regime. This leads to the second implication. In particular, our second assumption above says that the fixed exchange rate regime will stay in place only as long as reserves are non–negative. Since reserves are falling throughout, this implies that the fixed exchange rate regime must collapse in finite time.

2.1 Post-collapse dynamics

To determine the dynamics of a currency crisis in this model, it useful to first determine the dynamics of the economy contingent on a collapse, i.e., the dynamics post-collapse. Throughout we shall denote the collapse date by T. Of course, we shall solve for T at the end. The nominal interest rate post-collapse is given by

$$i_t = r + \pi_t \qquad \text{for all } t \geq T$$

while $\dot{m}_t = (\mu_T - \pi_t)\, m_t$. Substituting this into the first order condition for money balances we get

$$\frac{\dot{m}_t}{m_t} = r + \mu_T + s'(m_t)$$

This is the equilibrium differential equation which governs the dynamics of m in this economy. Note that λ is constant since $\rho = r$. It is easy to check that this is an unstable differential equation (recall that $s'' > 0$). Hence, unless m jumps to its steady state level instantaneously it will never get there. Hence, $\dot{m} = 0$ throughout in the post-collapse phase. This, in turn, means that $\pi = \mu_T$ post-collapse.

The preceding leaves open the question of what is the equilibrium rate of money growth after the collapse of the fixed exchange rate regime? The answer can be determined from the government flow constraint. Recall, that the fixed exchange regime collapses when R hits its lower level 0. Moreover, after a collapse the central bank shifts to a flexible exchange rate regime. Under a flexible exchange rate regime the central bank does not sell any reserves. Hence, $\dot{R} = 0$. Combining this with the fact that $R = 0$ after the collapse implies that the government's flow constraint in the post-collapse phase is

$$\dot{m}_t = \bar{g} - \pi_t m_t$$

Since $\dot{m} = 0$, this implies that the constant steady state rate of money growth after the collapse at date T is given by

$$\mu_T = \frac{\bar{g}}{m_T}$$

where m_T is the steady state level of real money balances and μ_T is the constant rate of domestic credit growth post-collapse.

Lastly, note that the first-order condition for optimal money demand implies that

$$\frac{-s'(m_{T_-})}{-s'(m_T)} = \frac{r}{r + \mu_T} < 1 \tag{2.3}$$

where $T-$ is the instant before the collapse. Hence, real balances must fall at the date of the collapse, i.e., $m_{T_-} > m_T$. Since i is constant (and equal to r) prior to the collapse, we also have $m_{T_-} = m_0$.

2.2 Crisis dynamics

On the date of the crisis private agents must reduce their money holding by exchanging them for foreign bonds at the going exchange rate. The peg collapses because the aggregate loss of central bank reserves drives international reserves with the central bank to zero. This episode is usually referred to as a speculative attack on reserves.

So, what is the size of the speculative attack? It is given by

$$\Delta R_T = m_{T-} - m_T$$

In the context of our perfect foresight model, the speculative attack must happen on the precise date at which a run will wipe out all remaining reserves. Moreover, on the date of the crisis there cannot be a jump in E. Since this is a perfect foresight economy, a jump in E at date T would be perfectly anticipated by all agents. Hence, there would be perfectly anticipated capital losses which cannot be an equilibrium. Hence, the fall in real money balances will happen through a decline in nominal money balances rather than a jump up in E.

From the central bank balance sheet we know that $R = m - d$. Real domestic credit at date T is given by

$$d_T = d_0 e^{\int_{t=0}^{T} \mu_t dt}$$

where μ_t denotes the rate of growth of nominal domestic credit at every date $t < T$. To solve for μ_t at each date we can combine the central balance sheet and the government flow budget constraint prior to the collapse to get

$$\mu_t = \frac{\bar{g} - rR_t}{m_0 - R_t}.$$

Collecting results, thus far we know the following: Consumption is constant pre and post-collapse. Real money balances are also constant pre and post collapse but with $m_0 > m_T$. Reserves fall secularly prior to the collapse at date T with $R = 0$ after the collapse.

The key undetermined variable thus far is the date of the collapse T. To solve for this note that a successful attack must wipe out reserves at date T. Reserves an instant before the crisis are given by R_{T-}. Since the speculative attack involves exchanging money balances for reserves (while E remains unchanged), a successful attack must imply

$$m_0 - m_T = R_{T-}.$$

where m_0 and m_T are determined from the first order condition for money: $-s'(m_0) = r$ and $-s'(m_T) = r + \mu_T$.

One can now use the lifetime budget constraint for the government to get

$$\bar{g} = rR_0 + e^{-rT}\left[(r + \mu_T)\, m_T - rm_0\right]$$

Since m_0 and m_T are functions of r and $r + \mu_T$, we can substitute those solutions into the above and then solve the resultant equation for T. In particular,

$$e^{rT} = \frac{(r + \mu_T)\, m_T - rm_0}{\bar{g} - rR_0}$$

Hence, the bigger the initial deficit the sooner that attack must happen.

Note that the attack cannot happen before T since prior to that date reserves exceed the size of the run. Hence, an attack cannot drive reserves to zero which means that the attack cannot succeed. The attack cannot happen any later than date T. If it did then some agents would not get any dollars in exchange for their domestic money balances. Hence, they would suffer a capital loss on their assets. In anticipation of these capital losses all agents would attack sooner.

What is the steady state level of consumption? To determine this, combine the flow constraints of the government and households to get

$$\dot{f} = rf + y - c - s(m).$$

Since c is constant before and after the collapse while y is constant, we can integrate this equation to get

$$c_0 = rf_0 + y - \left(1 - e^{-rT}\right) s(m_0) - e^{-rT} s(m_T) \tag{2.4}$$

Given a specific transaction costs function, one can solve for m_0 and m_T as functions of r and $r + \mu_T$. Combining this with the resource constraint gives the solution for c_0. Recall that μ_T is determined from the government flow constraint post-collapse along with the requirement that the government have a balanced budget post-collapse.

2.3 Interest rate defense

The model above had the slightly unrealistic characteristic that the government/central bank just stood around and watched its reserves dwindle to zero over time. Typically, when an exchange rate peg is under attack, central banks fight it tooth and nail. The standard instrument that they use to do the fighting is the domestic interest rate. In particular, during attacks the first line of defense employed by central banks is to raise domestic interest rates.[2] Here we follow Lahiri and Vegh (2003) and formalize a type of interest rate defense.

We change the model slightly here. In particular, here we shall assume that the government chooses a constant rate of domestic credit growth throughout, i.e.,

$$\frac{\dot{D}_t}{D_t} = \mu \qquad \text{for all } t.$$

The central bank continues to fix the exchange rate as before and continues to do so as long as central bank reserves are positive. If and when reserves go to zero the peg is abandoned

[2] On IMF policies, it is worth quoting from Stan Fischer's (1998) remarks on the IMF-supported programs in Asia in the late 1990s: "Are the programs too tough? In weighing this question, it is important to recall that when they approached the IMF, the reserves of Thailand and Korea were perilously low, and the Indonesian rupiah was excessively depreciated. Thus, the first order of business was, and still is, to restore confidence in the currency. To achieve this, countries have to make it more attractive to hold domestic currency, which, in turn, requires increasing interest rates temporarily, even if higher interest costs complicate the situation of weak banks and corporations. This is a key lesson of the tequila crisis in Latin America 1994-95, as well as from the more recent experience of Brazil, the Czech Republic, Hong Kong and Russia, all of which have fended off attacks on their currencies in recent months with a timely and forceful tightening of interest rates along with other supporting policy measures. Once confidence is restored, interest rates can return to more normal levels."

Jeff Sachs, in particular, has been a loud critic of the IMF's high interest rates policies in Asia. In a New York Times article (November 3, 1997), for example, he states that "[the] International Monetary Fund has just announced a second bailout package for the region, about $20 billion for Indonesia. That should, in principal, boost confidence. But if it is tied to orthodox financial conditions, including budget cuts and sharply higher interest rates, the package could do more harm than good, transforming a currency crisis into a rip-roaring economic downturn."

and the central bank shifts to a flexible exchange rate regime.

We now introduce an additional policy instrument for the government. Suppose the central bank can pay interest on money, $i^g < i$. We assume that the policy interest rate is set to zero during the life of the peg, i.e., $i_t^g = 0$ for all $t < T$ where T is the date of the speculative attack. We also assume that the government announces the following rule for the domestic interest rate:

$$\Delta i^g = \gamma \Delta i \tag{2.5}$$

The announced rule says that the government raises the domestic interest by a proportion γ of any change in the market nominal interest rate. Note that $\gamma = 0$ implies the case that we analyzed before – the passive interest rate policy case – while $\gamma = 1$ is the polar opposite case wherein the government raises the domestic interest rate one-for-one with changes in the market nominal interest rate. Now the fiscal authority is passive and chooses fiscal transfers to accommodate the monetary authority's chosen rate of domestic credit growth. The government's flow constraint is

$$\dot{R} = rR + \dot{m} + (\pi - i^g)m - g.$$

The flow constraint of private households is given by

$$\dot{b} = rb + y + g - c - \dot{m} - (\pi - i^g)\, m - s(m)$$

where i^g is the interest paid on real balances. Using the definition $a = b + m$ we can rewrite the flow constraint as:

$$\dot{a} = ra + y + g - c - s(m) - (i - i^g)m \tag{2.6}$$

The first-order conditions are

$$\begin{aligned} u'(c) &= \lambda \\ \dot{\lambda} &= (\rho - r)\, \lambda \\ -s'(m) &= i - i^g \end{aligned}$$

A key feature of the model now is that the path of real domestic credit is given exogenously since μ is fixed and the central bank pegs the exchange rate. Specifically, $\dot{d}_t = \mu d_t$ for all

$t < T$ where T is the date of the crisis. The central bank balance sheet then implies that $\dot{R} = -\dot{d} < 0$ since m is constant pre-collapse. Hence, reserves go to zero in finite time and a collapse is a probability one event.

Given that the peg is unsustainable, what is the implied path for the opportunity cost of money, $i - i^g$? This is given by

$$i_t - i_t^g = \begin{cases} r & \text{for } t < T \\ r + \mu(1-\gamma) & \text{for } t \geq T \end{cases}$$

Note that the economy is stationary from T onward. It is easy to check that in the stationary economy post-collapse we must have $\pi = \mu$ at all times. Hence, the nominal interest rate post-collapse must be $r + \mu$. So, the path for real money balances must be given by

$$m_t = \begin{cases} L[r], & t < T \\ L[r + \mu(1-\gamma)], & t \geq T \end{cases}$$

Clearly, for all $\gamma \leq 1$ we must have $m_T \leq m_0$.

2.4 Special case: $\gamma = 1$

It is clear from the path of money balances that if $\gamma = 1$ then $m_T = m_0$. Since a speculative attack occurs through a run on reserves which is effected by private agents swapping their domestic money balances, the case of $\gamma = 1$ must correspond to a case where there is no run since money balances post collapse are the same as real balances pre-collapse. Effectively, this case corresponds to a case wherein the exchange rate peg collapses but without a run on reserves. The end comes peacefully with the regime just switching from a peg to a free float.

What is the effect of this policy on the date of the crisis? Recall that on the date of the crisis we must have

$$R_{T-} = m_0 - m_T.$$

Hence, the crisis must happen when reserves exactly hit zero. At every date $t < T$, reserves are given by

$$R_t = m_0 - d_t = m_0 - d_0 e^{\mu t}.$$

The crisis must happen on the date T at which

$$d_0 e^{\mu T} = m_0 = L(r).$$

Note that instead if $\gamma = 0$, then the crisis would have happened on the date where

$$d_0 e^{\mu T} = m_T = L(r + \mu)$$

It is easy to check that the date of the crisis in the case $\gamma = 0$ must be smaller than the T for $\gamma = 1$. Hence, an interest rate defense successfully delays a crisis.

Intuitively, the mechanism at work here is that by announcing higher domestic interest rates in the event of a crisis, the government reduces the increase in the opportunity cost of holding money contingent on a successful attack. This makes people more willing to hold on to their pre-collapse money balances longer. Lastly, one can verify that welfare is an increasing function of γ. Hence, in this case, raising domestic interest rates in the event of a crisis is the optimal thing to do.

2.5 Multiple equilibria

The first-generation currency crises models that we have analyzed till now have been characterized by unique equilibrium: the time of the attack and the size of the attack are unique. Some researchers have argued that recent currency crisis episodes are better characterized by multiple equilibria models. The first generation model itself can be adapted to yield multiple equilibria fairly easily. Here we sketch out a general equilibrium version of Obstfeld (1986) which was the first paper to make this point.

Consider the standard small open economy model that we have been analyzing. The representative agent maximizes lifetime welfare given by

$$V = \int_{t=0}^{\infty} e^{-\rho t} u(c) dt$$

subject to the flow constraint given by

$$\dot{b} = rb + y + g - c - \dot{m} - \pi m - s(m)$$

where we have suppressed time subscripts to economize on notation. $s(m)$ denotes transactions costs faced by households. Assume that $s' < 0, s'' > 0$. Hence, transactions costs are strictly decreasing and strictly convex in m. Using the definition $a = b + m$ we can rewrite the flow constraint as:

$$\dot{a} = ra + y + g - c - s(m) - im \tag{2.7}$$

The first-order conditions are

$$\begin{aligned} u'(c) &= \lambda \\ \dot{\lambda} &= (\rho - r)\,\lambda \\ -s'(m) &= i \end{aligned}$$

We continue to assume that $\rho = r$ throughout. Hence, λ is constant over time which implies that c is constant over time. Also, along all paths with a constant i, real money balances must be constant as well.

The government prints money, holds reserves and makes lump-sum transfers. We assume that the central bank announces a path for the nominal exchange rate. In particular, it picks a fixed exchange rate for the domestic currency:

$$E_t = \bar{E} \qquad \text{for all } t.$$

As we noted in earlier lectures, a fixed exchange rate regime entails that the central bank stands ready to buy and sell foreign currency in exchange for domestic currency at the pre-announced exchange rate. But this requires the central bank to have adequate foreign reserves at all times to defend the announced fixed exchange rate regime. We assume that the central bank defends the exchange rate peg as long as $R > 0$. If $R \leq 0$ then the central bank abandons the peg and shifts to a free float. We assume that the central bank also announces the following policy for nominal domestic credit:

$$\frac{\dot{D}}{D} = \begin{cases} 0 & \text{if } E_t = \bar{E} \\ \mu & \text{if } E_t = \tilde{E}_t \end{cases}$$

where $\tilde{E}_t$ is the level of the exchange rate if the government pursues a flexible exchange rate policy at date t.

The government's fiscal spending is given by g. Hence, the government's flow constraint is given by

$$\dot{R} = rR + \dot{m} + \pi m - g.$$

Note that g is not constant over time in this example. The law of one price combined with the fixed exchange rate implies that during the fixed peg we must have

$$\pi_t = 0 \qquad \text{for all } t.$$

As we saw before, in the event of a successful attack on foreign exchange reserves at date T the economy will shift to a free float. The economy will be stationary from that date onward with the nominal interest rate being given by $i_t = r + \mu$ from T onward. Hence, the path for the nominal interest rate is

$$i_t = \begin{cases} r, & t < T \\ r + \mu, & t \geq T \end{cases}$$

Hence, the path for money demand is given by

$$m_t = \begin{cases} L(r), & t < T \\ L(r + \mu), & t \geq T \end{cases}$$

Thus, in the event of a crisis, real money balances will jump down.

The key question here is whether or not the crisis will happen in this economy? Note that $\dot{D}/D = 0$ as long as the exchange rate stays fixed. Hence, prior to the collapse we must have

$$\dot{R} = \dot{m} - \dot{d} = 0.$$

Thus, reserves are constant during the fixed exchange rate period which implies that the peg is potentially sustainable forever. To make matters concrete first assume that

$$R_0 > L(r) - L(r + \mu).$$

It should be clear that in this case a run can never be rationalized. Reserves are too big to start with itself to ever put the peg in peril.

Now suppose instead that

$$R_0 \leq L(r) - L(r + \mu).$$

In this case, a run would wipe out all the reserves of the central bank and thereby provoke an abandonment of the peg. Moreover, since reserves are constant under a peg, this situation would characterize all periods under the peg. In other words, in this case, at every date one can rationalize both a permanent peg as well as a successful attack on foreign reserves. Hence, there are a continuum of equilibria here, the continuum being over the date of the attack. These potential currency crises episodes are self-fulfilling: if everyone thinks that the peg will collapse then everyone will attack, hence the peg will collapse which will validate the initial assumption. The opposite logic is easy to check.

3 Second-generation models

The onset of the ERM crisis in Europe in the early 1990s led to a number of commentators suggesting that the first-generation models were incapable of explaining crises such as the ERM crisis. These commentators suggested these crises were better described as the outcome of self-fulfilling speculative attacks and herding behavior by investors rather than the unique equilibrium described by the first-generation models. Instead of an inconsistency between the fiscal and monetary stance of the government as the key to factor behind crises, the ERM episode was viewed by many as the outcome of a game between investors on one side and governments on the other with each having well defined goals and objectives. Since the first generation models formalize the government as a passive entity whose objective function is unclear, the second-generation of models focus closely on the behavior of the government. In particular, they formulate an explicit choice-theoretic framework for the government and follow through the consequences of different government optimization choices.

Here we sketch out a representative model in this class due to Sachs, Tornell, and Velasco (1996). Most of these models owe their ancestry to Barro and Gordon (1983) and, in the currency crisis context, Obstfeld (1994). Consider a policymaker in a small open economy who attempts to choose inflation π and distortionary taxes x to minimize the quadratic loss function

$$L = \frac{1}{2}\left[a\pi^2 + x^2\right]$$

subject to the budget constraint

$$Rb = x + \theta(\pi - \pi^e)$$

where π^e denotes inflationary expectations. For the purposes of this example consider $\theta(\pi - \pi^e)$ to be revenues from the inflation tax. The idea here is that the policymaker dislikes x because it distorts output while it dislikes π because it is inflation. Note that this being a small open economy, we shall refer to π as being both the inflation rate and the devaluation rate.

Using the budget constraint, one can rewrite the loss function as

$$L = \frac{1}{2}\left[a\pi^2 + \{Rb - \theta(\pi - \pi^e)\}^2\right]$$

The policymaker's optimal choice for π must satisfy

$$a\pi = \theta\left[Rb - \theta(\pi - \pi^e)\right]$$

This can be rearranged to give

$$\theta\pi = \left(\frac{\theta^2}{a+\theta^2}\right)(Rb + \theta\pi^e)$$

Define $\lambda \equiv \frac{a}{a+\theta^2}$. Using this definition, we can rewrite the budget constraint as

$$\begin{aligned} x &= \lambda\,(Rb + \theta\pi^e) \\ \theta\pi &= (1-\lambda)\,(Rb + \theta\pi^e) \end{aligned}$$

Using these relations, we can now rewrite the loss function as

$$\begin{aligned} L &= \frac{1}{2}\left[\frac{a\theta^2}{\left(a+\theta^2\right)^2} + \lambda^2\right](Rb + \theta\pi^e)^2 \\ &= \frac{1}{2}\left[\lambda(1-\lambda) + \lambda^2\right](Rb + \theta\pi^e)^2 . \end{aligned}$$

Simplifying this equation yields the parsimonious term:

$$L = \frac{1}{2}\lambda\,(Rb + \theta\pi^e)^2$$

The rational expectations equilibrium must have $\pi = \pi^e = \pi^*$. Since, $\theta\pi = (1-\lambda)\,(Rb + \theta\pi^e)$, we can deduce that

$$\pi^* = \left(\frac{1-\lambda}{\lambda\theta}\right) Rb = \frac{\theta}{a} Rb$$

The key question that we want to answer is whether or not the policymaker will choose a fixed exchange rate or a flexible exchange rate. Assume that if the policymaker devalues then she faces a cost $c > 0$. Let he loss from devaluing be L^d and the loss under a fixed exchange rate be L^f. Hence, she will devalue if and only if

$$L^d + c < L^f.$$

Rearranging, this implies that a devaluation will occur if and only if

$$L^f - L^d > c.$$

Now, under a fixed peg $\pi = 0$. Hence,

$$L^f = \frac{1}{2}\left(Rb + \theta\pi^e\right)^2$$

Similarly, loss under a devaluation is

$$L^d = \frac{\lambda}{2}\left(Rb + \theta\pi^e\right)^2$$

Thus, a devaluation is optimal if and only if

$$Rb + \theta\pi^e > \left(\frac{2c}{1-\lambda}\right)^{1/2} \equiv k$$

It is easy to see that a fixed exchange rate is optimal if $Rb < k$. For a devaluation to be optimal we must have

$$Rb + \theta\pi^* > k$$

Using the relation $\pi^* = \frac{\theta}{a}Rb$ in the above and simplifying shows that a devaluation is optimal if and only

$$Rb > \lambda k$$

It then follows that this model can generate self-fulfilling equilibria if

$$\lambda k < Rb < k.$$

Hence, if initial debt is in this intermediate range then both a devaluation and a fixed exchange rate are rationalizable as equilibria.

3.1 Global games and unique equilibrium

This class of models of self-fulfilling crises have been critiqued in influential recent work by a host of authors starting with Morris and Shin (1998). Morris-Shin showed that the genesis of the multiple equilibria in these models is the public knowledge of fundamentals. They show that if one introduces a little bit of uncertainty about the true fundamentals of the economy so that each investor only observes a noisy signal of the true fundamental, the equilibrium becomes unique. Intuitively, under some uncertainty about the true fundamentals, investors have to form expectations regarding the expectations of other investors since payoffs from taking long or short positions on the currency are dependent on the actions of other investors. While one's own signal may indicate that fundamentals are really bad, one can never be sure that other investors have the same information and hence that they will attack the currency. Consequently, investors form expectations regarding the expectations of other investors which generally leads to a unique equilibrium based on a threshold value for the noisy signal that is received by investors.

4 Third-generation models: Twin crises

The onset of the Asian crisis of the late 1990s led to a third class of models being proposed. This was brought on by increasing evidence of a close link between banking sector problems and currency crises episodes. Influential work by Kaminsky and Reinhart (1999) found that currency crisis episodes were often accompanied by banking sector turmoil. In fact, this aspect of crisis episodes had been pointed out by Diaz-Alejandro (1985) as well and was labelled as "Twin crises" by Kaminsky-Reinhart.

Studying a dataset spanning the period 1970-1995, covering both industrial and developing countries with 76 currency crises and 26 banking crises episodes, Kaminsky-Reinhart showed that currency crises often followed periods of severe banking sector problems. Moreover, they also found that currency crises had a feedback effect that made banking problem even more severe. Interestingly, they found that while banking crises typically precede currency crises, they are not causal. Rather, the two crises are often responses to a third common factor. They also found that twin crises were typically preceded by rapidly deteriorating fundamentals on output, debt and the current account. Moreover, banking crises episodes were often preceded by financial liberalizations.

Third-generation crisis models formalize twin crises episodes. There are a number of contributors to this literature including Burnside, Eichenbaum, and Rebelo (2004), Caballero and Krishnamurthy (2001), etc.. They all formalize different aspects of this phenomenon. The Burnside, Eichenbaum, and Rebelo (2004) structure essentially combines Obstfeld (1986) with banking bailouts. Obstfeld had shown that if agents expect a switch to an expansionary monetary policy in the event the exchange rate regime collapses then there could be self-fulfilling runs. Burnside et al provide microfoundations for such a switch.

They argue that if private agents expect banks to fail in the event of a currency collapse and they also expect that the central bank has an implicit bailout guarantee for commercial banks, then a run could be self-fulfilling. Effectively, a run would cause banks to fail thereby triggering the bailouts. This would trigger a switch to an expansionary monetary policy which would validate the run.

We will study an influential contribution of Chang and Velasco (2000) who stressed the illiquidity of bank portfolios as the primary catalyst for banking problems and examined their interaction with different exchange rate regimes. Their structure also permits an explicit assessment of welfare under different exchange rate and banking policy regimes.

Consider a one good, small open economy with 3-period lived agents of measure one. There is free goods and capital mobility. Free goods mobility implies the law of one price holds so that $P = E$ or the domestic currency price of the good is just the nominal exchange rate. We are normalizing the foreign currency world price of the good to unity here. Agents are born in period 0 and die at the end of period 2. All agents are born identical in period 0 with an endowment of $e > 0$ units of the good. The endowment can be invested at home in a technology which yields per unit of investment $r < 1$ units of consumption in period 1 and $R > 1$ units of consumption in period 2. The endowment can also be invested abroad at time 0 which yields a constant one unit of consumption in either period 1 or period 2. We shall maintain the assumption throughout that domestic agents can invest but cannot borrow abroad. It is feasible to relax this assumption but at the cost of considerable analytical complication.

Agents receive a "type" shock in period 1 which divides them into two types. A fraction λ are impatient who get utility from consumption in period 1 while the remaining proportion $1 - \lambda$ are patient who value real money balances in period 1 and consumption in period 2. These "type" draws however are private information and not publicly observed. Money is created and destroyed costlessly by a monetary authority.

The expected utility of an agent born at time 0 then is given by

$$V = \lambda u(x) + (1 - \lambda) u\left[g(M/E_2) + y\right] \tag{4.8}$$

where x denotes consumption if the agent is impatient, y is consumption if the agent is patient and M denotes nominal money balances chosen in period 1. The functions u and g are strictly increasing and concave in their arguments. Moreover, we also assume that $g(0) = 0$, $g'(0) = \infty$ and $g'(\bar{m}) = 0$. The last assumption implies that $\bar{m}$ is the satiation level of real balances for the agent. It is worth noting that the preference specification for money essentially says that money holdings chosen in period one yield utility in period 2 based on their real value on that date.

This environment is essentially the celebrated framework of Diamond and Dybvig (1983) with the caveat that it is an open economy and agents can both invest abroad as well potentially derive utility from money holdings. This last feature allows the model to speak to issues related to exchange rate regimes and their implications for financial fragility. We shall analyze alternative exchange rate and banking arrangements below.

4.1 Currency board and Autarky

The first arrangement that we shall analyze is one where agents operate in autarky,i.e., they do the best they can on their own with transactions only with foreign capital markets and the domestic central bank. The central bank follows a currency board arrangement whereby it maintains an exchange rate of one of the local currency (Peso) against the dollar. Moreover, the central bank only issues a new peso when it receives a dollar in deposits. Hence, as long as the currency board is in place the exchange rate of one of the peso is fully sustainable.

The problem $P0$ being solved by each agent is to maximize 4.8 subject to

$$b + k \leq e \tag{4.9}$$

$$x \leq b + rk \tag{4.10}$$

$$M \leq b + rl \tag{4.11}$$

$$y \leq M + R(k - l) \tag{4.12}$$

$$l \leq k \tag{4.13}$$

The solution to this problem is clearly not going to be optimal since there will be some inefficient liquidation of long-term investments by agents who discover themselves to be impatient. Note that l is the amount of the long term investment that will be liquidated in period 1 after the revelation of the consumer type.

4.2 Currency board and Pooling

A second arrangement of interest is one where all agents pool their resources to eliminate idiosyncratic "type" risk. Notice that there is no aggregate risk here since there is always a share λ who are impatient and $1 - \lambda$ who are patient. Under the pooling arrangement, the problem being solved by the "bank" that is pooling resources is to maximize 4.8 subject to equation 4.9, 4.13 and

$$\lambda x + (1 - \lambda) M \leq b + rl \tag{4.14}$$

$$(1 - \lambda) y \leq (1 - \lambda) M + R (k - l) \tag{4.15}$$

$$x \leq g (M) + y \tag{4.16}$$

Equations 4.14 and 4.15 are essentially the resource constraints facing the "bank" in periods 1 and 2. Equation 4.16 is the truth-telling constraint which says that all admissible allocations under this arrangement have to make it unprofitable for the patient type to lie about his/her type in period 1. This is often also called the incentive compatibility constraint.

Two features of the optimality conditions are important. First, it is easy to see that there will be no inefficient liquidation l under this arrangement because there is no aggregate uncertainty facing the "bank", $l = 0$. Second, one can check that at an optimum we must have

$$g' (M) = R - 1$$

This defines the optimal demand for pesos in period 1 $\tilde{M}$. Under this allocation for M, the period 1 and period 2 resource constraints can be combined to yield

$$\lambda x + \frac{(1 - \lambda) y}{R} = e - (1 - \lambda) \tilde{M} \left(\frac{R - 1}{R} \right) \tag{4.17}$$

Maximizing 4.8 subject to equation 4.17 gives the optimality condition

$$\frac{\lambda u' (\tilde{x})}{(1 - \lambda) u' \left(g \left(\tilde{M} \right) + \tilde{y} \right)} = R \left(\frac{\lambda}{1 - \lambda} \right) \tag{4.18}$$

This condition says that at a social optimum the slope of the social transformation curve should equal the slope of the social indifference curve between first and second period consumption. Note that since $R > 1$ and g and u are both strictly concave, this condition guarantees that the truth telling condition does not bind at an optimum. In the following we shall refer to allocations under this arrangement as the social optimum and denote them with tildes on top.

4.3 Currency board and deposit banks

A third arrangement to organize this economy is one with commercial banks who accept deposits from households which can be redeemed on demand, i.e., they accept demand deposits. The rest of the economy is as before. There is a currency board arrangement with $E = 1$. The question is can this arrangement implement the socially optimum allocation we derived in the previous subsection? The specific arrangement with commercial banks is a contract wherein agents deposit their endowment e with the bank in period 0. The bank invests this deposit in foreign bonds b and long term assets k to maximize the payment to the depositors who can redeem their deposits in either period 1 or 2. The key wrinkle here is that depositors can redeem their deposits on demand and are not constrained by their specific type realization. The contract stipulates the payment that the bank will make contingent on the type that the depositor reports herself to be.

To make this problem concrete we make some additional assumptions. First, we shall assume that all deposits with commercial banks are in terms of pesos and the withdrawals from commercial banks are also in terms of pesos. Hence, acquisition of dollars requires a visit to the central bank with a corresponding number of pesos in hand. Second, we shall impose sequential service constraints. Specifically, banks will redeem deposits of depositors on a first-come first-serve basis. Central banks will exchange dollars for pesos also on a first-come first-serve basis. Commercial banks service depositors as long as they have assets to liquidate. Similarly, the central bank keeps redeeming pesos for dollars as long as it has dollars to sell. When either of these agencies run out of their respective assets they effectively "disappear".

In period 1, depositors arrive at the bank in random order and announce their type. If they announce themselves to be patient, the bank gives them M pesos which they must hold till period 2. If they announce that they are impatient then the bank gives them x pesos which they can exchange for dollars at the central bank. In period 2 patient agents again visit the bank if the bank did not close in period 1 and receive $y - M$ pesos which can then be converted into dollars at the central bank. All of these withdrawals are contingent on the commercial bank and/or the central bank remaining open after period 1 and when they arrive there in period 2.

To see how the currency board arrangement and domestic money creation process works in this economy, notice that in period 1 commercial banks liquidate their foreign bonds and deposits the resulting dollars with the central bank which issues a corresponding number of domestic pesos in exchange. These are the pesos that are then given by the commercial banks to their depositors. In period 2, the remaining number of pesos M get retired from the system as patient agents convert them one-for-one at the central bank for dollars.

Proposition 4.1 *There exists an Honest equilibrium of this anonymous game. In period 1 impatient agents retire* $\tilde{x}$ *pesos and patient agents retire* $\tilde{M}$ *pesos. The bank does not fail and pays* $\tilde{y} - \tilde{M}$ *pesos to patient agents in period* 2*. The central bank ends period 1 with* $(1 - \lambda)\tilde{M}$ *dollars which it uses to redeem the money supply in period* 2*.*

The key piece of the proof is to check whether or not there is any benefit from lying. If impatient investors pretend to be patient they get $\tilde{M}$ pesos which they have to hold till period 2 when they have no consumption value whatsoever for them. Hence, impatient investors have no incentive to lie. If patient investors lie and pretend to be impatient they receive $\tilde{x}$ pesos in period 1. On the other hand, if they reveal their type truthfully then their utility value is $u\left(g\left(\tilde{M}\right) + \tilde{y}\right)$. As long as $\tilde{x} \leq g\left(\tilde{M}\right) + \tilde{y}$ (the truth-telling constraint), there is no incentive to lie given that the u function is strictly concave.

Proposition 4.2 *There exists a run equilibrium if and only if* $\tilde{x} \geq b + rk$*. In this equilibrium the bank fails in period* 1 *but the central bank and currency board regime survives.*

The key to the sufficiency part of the result is to see why it is optimal for patient investors to run in period 1 withdraw $\tilde{x}$ from the commercial bank. Suppose a patient investor arrives at the commercial bank in period while it is still open. His options are to either withdraw $\tilde{x}$ or $\tilde{M}$ pesos. The $\tilde{x}$ pesos can be converted to dollars in period 1 itself but the $\tilde{M}$ dollars have to be held till period 2. But since the commercial bank will close in period 1 and the central bank will also close after redeeming all the outstanding pesos for dollars at an exchange rate of one, there will be no central bank to redeem the $\tilde{M}$ pesos for dollars in period 2. Consequently, the value of those pesos will be zero. Hence, there is no incentive for patient investors to reveal their true type. The proof of the necessity part is a bit more complicated. However, it is described fairly clearly in Chang and Velasco (2000).

The main summary of this part is that a run equilibrium can arise under a deposit banking system even though this system can improve upon the allocations under autarky. The run outcome arises though precisely because banks provide the improvement in outcomes by holding more in long term securities than their aggregate potential short term liabilities. Effectively, there are multiple equilibria in Diamond-Dybvig style environment.

4.4 Fixed exchange rate and deposit banks

The analysis above has the feature that central banks provided no liquidity support to commercial banks. This made bank runs feasible. The reason for this missing feature is the requirement of a currency board which precludes the issuance of any domestic currency beyond the existing dollar reserves of the central bank. One might suspect that relaxing this could generate better outcomes. To facilitate such an analysis we now analyze a fixed exchange rate regime where the exchange rate is fixed at unity. But now we allow commercial banks to have access to central bank credit.

4.4.1 The social optimum

We start with an analysis of the social optimum which describes the best allocations that can be achieved by the central bank and commercial banks acting together. In describing this environment we shall start by noting that the cost of printing pesos for the central bank is zero. The social optimal will maximize 4.8 subject to equations 4.9, the incentive compatibility constraint $x \leq g(M) + y$, non-negativity constraints on x, y, M, k, b and

$$\lambda x \leq b$$

$$(1 - \lambda) y \leq Rk$$

Since the cost of printing pesos is zero for the central bank, the optimal allocation dictates that

$$g'\left(\bar{M}\right) = 0 \tag{4.19}$$

Since the exchange rate is one, note that $\bar{M} = \bar{m}$ which implies that the optimal allocation achieves the satiation level of money holdings. The social transformation curve in this case is

$$R\lambda x + (1 - \lambda) y = eR$$

while the optimality condition that determines the optimal allocation is given by

$$u'(\bar{x}) = Ru'(g(\bar{m}) + \bar{y}) \tag{4.20}$$

The difference between this outcome and the social optimum under the pooling arrangement is the optimal allocation of real balances. There the optimal allocation was given by the condition $g'\left(\bar{M}\right) = R - 1 > 0$ which implied that real balances were lower. Hence, welfare is higher in this case where the central bank satiates agents with money. The difference is due to the fact that under a currency board the central bank was equating the issuance of money to the opportunity cost of acquiring dollars which was $R - 1$ as opposed to the true social cost of printing pesos which is zero. We shall henceforth denote the socially optimum allocations with bars.

4.4.2 The decentralization: central bank credit

The social optimum under fixed exchange rates described above can be decentralized using the institution of central bank credit to commercial banks. Specifically, the central bank has a lending facility wherein it lends interest-free to commercial banks in period 1 with repayments due in period 2. We will denote this central bank lending by h. To keep things bounded we assume that central banks mandate a minimum reserve requirement

$$\lambda x \leq b \tag{4.21}$$

Absent this condition, commercial banks would invest their entire deposit base in period zero in long term assets and use central bank credit to service all payments in period 1. The commercial bank maximizes 4.8 subject to equations 4.9, the incentive compatibility constraint $x \leq g(M) + y$, non-negativity constraints on x, y, M, k, b, h and

$$\lambda x + (1 - \lambda) M \leq b + h$$

$$(1 - \lambda) y \leq (1 - \lambda) M + Rk - h$$

Under the constraint $\lambda x \leq b$ it is easy to see that central bank credit to commercial banks in period 1 essentially covers their payments to the patient agents, i.e., $(1 - \lambda) M = h$. Hence, this problem reduces to exactly the problem we solved to determine the social optimum. Moreover, the solution is also a Pareto improvement relative to the currency board regime. To see this note that banks had to invest enough of their initial deposits in foreign bonds b so as to cover both the withdrawals of the impatient and patient agents in period 1. Under the arrangement here, they only put aside enough in foreign bonds so as to cover the demands of the impatient agents. The patient agents are services through interest free loans from the central bank. Consequently, a greater share of initial deposits are invested in the long term asset which raises total consumable resources of the private sector.

4.4.3 Demand deposits

Can the solution derived above be implemented through commercial banks that accept demand deposits? Again, a demand deposit under a fixed exchange rate regime is a contract that requires depositors to deposit e in period 0. The bank invests b in foreign bonds and k in long term assets. Depositors can withdraw at their own discretion either x pesos in period 1, or M pesos in period 1 and $y - M$ pesos in period 2.

Suppose first that central banks only provide interest free credit to commercial banks to service the demands of patient agents. In other words, there is no lender of last resort function of the central bank here. In this case, the environment looks exactly like the environment the one we just analyzed. The central bank always has enough reserves to defend the currency peg. So the fixed exchange rate regime never collapses, i.e., the central bank never fails or shuts down. There are two equilibria of this anonymous game. The first is the Honest equilibrium whose outcome coincides with the social optimum. There is however another run equilibrium when $\bar{x} > \bar{b} + r\bar{k}$ where everyone withdraws their deposits in period 1, the commercial bank fails. However, the central bank survives. A key result that Chang and Velasco (2000) prove is that if $\tilde{x} > \tilde{b} + r\tilde{k}$ then $\bar{x} > \bar{b} + r\bar{k}$ but not the other way around. In other words, if the runs are possible under a currency board then they are feasible under fixed exchange rates but not the other way around. Hence, fixed exchange rates are more prone to bank runs than are currency boards. Intuitively, it comes from the fact that more tends to get invested in the long term asset under a fixed exchange rate system with central bank credit than under a currency board with no central bank credit. However, fixed exchange rates also implement a Pareto superior allocation relative to the currency board regime.

4.4.4 Lender of last resort

An alternative, possibly more realistic environment is one where the central bank also acts as a lender of last resort for banks which come under panic attacks (i.e., suffer bank runs). In concrete terms the central banks offers to provide unlimited lending to commercial banks in period 1 if more than λ share of depositors ask for payment claiming to be impatient. It is easy to see the outcome under this arrangement. The lender of last resort facility essentially makes commercial banks immune from failing in the even of a run. There are again two equilibria: an honest one where neither the commercial bank nor the central bank fails and allocations coincide with the social optimum; a run equilibria where everyone claims to be impatient in period 1. The commercial bank services all demand by liquidating its assets and through borrowing from the central bank. However, the central bank is unable to service its demands for dollars and consequently fails. Effectively, the lender of last resort facility under a fixed exchange rate system just pushes the crisis from commercial banks to the central bank and fixed exchange rate regime. The source of the problem is the same: the banking system creates value for its customers by investing in more long term assets and less liquid assets relative to its total demand liabilities. This opens the system to systemic crises.

4.5 Flexible exchange rates

In this environment it turns out the first-best allocation can be implemented by a flexible exchange rate arrangement and a lender of last resort facility. This system insulates the economy from both bank runs and currency crisis. Intuitively, the lender of last resort insulates the banking system while the flexible exchange rate implies that currency attacks are not profitable as the exchange rate depreciates in period 1 if patient agents run. This depreciation makes it unnecessary for long term assets to be liquidated. Recall that the dollar equivalent of the pesos that patient depositors who pretend to be impatient depends on the exchange rate. If that depreciates, the returns from running decline and makes an actual devaluation an off-equilibrium outcome. This is of course contingent on the central bank playing the role of a lender of last resort. Absent this, bank runs could still occur.

5 Debt Crises

The class of models that we analyzed in our discussion of second-generation crisis models above has also been used to formalize accounts of debt crises which are expectation driven. Narratives surrounding debt crisis episodes wherein sovereign governments default on their outstanding debt commitments often involve creditor panics and pessimistic expectations. These descriptions suggest that these events are often characterized by multiple, self-fulfilling equilibria. We now study one such environment due to Calvo (1988). The heart of the exercise is to show that in many realistic environments one can easily have multiple perfect foresight equilibria – one with low interest rates and no default and another with high interest rates and default. The primary reasoning behind the result is the existences of government liabilities. Expected repudiation would be built into current interest rates. But higher interest rates imply higher debt service which, in turn, increase the temptation to repudiate the debt. This validates the expected repudiation and the corresponding high interest rate.

Consider a two period model of a closed economy. There is a government which borrows in period 0 and repays in period 1. The risk-free gross return for private investors from the equity market is R. Hence, no-arbitrage between private capital and government bonds would require that their returns be equalized, i.e.,

$$R = (1 - \theta) R_b, \tag{5.22}$$

where $\theta \in [0, 1]$ is the degree of repudiation on public debt. Note that there is perfect foresight in this model. Hence, this condition must hold both ex-ante and along any rational expectation equilibrium.

The budget constraint of the government in period 1 is

$$x = g + (1 - \theta) R_b b + \alpha\theta R_b b, \tag{5.23}$$

where g is public spending, x is a distortionary tax, and $\alpha \in (0, 1)$ is the proportional cost of repudiating on one unit of outstanding debt. Note that this specification implies that the government incurs a cost of repudiating debt which is proportional to the amount of repudiation.

In period 1 private agents consume their entire wealth since it is the last period. Hence, their period 1 constraint is:

$$c = y - z(x) + Rk + (1 - \theta) R_b b - x, \tag{5.24}$$

where k is capital, b is outstanding public debt in period 1, R and R_b denote the interest factor on capital and bonds respectively. Note that $z(.)$ is the deadweight loss due to the distortionary tax x. y is endowment income.

From equation (5.23) we get

$$\theta = \frac{g + R_b b - x}{(1 - \alpha) R_b b}. \tag{5.25}$$

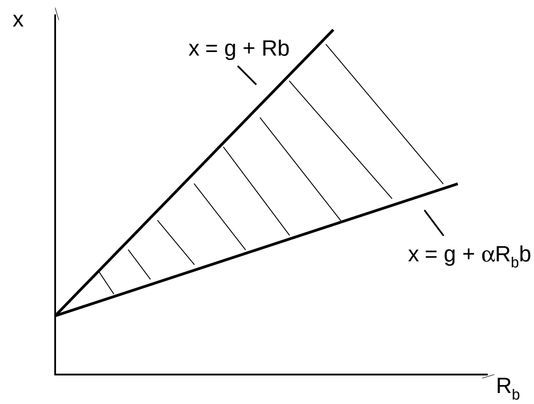
Since $\theta \in [0, 1]$, we must have two implied restrictions:

$$\begin{aligned} R_b b + g &\geq x, \\ \alpha R_b b + g &\leq x. \end{aligned}$$

Any permissible x must lie in the shaded region of Figure 1. Above the line $R_b b + g = x$, x is so high that $\theta < 0$. Conversely, below the line $\alpha R_b b + g = x$, x is so low that $\theta > 1$. Both these ranges for x are not permissible.

In order to guarantee an interior solution we need some conditions on the deadweight

Figure 1: Permissible range for x



loss function z. In particular, we assume that

$$\begin{aligned} z(0) &= z'(0) = 0, \\ z'(\infty) &= \infty = -z'(\infty), \\ z''(x) &> 0 \text{ for all } x. \end{aligned}$$

Substituting equation (5.25) in the private budget constraint (5.24) and simplifying the result gives

$$c = y + Rk - \frac{1}{1-\alpha}(\alpha R_b b + g) + \frac{\alpha}{1-\alpha}x - z(x).$$

A social welfare maximizing government will choose x to maximize the above. Hence, the optimal x^* must satisfy

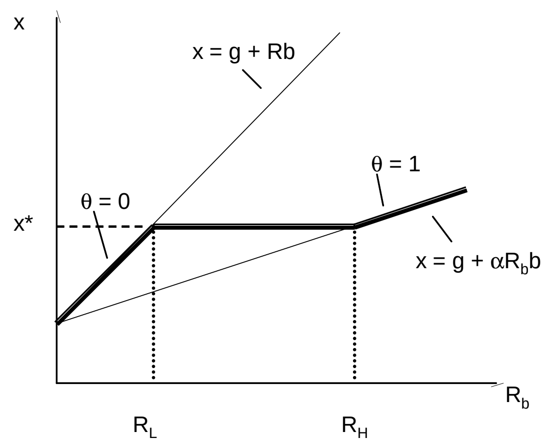
$$z'(x^*) = \frac{\alpha}{1-\alpha}.$$

Hence, the equilibrium tax schedule is given by

$$\hat{x} = \begin{cases} R_b b + g & \text{for } x^* \geq R_b b + g \\ x^* & \text{for } x^* \in (\alpha R_b b + g, R_b b + g) \\ \alpha R_b b + g & \text{for } x^* \leq \alpha R_b b + g \end{cases}$$

Figure 2 depicts the government's reaction function as the dark shaded schedule. Clearly, it is non-convex. Note that this schedule gives the optimal response of the government in period 1.

Figure 2: Government reaction function



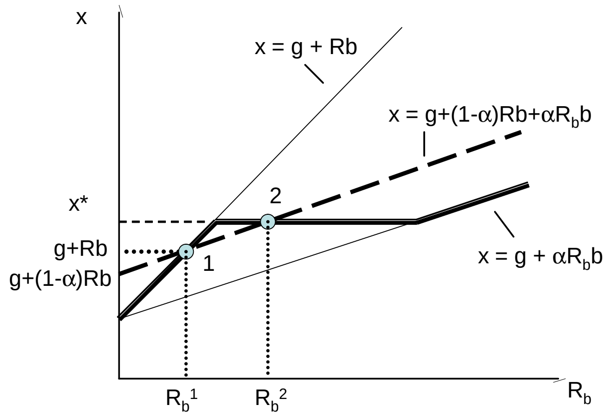
Along any time consistent, rational expectations equilibrium for this economy, at time $t = 0$ private agents must correctly predict government policy about x and θ at time $t = 1$. Clearly, along a rational expectations equilibrium path we must have $R_b(1 - \theta) = R$. Substituting this into the government's budget constraint (5.23) gives

$$x = g + (1 - \alpha)\, Rb + \alpha R_b b \tag{5.26}$$

This is a consistency condition that must be satisfied in equilibrium. Hence, rational expectations equilibria for this economy occur at points of intersection of this consistency condition with the government reaction function.

Figure 3 shows one possibility wherein there are two equilibria. In equilibrium 1, there is no default. In this equilibrium the interest rate on public debt is low so that the taxes required to service the debt are not too high. Hence, the distortionary cost of debt repayment is low enough relative to the cost of repudiation. Hence, there is full repayment. This validates the low interest rates that the market charges the government on its debt. In equilibrium 2 there is partial repudiation. The interest cost of the debt contracted in period 0 is high enough so that the taxes required to finance full repayment are too high relative

Figure 3: Rational expectations equilibria



to the cost of repudiation. Hence, there is partial repudiation. This justifies the higher interest cost of borrowing in period 0.

The figure has been drawn for the case $x^* > Rb + g$. If, on the other hand, $x^* < Rb + g$ then the the only equilibrium would be $\theta = 1$, i.e., full repudiation. Since x^* is an increasing function, this implies that for all initial levels of b there is a critical $\hat{\alpha}$ such that for all $\alpha < \hat{\alpha}$ there is full repudiation. Thus, punishment must be strong enough to sustain any public debt this model.

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