THE OLIGOPOLY SOLUTION CONCEPT IS IDENTIFIED *

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Oligopoly theory predicts that market price will be at least as high as the competitive price and no higher than the monopoly price. Particular oligopoly solution concepts offer more exact predictions, but it is difficult to know which solution concept holds in any real market. This paper shows that the oligopoly solution concept can be estimated econometrically.

1. Introduction

Recent case studies of concentrated industries have attempted to use structural econometric models to tell Cournot from Bertrand from Collusion. (See my working paper for a bibliography.) This note shows that the theoretical underpinnings of these studies are sound. A parameter indexing the oligopoly solution concept is econometrically identified. It is identified by standard econometric methods, even when no cost or profit data are available, and even when the demand and cost curves must be estimated as well. That is, the comparative statics of equilibrium, as price and quantity are moved by exogenous variables, reveal the degree of market power.

The models we treat will all have market price and quantity determined by the intersection of demand function and a supply relation. The demand function presumes price taking buyers. The supply relation may be a supply function, a solution of \( P = MC \). More general supply relations arise where the sellers may have some market power. They take

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the form $MR_p = MC$, perceived marginal revenue equals marginal cost. When $MR_p = P$, competition is present. When $MR_p = MR$, sellers are changing the monopoly price. When $MR_p < P$, there is some element of market power. The question here is whether observations on industry prices and quantities can reveal whether price or some smaller number is being set equal to $MC$, given that the demand function and the cost function are unknown a priori.

2. The model

Let buyers have a typical demand function:

$$Q = D(P, Y, \alpha) + \epsilon,$$

where $Q$ is quantity, $P$ price, $Y$ an exogenous variable, and $\alpha$ parameters of the demand system to be estimated. $\epsilon$ is the econometric error term. The selling side of the market equilibrium model is more complex. When sellers are price-takers, we can write

$$P = c(Q, W, \beta) + \eta,$$

where $W$ are exogenous variables on the supply side, $\beta$ the supply-function parameters, and $\eta$ the supply error. $c( )$ is marginal cost. When firms are not price takers, perceived marginal revenue, not price, will be equal to marginal cost. This in general will take the form

$$P = c(Q, W, \beta) - \lambda \cdot h(Q, Y, \alpha) + \eta,$$

where $P + h( )$ is marginal revenue, and $P + \lambda h( )$ is $MR$ as perceived by the firm. The demand-side parameters and exogenous variables are in $h( )$ because they affect marginal revenue. $\lambda$ is a new parameter indexing the degree of market power. $\lambda = 0$ is perfect competition, $\lambda = 1$ is a perfect cartel, and intermediate $\lambda$‘s correspond to other oligopoly solution concepts. For example, Cournot equilibrium has $\lambda = 1/n$.¹ In general, the econometrician will estimate (1) and (2') simultaneously, treating both price and quantity as endogenous in both. The formal

¹ Applied work has often concentrated on models in which each individual firm has its own $\lambda$. In this way Stackelberg, dominant firm, etc., can be handled.
question is then whether \( \lambda \) is identified in (1) (2'). This is the question: Are competition and a cartel observationally distinct?

**The problem**

Let demand and \( MC \) be linear. The demand function is now

\[
Q = \alpha_0 + \alpha_1 P + \alpha_2 Y + \epsilon
\]

(3)

and the marginal-cost function is

\[
MC = \beta_0 + \beta_1 Q + \beta_2 W.
\]

(4)

The supply relation is

\[
P = \lambda(\frac{-Q}{\alpha_1}) + \beta_0 + \beta_1 Q + \beta_2 W + \eta,
\]

(5)

since \( MR = P - Q/\alpha_1 \).

The demand equation is identified no matter which form the supply relation takes. The demand eq. (3) has only one included endogenous variable, \( P \), and there is an excluded exogenous variable, \( W \), so the equation is identified.

The supply relation is also identified. But the degree of market power is not. To see the first assertion, rewrite (5) as

\[
P = \beta_0 + \gamma Q + \beta_2 W + \eta,
\]

(6)

where \( \gamma = \beta_1 - \lambda/\alpha_1 \). Clearly (6) is identified. Only \( Q \) is included and endogenous, while \( Y \) is excluded. But how would we know whether we were tracing out \( P = MC \) or \( MR = MC \)? The thing we can estimate, \( \gamma \), depends on both \( \beta_1 \) and \( \lambda \). We cannot determine both of these from knowledge of \( \gamma \), even though we can treat \( \alpha_1 \) as known (since the demand curve can be estimated).

Fig. 1 may clarify the issues. Look first at \( D_1, MR_1 \) and \( E_1 \). The demand curve is linear, so the \( MR \) is linear and twice as steep. Note that \( E_1 \) could be an equilibrium either for a cartel or monopolist with cost \( MC" \) (by \( MR_1 = MC" \)), or for a perfectly competitive industry with cost \( MC' \) (by \( P = MC' \)). Increase \( Y \) to shift the demand curve out to \( D_2 \), and note that both the monopoly and the competitive equilibria move to \( (P_2, \)
In fact, $MC^c$ is the supply relation either for the competitor for whom $MC^c$ is marginal cost, or for the cartel with the lower, flatter marginal cost $MC^m$. Unless we know marginal costs, for this example there is no observable distinction between the hypotheses of competition and monopoly.

3. Solution

Solving the problem posed involves generalizing the demand function so that movements in the exogenous variables do more than shift its intercept up and down. Some exogenous variables must also be capable of changing the demand slope.

The argument is made graphically in fig. 2. The demand system $D_1 - MR_1$ and the two cost curves are as before. But now instead of shifting the demand curve vertically (to get $D_2 - MR_2$ in fig. 1) we rotate it around $E_1$ to get $D_3 - MR_3$. If the supply relation is a supply curve, then this will have no effect on the equilibrium. That is, if $MC^c$ is the marginal cost curve and competition is perfect, $E_1$ should be the equilibrium under either $D_1$ or $D_3$. But if $MC^m$ were the marginal cost curve, and supply were monopoly, then equilibrium shifts to $E_3$, where $MR_3 = MC^m$. Thus, if we can rotate as well as shift the demand function, the
Formally, change the demand equation to

$$Q = \alpha_0 + \alpha_1 P + \alpha_2 Y + \alpha_3 PZ + \alpha_4 Z + \epsilon,$$

where $Z$ is a new demand-side exogenous variable. They key feature is that $Z$ enters interactively with $P$, so that changes in $Y$ and $Z$ combine elements both of rotation and of vertical shifts in demand. $Z$ might best be viewed as the price of a substitute good, which makes the interaction natural, while $Y$ might be interpreted as income.

Now the supply relation has been altered to be

$$P = \frac{-\lambda}{\alpha_1 + \alpha_3 Z} Q + \beta_0 + \beta_1 Q + \beta_2 W + \eta.$$  

Clearly $\lambda$ is identified. The demand side is still identified. So in attempting to disentangle $\lambda$ and $\beta$ in (8), we treat $\alpha_1$ and $\alpha_3$ as known. Writing $Q^* = -Q/(\alpha_1 + \alpha_3 Z)$, there are two included exogenous variables, $Q$ and $Q^*$. And there are two excluded exogenous variables $Z$ and $W$. Thus, $\lambda$ is identified as the coefficient of $Q^*$.

The logic of this argument holds up even if the curves are not linear. Translation of the demand curve will always trace out the supply
relation. Rotations of the demand curve around the equilibrium point will reveal the degree of market power. Conditions on the demand system in which movements in the exogenous variables can do this have been worked out exactly. See the companion paper by Lau (1981). In general, such rotations will have no effect on the equilibrium if pricing is competitive, but will have an effect if there is market power. Thus the hypotheses of competition and monopoly are distinct. In any applied study the model of supply and demand will be more complex than that here. Considerations of product differentiation or of the fixity of capital, for example, might enter. The logic of the result here seems likely to be robust to such considerations.

References


Lau, Lawrence J., 1981, On identifying the degree of competitiveness from industry price and output data, see this issue.