

Working paper

Monetary Policy in Pakistan

A Dynamic
Stochastic General
Equilibrium Analysis

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Abstract

A small-scale DSGE model for Pakistan is developed to analyze monetary policy in Pakistan. The model includes a financial sector and distinguishes between high-income households who participate in the financial sector and low-income households who face borrowing constraints. In evaluating different monetary policy options, the model takes into account the constraint that the State Bank has to satisfy the long-term borrowing needs of the government, and thus cannot independently determine the long-run rate of inflation. The baseline model assumes that fiscal policy adjusts primary budget surplus to stabilize government debt to GDP ratio around a feasible target level. This model is used to examine macroeconomic adjustment to various shocks and to compare the macroeconomic performance of alternative monetary policy rules. In this regime, monetary policy can play an important role in stabilizing inflation and output. The paper also considers an alternative policy regime in which fiscal policy does not attempt to stabilize government debt. In this case, monetary policy is constrained further by the need to use interest rates to control the growth of government debt. Feasible monetary rules under this constraint are found to produce much larger variability in inflation.

1. Introduction

Pakistan has recently experienced high inflation persisting in double-digits, fiscal imbalances, low private sector credit growth and stagnant economic growth. This paper explores how monetary policy should be formulated in this difficult macroeconomic environment.

Dynamic Stochastic General Equilibrium (DSGE) models have emerged recently as the standard tool for evaluating monetary policy and are increasingly used for policy advice to central banks in developed and many emerging countries. We develop a small-scale DSGE model for Pakistan and use it to evaluate different monetary policy options. Our model follows the standard framework of the new open economy macroeconomic models, but introduces certain variations that are appropriate for monetary policy analysis in Pakistan.

First, we include a banking sector to incorporate financial frictions in the model. There are a number of models that introduce financial frictions of one type or another in DSGE models (e.g., Bernanke, Gertler and Gilchrist, 1999; Goodfriend and McCallum, 2007; Canzoneri et al., 2008), but the focus of this literature is on modelling financial markets in developed countries. In this paper, we use a variant of the Canzoneri et al.'s (2008) model to examine how financial frictions interact with monetary policy in a developing economy like Pakistan. Second, we depart from the representative-agent setup and distinguish between households with high and low incomes. High-income households participate in the financial market (e.g., hold bank deposits and purchase government bonds). Low-income households, on the other hand, face liquidity constraints (do not borrow or lend) and thus do not interact with financial markets. A number of recent DSGE models (e.g., Gali et al., 2007) have included liquidity-constrained households (also referred to as "non-Ricardian" households or "rule-of-thumb" consumers) to allow departures from the Ricardian equivalence proposition, which implies that a debt-financed tax

cut would not affect consumption. We include such households also to explore whether monetary policy actions impact households at low and high income levels differently. Finally, as financial markets in Pakistan are not well integrated with foreign financial markets, we do not assume an interest parity relation linking the domestic and foreign interest rates.

One major constraint for monetary policy in Pakistan arises from the need of the government to continuously borrow from the State Bank. If fiscal policy relies on a permanent flow of revenue from money creation (seignorage), the inflation rate in the long run is determined by the rate of growth of the monetary base needed to yield the long-run level of seignorage. In this case, monetary policy cannot independently determine a long-run inflation target. A crucial question is whether fiscal policy is prepared to adjust primary deficit to stabilize government debt at some target level. If fiscal policy stabilizes government debt, then monetary policy can stabilize inflation and output by following a conventional (Taylor-type) rule whereby the real interest rate is increased in response to an increase in inflation above the long-run rate (determined by seignorage requirements) and to an increase in the output gap. However, the conventional interest rate response to inflation may not be desirable or even feasible if the government is not willing or able to adjust primary deficit in response to debt growth. Such an inflexible fiscal policy - - that subordinates monetary policy to fiscal needs - - is referred to as "fiscal dominance" in the literature. Under fiscal dominance, a tightening of monetary policy in response to higher inflation has been shown to lead to perverse results (Sargent and Wallace, 1981; Woodford, 2001). Kumhof, Nunes and Yakadina (2008) explore feasible interest rate rules under fiscal dominance for a closed economy model. They show that it is beneficial to include government debt as an argument in the interest rate rule and an optimal rule would require lowering (instead of raising) the real interest rate in response to higher inflation. Fiscal

dominance, moreover, leads to greater inflation variability and loss of welfare than a fiscal policy that stabilizes government debt.

In evaluating monetary policy rules for Pakistan, we consider both fiscal regimes. In our baseline case, we assume that fiscal policy determines the long-run value of seignorage, but adjusts taxes in response to the deviation of government debt from its target level. The economy is assumed to be subject to several internal and external shocks including shocks to government expenditures. We explore what monetary rule is appropriate under these conditions. In the conventional DSGE model based on the New-Keynesian framework without financial frictions, monetary aggregates do not play a role in the formulation of monetary policy (Clarida, Gali, and Gertler, 1999; Woodford, 2003). Our model, however, incorporates financial frictions, and we examine how such frictions influence the transmission mechanism for monetary policy. The model is also used to investigate some issues that have been widely debated. For example, it has been argued that supply and foreign price shocks have been an important source of inflation in Pakistan. We use impulse response functions derived from the model to identify the contribution of such shocks to inflation. There is an ongoing debate in Pakistan that huge borrowing requirements of the government significantly crowd out bank lending to the private sector and impede investment. We also use impulse response analysis to examine the impact of shocks to government expenditure on credit and investment.

We also explore monetary policy options under fiscal dominance. We consider monetary rules similar to Kumhof et al. (2008). As our model differs from theirs, especially in including financial frictions and liquidity-constrained households, we explore what rules are feasible and appropriate in our model. Monetary rules in this regime require a decrease in the real interest rate

in response to an increase in real government debt and this policy is found to produce excessive inflation variability.

The model is presented in Section 2 and calibrated to Pakistan's economy in Section 3. Section 4 analyzes monetary policy in Pakistan and Section 5 concludes the paper.

2. The Model

Our model is based on the standard new-Keynesian framework, but introduces a number of variations to address certain monetary policy issues in Pakistan. There is one composite good (consisting of differentiated home and foreign varieties), which is used by households, investors and government. There are two types of households denoted by H and L . Households of type H have higher wage income, own firms and participate in financial markets: buy government bonds, hold bank deposits and take bank loans to finance fixed expenditures on nondurables. Households of type L get lower wage income, are liquidity constrained, and do not transact in the financial markets (hold no assets except currency). Capital goods producers undertake investment decisions subject to adjustment costs and supply (installed) capital to capital leasing firms who finance the additions to capital by loans from banks. Banks require cash reserves and government bonds to provide convertibility services for deposits and use labor to monitor loans. Government uses lump-sum taxes to raise revenue.¹ Domestic financial markets are not integrated with global financial markets, and households and banks are assumed not to hold foreign bonds. Finally, nominal rigidities are introduced by assuming that there are adjustment costs for both prices and wages as in Rotemberg (1982).

¹ As our focus is on monetary policy issues and not on the efficiency costs of distortionary taxes, such taxes are not included in the model to simplify the analysis..

Real variables are denoted by lower case letters and nominal variables by upper case letters. An asterisk is used to denote foreign variables.

2.1 Households

Assume that there be a continuum of households of type H and L , indexed by $h \in (0,1)$ and $l \in (0,1)$, respectively. The Utility function for household h of type H is

$$U_{H,t}(h) = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{c_{H,s}^{1-\theta}(h)}{1-\theta} + \frac{\xi_{HC} (CU_{H,s}(h) / P_s)^{1-\chi}}{1-\chi} + \frac{\xi_{HD} (D_{H,s}(h) / P_s)^{1-\kappa}}{1-\kappa} - \frac{\xi_{HN} n_{H,s}^{1+\nu}(h)}{1+\nu} \right), \quad (1)$$

where, in period t , $c_{H,t}(h), n_{H,t}(h)$ are the household's consumption and labor supply while $CU_{H,t}(h)$, and $D_{H,t}(h)$ are the (end of period) holdings of currency and bank deposits. The budget constraint for the household is

$$c_{H,t}(h) + \tau_{H,t}(h) + \frac{CU_{H,t}(h) + D_{H,t}(h) + B_{H,t}(h)}{P_t} = \frac{W_{H,t} n_{H,t}(h) (1 - AC_{WH,t}(h)) + PR_t(h)}{P_t} + \frac{CU_{H,t-1}(h) + (1 + R_{D,t-1}) D_{H,t-1}(h) + (1 + R_{t-1}) B_{H,t-1}(h) - (1 + R_{L,t-1}) \bar{L}(h)}{P_t}, \quad (2)$$

where $B_{H,t}(h)$ is the stock of government bonds (at the end of the period); $\bar{L}(h)$ is the fixed amount of bank loans; P_t is the price of one unit of the composite good; $PR_t(h)$ represents the household's share of profits; $\tau_{H,t}(h)$ stands for (lump sum) real taxes. $R_{D,t}$, R_t and $R_{L,t}$ are the interest rates on bank deposits, government bonds and bank loans; $W_{H,t}(h)$ is the wage rate; and

$AC_{WH,t}(h) = \frac{\omega_H}{2} \left(\frac{W_{H,t}(h) / W_{H,t-1}(h)}{P_{t-1} / P_{t-2}} - 1 \right)^2$ is the adjustment cost for wages. This adjustment cost

function is based on the extension of the basic Rotemberg model by Laxton and Pesenti (2003)

and accounts for the presence of inflation. We use a similar function below for price adjustment costs.

The household chooses $c_{H,t}(h)$, $CU_{H,t}(h)$, $D_{H,t}(h)$, and $W_{H,t}(h)$ to maximize utility subject to the budget constraint and the demand for its labor service (discussed below) with wage elasticity equal to ε . Optimization by the household implies the following conditions:

$$c_{H,t}^\theta(h) = E_t c_{H,t+1}^\theta(h) \frac{E_t P_{t+1}}{(1+R_t)\beta P_t}, \quad (3)$$

$$\left(\frac{CU_{H,t}(h)}{P_t} \right)^{-\chi} = \frac{c_{H,t}^{-\theta}(h)}{\xi_{HC,t}} \left(\frac{R_t}{1+R_t} \right), \quad (4)$$

$$\left(\frac{D_{H,t}(h)}{P_t} \right)^{-\kappa} = \frac{c_{H,t}^{-\theta}(h)}{\xi_{HD,t}} \left(\frac{R_t - R_{D,t}}{1+R_t} \right), \quad (5)$$

$$\begin{aligned} (1 - AC_{WH,t}(h))(\varepsilon - 1)W_{H,t}(h) &= \varepsilon \xi_{HN} n_{H,t}^\nu(h) P_t c_t^\theta(h) - (W_{H,t}(h))^2 \frac{\partial AC_{WH,t}(h)}{\partial W_{H,t}(h)} \\ &\quad - \beta \frac{P_t c_t^\theta(h)}{P_{t+1} c_{t+1}^\theta(h)} \frac{n_{H,t+1}(h)}{n_{H,t}(h)} W_{H,t}(h) W_{H,t+1}(h) \frac{\partial AC_{WH,t+1}(h)}{\partial W_{H,t}(h)}. \end{aligned} \quad (6)$$

Equation (3) is the standard Euler equation for intertemporal consumption choice. Equations (4) and (5) represent, respectively, the demand for real currency and real bank deposits as a function of consumption and the opportunity cost. Equation (6) determines the dynamics of wage adjustment in the presence of adjustment costs. Note that in steady state, $AC_{WH}(h) = 0$,

$$\frac{\partial AC_{WH,t}(h)}{\partial W_{H,t}(h)} = 0 \text{ and } \frac{\partial AC_{WH,t+1}(h)}{\partial W_{H,t}(h)} = 0, \text{ and (6) simplifies to } \frac{W_{H,t}(h)}{P_t} = \frac{\varepsilon \xi_{HN} n_{H,t}^\nu(h) c_{H,t}^\theta(h)}{(\varepsilon - 1)}.$$

The corresponding utility function and the budget constraint for household l are

$$U_{L,t}(l) = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{c_{L,s}^{1-\theta}(l)}{1-\theta} + \frac{\xi_{LC} (CU_{L,s}(l) / P_s)^{1-\chi}}{1-\chi} - \frac{\xi_{LN} n_{L,s}^{1+\nu}(l)}{1+\nu} \right), \quad (7)$$

$$c_{L,t}(l) + \tau_{L,t}(l) + \frac{CU_{L,t}(l)}{P_t} = \frac{W_{L,t}(l)n_{L,t}(l)(1 - AC_{WL,t}(l)) + CU_{L,t-1}(l)}{P_t}, \quad (8)$$

where $AC_{WL,t}(l) = \frac{\omega_L}{2} \left(\frac{W_{L,t}(l)/W_{L,t-1}(l)}{P_{t-1}/P_{t-2}} - 1 \right)^2$. Note that household l does not hold bank deposits

or government bonds, and does not receive any profits. Utility maximization by the household subject to the budget constraint and labor demand (also discussed below) implies that the choice of $CU_{L,t}(l)$ and $W_{L,t}(l)$ satisfies

$$\left(\frac{CU_{L,t}(l)}{P_t} \right)^{-\chi} = \frac{c_{L,t}^{-\theta}(l)}{\xi_{LC}} \left(1 - \frac{\beta E_t c_{L,t+1}^{-\theta}(l) P_t}{c_{L,t}^{-\theta}(l) E_t P_{t+1}} \right), \quad (9)$$

$$\begin{aligned} (1 - AC_{WL,t}(l))(\varepsilon - 1)W_{L,t}(l) &= \varepsilon \xi_{LN} n_{L,t}^{\nu}(l) P_t c_t^{\theta}(l) - (W_{L,t}(l))^2 \frac{\partial AC_{WL,t}(l)}{\partial W_{L,t}(l)} \\ &\quad - \beta \frac{P_t c_t^{\theta}(l)}{P_{t+1} c_{t+1}^{\theta}(l)} \frac{n_{L,t+1}(l)}{n_{L,t}(l)} W_{L,t}(l) W_{L,t+1}(l) \frac{\partial AC_{WL,t+1}(l)}{\partial W_{L,t}(l)}. \end{aligned} \quad (10)$$

Note that household l 's demand for real currency (9) is of a different form than that of household h because the two households face different opportunity costs. In steady state, the wage-setting equation (10) also simplifies to $\frac{W_{L,t}(l)}{P_t} = \frac{\varepsilon \xi_{LN} n_{L,t}^{\nu}(l) c_{L,t}^{\theta}(l)}{(\varepsilon - 1)}$. Since the household cannot borrow

or lend, consumption, $c_{L,t}(l)$, is determined by the constraint (8).

2.2 Banks

The specification of the banking sector is based on Canzoneri et al. (2008). In their paper, bank loans finance a fixed amount of loans to households. We assume that bank loans are also used to finance investment. Deposit creation and production of loans by a bank is determined by the following liquidity and monitoring function:

$$\frac{D_{H,t}}{P_t} = \xi_{BD} \left(\frac{CR_t}{P_t} \right)^\gamma \left(\frac{B_{B,t}}{P_t} \right)^{1-\gamma}, \quad (11)$$

$$\frac{L_{B,t}}{P_t} = \xi_{BL} n_{HB,t}, \quad (12)$$

where CR_t and $B_{B,t}$ represents cash reserves and government bonds held by banks, $L_{B,t}$ are bank loans, and $n_{HB,t}$ is a bundle of labor services of H type households defined as

$$n_{HB,t} = \left[\int_0^1 n_{HB,t}(h)^{(\varepsilon-1)/\varepsilon} dh \right]^{\varepsilon/(\varepsilon-1)}. \text{ We assume, for simplicity, that banks employ only } H \text{ type}$$

households. The balance sheet of the banking sector is given by

$$\frac{CR_t + B_{B,t} + L_{B,t}}{P_t} = \frac{D_{H,t}}{P_t}. \quad (13)$$

Express the discounted value of profits for a bank as

$$E_t \sum_{s=t}^{\infty} \Delta_{t,s} \left(\frac{CR_s + (1+R_s)B_{B,s} + (1+R_{L,s})L_{B,s} - (1+R_{D,s})D_{H,s}}{E_t P_{s+1}} - \frac{W_{H,s}}{P_s} n_{HB,s} \right), \text{ where } \Delta_{t,s} \text{ denote}$$

the discount factor, and $W_{H,t} = \left[\int_0^1 W_{H,t}(h)^{1-\varepsilon} dh \right]^{1/(1-\varepsilon)}$ represents the wage rate for the labor

bundle. Banks choose the ratios, $B_{B,t} / D_{H,t}$ and $CU_{H,t} / D_{H,t}$, to maximize this value subject to

the balance-sheet constraint (13), and the liquidity and monitoring relations (12) and (13). The optimal choice by banks implies that

$$\frac{B_{B,t}}{D_{H,t}} = (1 - \gamma) \left[\frac{(P_t / E_t P_{t+1})(R_{L,t} - R_{D,t}) - W_{H,t} / (\xi_{BL,t} P_t)}{(P_t / E_t P_{t+1})(R_{L,t} - R_t) - W_{H,t} / (\xi_{BL,t} P_t)} \right], \quad (14)$$

$$\frac{CR_t}{D_{H,t}} = \gamma \left[\frac{(P_t / E_t P_{t+1})(R_{L,t} - R_{D,t}) - W_{H,t} / (\xi_{BL,t} P_t)}{(P_t / E_t P_{t+1})(1 + R_{L,t} - 1) - W_{H,t} / (\xi_{BL,t} P_t)} \right], \quad (15)$$

where $W_{H,t} / (\xi_{BL,t} P_t)$ represents the marginal cost of making a loan (in real value). As (14) and (15) show, the securities to deposits and the cash reserves to deposits ratios are influenced by interest rate spreads and the expected inflation rate.

2.3 Capital goods producers and capital leasing firms

We assume a standard model of investment where capital producers make additions to installed capital in the presence of adjustment costs. However, to relate investment to bank loans, we introduce capital leasing firm who require bank loans to finance purchases of additional installed capital from capital producers. Let k_t represent the installed capital stock at the beginning of period t , and i_t investment in the period. In each period, capital goods producers buy previously installed capital (after depreciation), k_t , from capital leasing firms, produce and sell new installed capital, $k_{t+1} = k_t + i_t$. Investment is subject to the following adjustment cost:

$$AC_{I,t} = \frac{\omega_I}{2} \left(\frac{i_t}{k_t} - \delta \right)^2 k_t \quad (16)$$

where δ is the depreciation rate. Although the price of a unit of i_t is P_t (the same as the price of a unit of c_t), the price of a unit of installed capital is different because of capital adjustment costs and is denoted by Q_t . Capital goods producers maximize the discounted value of profits equal to $E_t \sum_{s=t}^{\infty} \Delta_{t,s} [Q_s(k_{s+1} - k_s) - P_s(i_s - AC_{I,s})]$. Substituting i_t for $k_{t+1} - k_t$, and using (16), the first order condition for this problem is

$$\frac{Q_t}{P_t} = 1 + \omega_I \left(\frac{i_t}{k_t} - \delta \right). \quad (17)$$

Capital leasing firms rent installed capital to firms producing the final good. In each period, they distribute income from previously installed capital to H households, and finance purchase of additional installed capital $[Q_t(k_{t+1} - k_t)]$ by a loan from banks. Their profits from the acquisition of additional installed capital in period $t+1$ are $[E_t RE_{t+1} + (1 - \delta)E_t Q_{t+1}]i_t - (1 + R_{L,t})Q_t i_t$, where RE_t denoted the rental rate for a unit of capital. The optimal choice for investment satisfies

$$1 + R_{L,t} = \frac{E_t RE_{t+1} + (1 - \delta)E_t Q_{t+1}}{Q_t}. \quad (18)$$

Capital accumulates as

$$k_{t+1} = i_t + (1 - \delta)k_t, \quad (19)$$

and investment is linked to bank loans as

$$Q_t i_t = L_{B,t} - \bar{L}, \quad (20)$$

where \bar{L} is the fixed amount of loans to all households.

2.4 Composite good producers

Assume that the composite good (used by household, government and investors) is a CES bundle of home and foreign varieties produced by a continuum of home firms indexed by $f \in (0,1)$, and foreign firms indexed by $f^* \in (0,1)$. Letting z_t represent the amount of the composite good, we have

$$z_t = c_{H,t} + c_{L,t} + i_t + g_t, \quad (21)$$

$$z_t = \left[(1-\psi)^{1/\eta} (z_{D,t})^{(\eta-1)/\eta} + \psi^{1/\eta} (z_{M,t})^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}, \quad (22)$$

$$z_{D,t} = \left(\int_0^1 z_{D,t}(f)^{(\sigma-1)/\sigma} df \right)^{\sigma/(\sigma-1)}, \quad (23)$$

$$z_{M,t} = \left(\int_0^1 z_{M,t}(f^*)^{(\sigma-1)/\sigma} df^* \right)^{\sigma/(\sigma-1)}. \quad (24)$$

It follows that the price of the composite good and the demand for the domestic and imported varieties is

$$P_t = \left[(1-\psi)(P_{D,t})^{(1-\eta)} + \psi(P_{M,t})^{(1-\eta)} \right]^{1/(1-\eta)} \quad (25)$$

$$z_{D,t} = (1-\psi)z_t(P_{D,t}/P_t)^{-\sigma}, \quad z_{M,t} = \psi z_t(P_{M,t}/P_t)^{-\sigma} \quad (26)$$

$$z_{D,t}(f) = z_{D,t}(P_{D,t}(f)/P_{D,t})^{-\sigma}, \quad z_{M,t}(f^*) = z_{M,t}(P_{M,t}(f^*)/P_{M,t})^{-\sigma} \quad (27)$$

Similarly, the foreign demand for exported variety is given by

$$z_{X,t} = \psi^* z_t^* (P_{X,t}^*/P_t^*)^{-\sigma}, \quad z_{X,t}(f) = z_{X,t}(P_{X,t}^*(f)/P_{X,t}^*)^{-\sigma} \quad (28)$$

Let S_t denote the (rupee/dollar) exchange rate. The prices of imported and exported varieties in the home and foreign markets are linked as

$$P_{M,t}(f^*) = S_t P_{M,t}^*(f^*), \quad (29)$$

$$P_{X,t}(f) = S_t P_{X,t}^*(f). \quad (30)$$

The home variety of the composite good is produced according to the following production function:

$$y_t(f) = \xi_{y,t} n_{HY,t}(f)^{\alpha_H} n_{L,t}(f)^{\alpha_L} k_t(f)^{1-\alpha_H-\alpha_L} \quad (31)$$

where $n_{HY,t}$ and $n_{L,t}$ are bundles of labor services defined as $n_{HY,t} = \left[\int_0^1 n_{HY,t}(h)^{(\varepsilon-1)/\varepsilon} dh \right]^{\varepsilon/(\varepsilon-1)}$,

and $n_{L,t} = \left[\int_0^1 n_{L,t}(l)^{(\varepsilon-1)/\varepsilon} dl \right]^{\varepsilon/(\varepsilon-1)}$. The optimal choice of inputs implies the following demand

functions:

$$n_{HY,t}(f) = \alpha_H y_t(f) MC_t / W_{H,t}, \quad (32)$$

$$n_{L,t}(f) = \alpha_L y_t(f) MC_t / W_{L,t}, \quad (33)$$

$$k_t(f) = (1 - \alpha_H - \alpha_L) y_t(f) MC_t / RE_t, \quad (34)$$

where $W_{L,t} = \left[\int_0^1 W_{L,t}(l)^{1-\varepsilon} dl \right]^{1/(1-\varepsilon)}$ is the wage rate for the L type labor bundle, and MC_t is the marginal cost of the composite output. Define $n_{H,t} = n_{HY,t} + n_{HB,t}$. The demand functions for labor services of households h and l [used in setting $W_{H,t}(h)$ and $W_{L,t}(l)$ in (6) and (10)] are

$$n_{H,t}(h) = n_{H,t} (W_{H,t}(h) / W_{H,t})^{-\varepsilon}, \quad n_{L,t}(l) = n_{L,t} (W_{L,t}(l) / W_{L,t})^{-\varepsilon}. \quad (35)$$

Output of the variety of a composite good equals

$$y_t(f) = z_{D,t}(f) + z_{X,t}(f). \quad (36)$$

We assume the following adjustment costs for domestic and export prices:

$$AC_{PD,t}(f) = \frac{\omega_P}{2} \left(\frac{P_{D,t}(f) / P_{D,t-1}(f)}{P_{t-1} / P_{t-2}} - 1 \right)^2 \quad \text{and} \quad AC_{PX,t}(f) = \frac{\omega_P}{2} \left(\frac{P_{X,t}(f) / P_{X,t-1}(f)}{P_{t-1} / P_{t-2}} - 1 \right)^2. \text{ Firms}$$

choose $P_{D,t}(f)$ and $P_{X,t}(f)$ to maximize the discounted value of the profits,

$$\sum_{s=t}^{\infty} \Delta_{t,s} \left[(P_{D,s}(f) - MC_s) z_{D,s}(f) (1 - AC_{PD,s}(f)) + (P_{X,s}(f) - MC_s) z_{X,s}(f) (1 - AC_{PX,s}(f)) \right],$$

subject to demand functions in (27) and (28). Noting that $\Delta_{t,t} = 1, \Delta_{t,t+1} = 1 / (1 + R_t)$, the optimal prices are

$$\begin{aligned} (1 - AC_{PD,t}(f)) \left[(\sigma - 1) P_{D,t}(f) - \sigma MC_t \right] &= -P_{D,t}(f) (P_{D,t}(f) - MC_t) (\partial AC_{PD,t}(f) / \partial P_{D,t}(f)) \\ &\quad - \frac{z_{D,t+1}(f)}{(1 + R_t) z_{D,t}(f)} P_{D,t}(f) (P_{D,t+1}(f) - MC_{t+1}) (\partial AC_{PD,t+1}(f) / \partial P_{D,t}(f)), \end{aligned} \quad (37)$$

$$\begin{aligned} (1 - AC_{PX,t}(f)) \left[(\sigma - 1) P_{X,t}(f) - \sigma MC_t \right] &= -P_{X,t}(f) (P_{X,t}(f) - MC_t) (\partial AC_{PX,t}(f) / \partial P_{X,t}(f)) \\ &\quad - \frac{z_{X,t+1}(f)}{(1 + R_t) z_{X,t}(f)} P_{X,t}(f) (P_{X,t+1}(f) - MC_{t+1}) (\partial AC_{PX,t+1}(f) / \partial P_{X,t}(f)). \end{aligned} \quad (38)$$

In steady state, both prices are the same and equal marginal cost multiplied by a markup factor:

$$P_{D,t}(f) = P_{X,t}(f) = \frac{\sigma}{\sigma-1} MC_t.$$

In symmetric equilibrium, $z_{M,t}(f^*) = z_{M,t}$, $z_{D,t}(f) = z_{D,t}$, $z_{X,t}(f) = z_{X,t}$, $y_t(f) = y_t$;

$P_{M,t}(f^*) = P_{M,t}$, $P_{D,t}(f) = P_{D,t}$, $P_{X,t}(f) = P_{X,t}$; $n_{H,t}(h) = n_{H,t} = n_{HY,t} + n_{HB,t}$, $n_{L,t}(l) = n_{L,t}$; and

$W_{H,t}(h) = W_{H,t}$, $W_{L,t}(l) = W_{L,t}$. Finally, the current account balance condition is

$$P_{M,t}z_{M,t} = P_{X,t}z_{X,t} + CF_t, \quad (39)$$

where CF_t is an exogenous net capital inflow (including remittances). Assuming that the home economy is small, foreign variables z_t^* , P_t^* and $P_{M,t}^*$ are exogenous.

2.5 Monetary and fiscal policy

Define $B_{P,t} = B_{H,t} + B_{B,t}$ as government bonds held in the private sector,

$CU_t = CU_{H,t} + CU_{L,t}$ as currency held by public, and $MB_t = CU_t + CR_t$ as the monetary base.

The government's flow budget constraint is

$$B_{Pt} = P_t(g_t - \tau_{H,t} - \tau_{L,t}) - (MB_t - MB_{t-1}) + (1 + R_{t-1})B_{P,t-1}, \quad (40)$$

where it is assumed that the government does not pay interest to the State Bank (i.e., interest income from the Bank's holding of government securities is transferred to the government). Real seignorage equals $(MB_t - MB_{t-1}) / P_t$.

We assume that the long-run real seignorage is determined by the fiscal authority, but we distinguish two policy regimes. In the first regime, fiscal policy adjusts taxes to stabilize debt at

some target level, and monetary policy can use a Taylor-type rule with an inflation target given by the long-run inflation rate determined by real seignorage. We call this regime seignorage-constrained monetary policy. In the second regime, fiscal policy does not adjust taxes in response to debt growth and monetary policy attempts to stabilize debt. This regime is referred to as fiscal dominance.

The first regime is described by the following tax and interest-rate rules:

$$\tau_{H,t} = \bar{\tau}_H + \phi_\tau \left(\frac{B_{P,t-1}}{P_{t-1}} - \frac{\bar{B}_P}{\bar{P}_t} \right), \quad \phi_\tau > 0, \quad (41)$$

$$\ln(1 + R_t) = \ln(1 + \bar{R}) + \phi_{r\pi} \ln(\Pi_t / \bar{\Pi}) + \phi_{ry} \ln(y_t / \bar{y}) + \ln \xi_{r,t}, \quad \phi_{r\pi} > 0, \phi_{ry} > 0, \quad (42)$$

where $1 + R_t = (1 + r_t)E_t P_{t+1} / P_t$ is the gross nominal interest rate, $1 + \bar{R} = (1 + \bar{r})\bar{\Pi}$, $\Pi_t = P_t / P_{t-1}$ (so that the inflation rate equals $\pi_t = \Pi_t - 1$), $\xi_{r,t}$ is a monetary policy shock, and an overbar over a variable denotes the value in steady state. In the tax rule, only the taxes for type H households are assumed to be adjusted.

In the second regime, there is no tax rule followed by the fiscal authority and the interest-rate rule is modified to include reaction to debt growth as follows:

$$\ln(1 + R_t) = \ln(1 + \bar{R}) + \phi_{r\pi} \ln(\Pi_t / \bar{\Pi}) + \phi_{ry} \ln(y_t / \bar{y}) + \phi_{rb} \left(\frac{B_{P,t-1}}{P_{t-1}} - \frac{\bar{B}_P}{\bar{P}_t} \right) + \ln \xi_{r,t}. \quad (43)$$

Note that the signs of the coefficients in (43) are not restricted to be positive as this restriction may no longer be feasible under fiscal dominance.

2.6 Shocks

We consider four shocks in the model: three internal and one external shocks. The internal shocks include shocks to real government expenditures (g_t), total factor productivity ($\xi_{y,t}$) and to monetary policy rule ($\xi_{r,t}$). The external shock is a shock to real foreign price of imports ($P_{M,t}^* / P_t^*$). Each shock is assumed to follow an AR (1) process, and the equations for the variables subject to shocks are given by

$$\ln g_t = (1 - \rho_g) \ln \bar{g} + \rho_g \ln g_{t-1} + x_{g,t}, \quad (44)$$

$$, \ln \xi_{y,t} = (1 - \rho_y) \ln \bar{\xi}_y + \rho_y \ln \xi_{y,t-1} + x_{y,t}, \quad (45)$$

$$\ln \xi_{r,t} = (1 - \rho_r) \ln \bar{\xi}_r + \rho_r \ln \xi_{r,t-1} + x_{r,t}, \quad (46)$$

$$\ln(P_{M,t}^* / P_t^*) = (1 - \rho_{PM}) \ln(\bar{P}_M^* / \bar{P}^*) + \rho_{PM} \ln(P_{M,t-1}^* / P_{t-1}^*) + x_{pm,t}, \quad (47)$$

where $x_{g,t}$, $x_{y,t}$, $x_{r,t}$, and $x_{pm,t}$ are white-noise shocks.

As the model variables in nominal values are non-stationary, they were converted to real values to obtain a steady state solution for the model. The real version of the model is summarized in Appendix A.

3. Calibration to Pakistan's Economy

The values of a number of model parameters were chosen by calibrating the model to the data for Pakistan's economy. Table 1 shows the average annual values for key financial and macro variables for Pakistan. The steady-state values of model variables were matched with this data by appropriate choice of certain parameters. The unit of time in the model equals a quarter, and (where needed) the data in the table were converted to quarterly values. The steady-state

quarterly inflation rate is assumed to be equal to 3% (12% annual rate). Survey data suggests that about 70% of the non-agricultural workers are in the informal sector. Considering the distribution of workers between the informal and formal sectors as a rough indicator of the relative size of L and H type households, we let $n_L = 2n_H$ in steady state. Household of type L earn a lower wage, and we assume that $w_L = w_H / 3$. We do not have information on what proportion of total currency is held by the two types of households (or in informal and formal sectors). We initially assume that the share of total currency held by L type households roughly corresponds to their relative wage income.

The values of model parameters that were not determined by calibration or the assumptions discussed above were selected from other studies. The quarterly value of the real interest rate (which determines the discount factor, β) is typically assumed to equal 0.01 in DSGE models and we use this value in our model.² The value of the risk aversion parameter (θ) is generally assumed to be close to one in a number of recent DSGE models for emerging economies and we let it equal 1.01.³ The inverse of the elasticity of labor supply (ν) is set equal to 2.0, which is within the range of values assumed in these models. The elasticity of real currency and real bank deposits with respect to real consumption ($\chi / \theta, \kappa / \theta$) are assumed to equal one,⁴ The substitution elasticities between domestic and foreign goods is assumed to be 2.0, which is consistent with the range of values typically used in open economy macro models. The elasticities of substitution between varieties of the (home or foreign) differentiated good and

² Ahmed, Haider and Iqbal's (2012) estimates suggest that the long-term real interest is lower in Pakistan. A lower value of this variable would not make a major difference to our results.

³ For example, see Ahmad et al.'s (2012) model for Pakistan, Gabriel et al.'s (2010) model for India, Shaari's (2008) model for Malaysia, and Castro et al.'s (2011) model for Brazil.

⁴ This is similar to the assumption of a unitary income elasticity of money demand. Also note that parameters χ and κ can be interpreted as the inverse of the elasticity of real currency and real bank deposits with respect to their opportunity cost.

differentiated labor services (of the two households types), σ and ε , are assumed to equal 6.0, which implies a markup of 20%.

Studies of the frequency of changes in prices and wages in Pakistan suggest that prices are less sticky than wages, and the degree of stickiness for both variables may be less than in developed economies. In the Rotemberg model of wage-price adjustment used in this paper, the degree of stickiness depends on ω_p for prices, and on ω_H and ω_L for wages. A value of 400 for these parameters, which is roughly equivalent to a four-quarter average contract length in a Calvo-type model, is typically assumed in DSGE models. We use a lower value of 200 for ω_H and ω_L , and a further-reduced value of 100 for ω_p .

In equations (44)-(47), we set the autoregressive coefficients equal to 0.9. We set the standard deviations of the white noise shocks to government expenditures and import prices ($x_{g,t}$ and $x_{pm,t}$) equal to 0.05.⁵ We set the standard deviation of the white noise shock to productivity ($x_{y,t}$) equal to 0.025 to bring the variability of output closer to the variability of real GDP in Pakistan. In our stochastic simulations comparing the performances of different monetary policy rules, we consider rules without shocks and set the monetary policy shock ($x_{r,t}$) equal to zero.

4. Monetary Policy Analysis

In analyzing monetary policy in this section, we focus on a policy regime where monetary policy is constrained by fiscal needs to raise some revenue from money creation. The fiscal authority, however, adjusts taxes to stabilize debt in the long run. This regime is

⁵ These values are suggested by the detrended annual data on real government expenditures and foreign import price since 1983. The autoregressive coefficient in the annual data is lower, but we use a higher value for our quarterly model.

represented by tax and interest rate rules (41) and (42) with the long-run inflation target in the interest rate rule determined by fiscal policy. We first examine the transmission mechanism of the model by deriving the dynamic response of key macro variables to different shocks under baseline policy rules. Next, we explore desirable rule by comparing the performance of the alternative rules. Finally, we discuss monetary rules in the case of fiscal dominance where the fiscal authority does not react to changes in government debt and the central bank adjusts the interest rate to stabilize government debt. This monetary policy regime is represented by the interest rate rule (43).

4.1 Transmission Mechanism in the Baseline Case

For the baseline case, we consider a monetary policy rule in which the interest rate reacts only to inflation and exhibits considerable inertia. We set the autoregressive coefficient (ϕ_{rr}) equal to 0.9, the inflation coefficient ($\phi_{r\pi}$) equal to 0.5, and the output gap coefficient (ϕ_{ry}) equal to zero. This rule is compared later with variations that allow smaller inertia, less or more aggressive response to inflation and reaction to output gap. In the baseline tax rule, we assume a weak response of taxes to debt growth and let the debt coefficient in the rule ($\phi_{\tau b}$) equal to 0.025.

To illustrate the transmission mechanism for monetary policy, we trace the dynamic effects of a one-quarter shock to the interest rate rule that produces a one percentage point reduction in the nominal interest rate (expressed as a percentage rate on an annual basis) in the first quarter. The interest rate adjusts in the following quarters according to the interest rate rule. The behavior of the interest rate and the dynamic responses of selected macro variables over 15 quarters are displayed in Figure 1. In the presence of nominal rigidities (based on adjustment

costs for wages and prices), the reduction in the nominal interest rate also lowers the real interest rate and temporarily increases both the inflation rate (expressed as an annual percentage rate) and the output gap (defined as a log deviation from the steady state level).

The introduction of the banking sector in the model allows monetary policy to influence interest rate spreads. To explore this channel, the figure also shows the response of the real spread between the bank loan rate and the interest rate as well as the ratio of bank loans to deposits. The figure shows that the interest rate reduction temporarily lowers the real loan rate spread and increases the loans to deposit ratio. Although the loan-deposit ratio increases by about one point initially, the decrease in the real loan rate spread is quantitatively very small. To see whether monetary policy affects low- and high-income households differently, the response of the consumption and employment of low-income relative to high-income households is also exhibited in the figure. In the short run, the relative consumption of low-income households increases marginally because of liquidity constraints which make the consumption of low-income households respond more strongly to changes in income. The effect on the relative employment, on the other hand, is negligible

We next examine the effects of a one-quarter shock to real government expenditures equal to 5%.⁶ The dynamic responses to this shock are shown in Figure 2. The fiscal shock leads to a small increase in both the inflation rate and the output gap in the short run. One concern about the increase in government expenditures is that they could crowd out private investment. The figure shows that investment indeed decreases in the short run, and this decrease offsets

⁶ The magnitude of this shock implies an initial increase in the real value of government expenditure roughly equal to 1% of GDP.

much of the increase in government expenditures. The loan-deposit ratio decreases as well. The fiscal shock increases the relative consumption of low-income household only marginally.

Figure 3 illustrates the effects of a one-quarter shock to productivity equal to -1%. Adverse supply shocks are often thought to be a major source of inflation. In the present model, however, the negative productivity shock has little effect on the inflation rate because of nominal rigidities. The shock does have a strong negative impact on output. It also leads to a significant decrease in investment and loan-deposit ratio. Interestingly, the effects of the negative productivity shock on these variables are similar to those of a positive shock to government expenditures.

Finally, the transmission of a one-quarter shock to foreign import price equal to 5% is exhibited in Figure 4. This shock temporarily increases the inflation rate by about $\frac{1}{2}$ percentage point. The effect of the import price increase on inflation is dampened somewhat by the monetary policy response that raises the interest rate. The import price increase does lead to a more significant negative effect on output and investment. The model suggests that adverse shocks to productivity and import prices are not an important source of inflation. However, they have a significant negative impact on economic activity.

4.2 Performance of Different Monetary Policy Rules

To explore what kind of monetary rules would be desirable, this section compares the performance of the baseline rule with alternative rules incorporating different responses to inflation, different degree of inertia and a reaction to output gap. We compute two types of performance indicators from stochastic simulations, in which the economy is subjected to shocks to government expenditures, productivity and import prices. The indicators of the first type are

based on the traditional approach, in which losses arise from the variability of inflation and output around their target values. The indicators of the second type use a welfare criterion based on household utility. Welfare effects are typically discussed for a representative household. In the present model, however, we can examine whether the welfare effects are different for the low-income (L) and high-income (H) households.

The results are presented in Table 2. First, we consider a rule with an inflation coefficient equal to 0.15, which is less aggressive in fighting inflation than the baseline rule.⁷ The less-aggressive rule increases the variability of inflation (as measured by the standard deviation of the gap between the actual and the fixed target value)), but decreases the variability of output (measured in the same way). This rule also worsens the welfare of both L and H households. To explore whether a more aggressive anti-inflation rule would perform better than the baseline rule, we also show the results for a rule with inflation coefficient equal to 0.85. This rule lowers the variability of inflation and improves the welfare of both households, but the quantitative improvement over the baseline rule is relatively small. The table also shows that a rule with less interest rate inertia (an auto-regressive coefficient of 0.6 instead of the baseline value of 0.9) would, like the less aggressive anti-inflation rule, increase the inflation variability, reduce the output variability and worsen welfare of each household.

We next examine the performance of rules that also react to the output gap. A rule with an output coefficient equal to 0.25 (and the same inflation and auto-regressive coefficients as the baseline case) significantly reduces output variability, but at the cost of a substantial increase in variability of inflation. This rule also affects the two households differently: relative to the

⁷ This value is close to the inflation coefficient in an interest rate rule estimated for Pakistan using data for a long period.

baseline case, the L household loses while the H household gains. These effects are strengthened if the output coefficient is increased to 0.5.

4.3 Fiscal Dominance

We now consider a fiscal regime that does not adjust its primary surplus in response to growth of government debt and relies on borrowing from the central bank to meet its fiscal needs. In order to avoid acceleration of money growth and inflation under these conditions, monetary policy attempts to stabilize government debt via interest rate changes. For this case, we represent monetary policy by the interest rate rule (43), which includes government debt as an argument. In this rule, the real interest rate must decrease in response to an increase in real government debt in order to reverse debt growth by lowering interest payments.

Figure 5 illustrates some feasible interest rate responses to an increase in debt brought about by a one-quarter shock to government expenditures equal to 5%. Part (a) of the figure shows the dynamic effects of this shock when the interest rate rule responds negatively to real government debt and does not react to inflation. We let $\phi_{r\pi} = 0$, and $\phi_{rb} = -0.1$. This rule brings about the necessary reduction in the real interest rate, but this adjustment also involves a sharp increase in inflation and output. In fact, the initial increase in inflation is sufficiently large (is over 5%) to cause government debt to decrease in real terms (and the nominal interest rate to increase) initially. We next examine an interest rate rule that still responds negatively to real government debt, but also responds positively to inflation. For this case, we let

$\phi_{r\pi} = 0.5$, and $\phi_{rb} = -0.1$.⁸ Part (b) of the figure shows that this rule decreases the impact on inflation and output, but it now takes longer for the real government debt to converge to its target

⁸ The positive response of the interest rate to inflation, however, cannot be so large that it prevents the needed real interest rate adjustment.

value. Finally, we also consider a rule that reacts only to inflation. In this case, the response to inflation has to be negative and sufficiently strong to stabilize real government debt. We now set $\phi_{r\pi} = -0.75$, and $\phi_{rb} = 0$. The dynamic response of inflation and output under this rule is large as in the case of the first rule (see part (c) of the figure).

As these examples suggest, interest rate rules that stabilize real government debt can lead to greater inflation variability. In stochastic simulations where the economy faces shocks to productivity and import prices as well as to government expenditures, we find that inflation variability under all three rules discussed above is much higher as compared to rules that do not target government debt.⁹ Successful implementation of monetary policy requires, moreover, that it is well understood and is credible. These conditions would be more difficult to meet for monetary rules designed to stabilize government debt, which require large fluctuations in inflation and real interest rates.

5. Conclusions

The paper develops a Dynamic Stochastic General Equilibrium Model to analyze monetary policy in Pakistan. At this time, monetary policy is constrained by the needs of the government to borrow from the State Bank. Facing this constraint, the State Bank cannot independently determine an inflation target. However, if the long-term needs of the government are clearly established and fiscal policy takes steps to stabilize government debt to GDP ratio around a feasible target level, monetary policy can follow an interest rate rule to stabilize inflation (around a long-run rate determined by fiscal needs) and output. For such policy regime,

⁹ The standard deviations of inflation for the first, second and third rule are, respectively, 0.0725, 0.0802 and 0.0930. The corresponding standard deviations for output are 0.1028, 0.0901 and 0.1398.

we use the model to examine macroeconomic adjustment to various shocks and compare the macroeconomic performance of alternative monetary policy rules.

The evidence in Pakistan suggests that wages and prices, especially the latter, are changed more frequently than developed economies. Even under the assumption of less wage and price stickiness, our analysis shows that monetary policy exerts significant real effects through interest rate changes. We also examine the macroeconomic effects of changes in government expenditures, and find that they lead to significant crowding out of private investment. We find, moreover, that supply shocks contribute little to inflation, but they do have an important impact on output.

The model is also used to explore how interest rates should respond to inflation and output. Estimation of the interest rate rule based on past data for Pakistan suggests a weak response to inflation (an inflation coefficient close to 0.1). Our results show that a stronger response (an inflation coefficient of 0.5 or more), would significantly reduce inflation variability and improve the welfare of both low- and high-income households. We also examine the appropriate interest rate response to output. Our analysis suggests a trade off between inflation and output variability, which has implications for the welfare of households at different income levels. For example, a positive response to output gap causes a decrease in output variability, but also leads to an increase in inflation variability. This policy also affects households at high and low income levels differently: high-income households gain while low-income households loose.

The paper also considers an alternative policy regime in which fiscal policy does not attempt to stabilize government debt. In this case, monetary policy is constrained further by the need to use interest rates to control the growth of government debt. Feasible monetary rules

under this constraint are found to produce much larger variability in inflation. These rules may also be more difficult to implement because they involve interest rate adjustments that might be misunderstood and not considered credible.

The model in the paper can be extended and modified to investigate a wide range of policy issues. Some valuable theoretical extensions would include developing a model of net capital inflows, introducing further financial frictions, and exploring different models of price expectations. There is also a need for empirical work to identify important empirical regularities in Pakistan and verify how well the model explains these regularities. Such work could lead to modifications of the model that would improve the model and make it more useful for policy analysis.

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Appendix A

Real Model

Note: The model is transformed to a real model by converting nominal values in the home economy to real values in terms of the home composite good. Real values are denoted by lower case letters. The gross real interest rate is related to the gross nominal interest rate as $1 + r_t = (1 + R_t)P_t / E_t P_{t+1}$, and $\Pi_t = P_t / P_{t-1}$. A star denotes foreign value. Foreign real values are expressed in terms of the foreign composite good. $s_t (= S_t P_t^* / P_t)$ denotes the real exchange rate.

Households

$$c_{H,t}^{-\theta} = \beta(1 + r_t)E_t c_{H,t+1}^{-\theta} \quad (\text{A1})$$

$$d_{H,t}^{-\kappa} = \frac{c_{H,t}^{-\theta}}{\xi_{HD,t}} \left(\frac{r_t - r_{D,t}}{1 + r_t} \right) \quad (\text{A2})$$

$$cu_{H,t}^{-\chi} = \frac{c_{H,t}^{-\theta}}{\xi_{HC,t}} \left(\frac{r_t + 1 - 1/E_t \Pi_{t+1}}{1 + r_t} \right) \quad (\text{A3})$$

$$c_{L,t} = w_{L,t} n_{L,t} + cu_{L,t-1} / \Pi_t - cu_{L,t} - \tau_{L,t} \quad (\text{A4})$$

$$cu_{L,t}^{-\chi} = \frac{c_{L,t}^{-\theta}}{\xi_{LC,t}} \left(1 - \frac{\beta E_t c_{L,t+1}^{-\theta} / E_t \Pi_{t+1}}{c_{L,t}^{-\theta}} \right) \quad (\text{A5})$$

Production and investment

$$y_t = \xi_{F,t} (n_{HY,t})^{\alpha_H} (n_{L,t})^{\alpha_L} (k_t)^{1-\alpha_H-\alpha_L} \quad (\text{A6})$$

$$k_t = (1 - \alpha_H - \alpha_L) y_t mc_t / re_t \quad (\text{A7})$$

$$n_{HY,t} = \alpha_H y_t mc_t / w_{H,t} \quad (\text{A8})$$

$$n_{L,t} = \alpha_L y_t m c_t / w_{L,t} \quad (\text{A9})$$

$$k_{t+1} = i_t + (1 - \delta)k_t \quad (\text{A10})$$

$$E_t r e_{t+1} = (1 + r_{L,t})q_t - (1 - \delta)E_t q_{t+1} \quad (\text{A11})$$

$$q_t = 1 + \omega_l \left(\frac{i_t}{k_t} - \delta \right) \quad (\text{A12})$$

Foreign sector

$$y_t = z_{D,t} + z_{X,t} \quad (\text{A13})$$

$$1 = \left[\psi p_{M,t}^{1-\eta} + (1 - \psi) p_{D,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (\text{A14})$$

$$z_{M,t} = \psi z_t p_{M,t}^{-\eta} \quad (\text{A15})$$

$$z_{D,t} = (1 - \psi) z_t p_{D,t}^{-\eta} \quad (\text{A16})$$

$$z_{X,t} = \bar{z}_t^* (p_{X,t}^*)^{-\eta} \quad (\text{A17})$$

$$p_{M,t} = s_t p_{M,t}^* \quad (\text{A18})$$

$$p_{X,t} = s_t p_{X,t}^* \quad (\text{A19})$$

$$p_{M,t} z_{M,t} = p_{X,t} z_{X,t} + f c_t \quad (\text{A20})$$

Banking sector

$$\frac{cr_t}{d_{H,t}} = \gamma \left[\frac{(r_{L,t} - r_{D,t}) - w_{H,t} \xi_{BL,t}}{(1 + r_{L,t} - 1/E_t \Pi_{t+1}) - w_{H,t} / \xi_{BL,t}} \right] \quad (\text{A21})$$

$$\frac{b_{B,t}}{d_{H,t}} = (1 - \gamma) \left[\frac{(r_{L,t} - r_{D,t}) - w_{H,t} / \xi_{BL,t}}{(r_{L,t} - r_t) - w_{H,t} / \xi_{BL,t}} \right] \quad (\text{A22})$$

$$d_{H,t} = \xi_{BD,t} (cr_t)^\gamma (b_{B,t})^{1-\gamma} \quad (\text{A23})$$

$$cr_t + b_{B,t} + l_{B,t} = d_{H,t} \quad (\text{A24})$$

$$l_{B,t} = \xi_{BL,t} n_{HB,t} \quad (\text{A25})$$

Price and wage setting

$$(1 - AC_{PD,t}) \left[(\sigma - 1) p_{D,t} - \sigma mc_t \right] = -p_{D,t} (p_{D,t} - mc_t) dac_{PD,t} \\ - \frac{z_{D,t+1}}{(1 + r_t) z_{D,t}} p_{D,t} (p_{D,t+1} - mc_{t+1}) dac1_{PD,t} \quad (\text{A26})$$

$$dac_{PD,t} = (\partial AC_{PD,t} / \partial p_{D,t}) P_t = \omega_P \left(\frac{p_{D,t} \Pi_t}{p_{D,t-1} \Pi_{t-1}} - 1 \right) \frac{\Pi_t}{p_{D,t-1} \Pi_{t-1}} \quad (\text{A27})$$

$$dac1_{PD,t} = (\partial AC_{PD,t+1} / \partial p_{D,t}) P_t = -\omega_P \left(\frac{p_{D,t+1} \Pi_{t+1}}{p_{D,t} \Pi_t} - 1 \right) \frac{p_{D,t+1} \Pi_{t+1}}{p_{D,t}^2 \Pi_t} \quad (\text{A28})$$

$$AC_{PD,t} = \frac{\omega_P}{2} \left(\frac{p_{D,t} \Pi_t}{p_{D,t-1} \Pi_{t-1}} - 1 \right)^2 \quad (\text{A29})$$

$$(1 - AC_{PX,t}) \left[(\sigma - 1) p_{X,t} - \sigma mc_t \right] = -p_{X,t} (p_{X,t} - mc_t) dac_{PD,t} \\ - \frac{z_{X,t+1}}{(1 + r_t) z_{X,t}} p_{X,t} (p_{X,t+1} - mc_{t+1}) dac1_{PX,t} \quad (\text{A30})$$

$$dac_{PX,t} = (\partial AC_{PX,t} / \partial P_{X,t}) P_t = \omega_P \left(\frac{p_{X,t} \Pi_t}{p_{X,t-1} \Pi_{t-1}} - 1 \right) \frac{\Pi_t}{p_{X,t-1} \Pi_{t-1}} \quad (A31)$$

$$dac1_{PX,t} = (\partial AC_{PX,t+1} / \partial P_{X,t}) P_t = -\omega_P \left(\frac{p_{X,t+1} \Pi_{t+1}}{p_{X,t} \Pi_t} - 1 \right) \frac{p_{X,t+1} \Pi_{t+1}}{p_{X,t}^2 \Pi_t} \quad (A32)$$

$$AC_{PX,t} = \frac{\omega_P}{2} \left(\frac{p_{X,t} \Pi_t}{p_{X,t-1} \Pi_{t-1}} - 1 \right)^2 \quad (A33)$$

$$(1 - AC_{WH,t})(\varepsilon - 1)w_{H,t} = \varepsilon \xi_{HN} n_{H,t}^\nu c_{H,t}^\theta - w_{H,t}^2 dac_{WH,t} - \frac{1}{1 + r_t} \frac{n_{H,t+1}}{n_{H,t}} w_{H,t} w_{H,t+1} dac1_{WH,t} \quad (A34)$$

$$dac_{WH,t} = \frac{\partial AC_{WH,t}}{\partial W_{H,t}} P_t = \omega_H \left(\frac{w_{H,t} \Pi_t}{w_{H,t-1} \Pi_{t-1}} - 1 \right) \frac{\Pi_t}{w_{H,t-1} \Pi_{t-1}} \quad (A35)$$

$$dac1_{WH,t} = \frac{\partial AC_{WH,t+1}}{\partial W_{H,t+1}} P_t = -\omega_H \left(\frac{w_{H,t+1} \Pi_{t+1}}{w_{H,t} \Pi_t} - 1 \right) \frac{w_{H,t+1} \Pi_{t+1}}{w_{H,t}^2 \Pi_t} \quad (A36)$$

$$AC_{WH,t} = \frac{\omega_H}{2} \left(\frac{w_{H,t} \Pi_t}{w_{H,t-1} \Pi_{t-1}} - 1 \right)^2 \quad (A37)$$

$$(1 - AC_{WL,t})(\varepsilon - 1)w_{L,t} = \varepsilon \xi_{LN} n_{L,t}^\nu c_{L,t}^\theta - w_{L,t}^2 dac_{WL,t} - \frac{1}{1 + r_t} \frac{n_{L,t+1}}{n_{L,t}} w_{L,t} w_{L,t+1} dac1_{WL,t} \quad (A38)$$

$$dac_{WL,t} = \frac{\partial AC_{WL,t}}{\partial W_{L,t}} P_t = \omega_L \left(\frac{w_{L,t} \Pi_t}{w_{L,t-1} \Pi_{t-1}} - 1 \right) \frac{\Pi_t}{w_{L,t-1} \Pi_{t-1}} \quad (A39)$$

$$dac1_{WL,t} = \frac{\partial AC_{WL,t+1}}{\partial W_{L,t+1}} P_t = -\omega_L \left(\frac{w_{L,t+1} \Pi_{t+1}}{w_{L,t} \Pi_t} - 1 \right) \frac{w_{L,t+1} \Pi_{t+1}}{w_{L,t}^2 \Pi_t} \quad (A40)$$

$$AC_{WL,t} = \frac{\omega_L}{2} \left(\frac{w_{L,t} \Pi_t}{w_{L,t-1} \Pi_{t-1}} - 1 \right)^2 \quad (\text{A41})$$

Equilibrium conditions

$$q_t i_t = l_{B,t} - \bar{l} \quad (\text{A42})$$

$$n_{H,t} = n_{HY,t} + n_{HB,t} \quad (\text{A43})$$

$$z_t = c_{H,t} + c_{L,t} + i_t + g_t \quad (\text{A44})$$

$$b_{Pt} = g_t - \tau_{H,t} - \tau_{L,t} - (mb_t - mb_{t-1} / \Pi_t) + (1 + r_{t-1}) b_{P,t-1} \quad (\text{A45})$$

$$mb_t = cu_{H,t} + cu_{L,t} + cr_t \quad (\text{A46})$$

Basic monetary and fiscal rule

Regime 1

$$\ln(1 + R_t) = \ln(1 + \bar{R}) + \phi_{r\pi} \ln\left(\frac{\Pi_t}{\bar{\Pi}}\right) + \phi_{ry} \ln\left(\frac{y_t}{\bar{y}}\right) + \ln \xi_{R,t} \quad (\text{A47})$$

$$1 + R_t = E_t \Pi_{t+1} (1 + r_t) \quad (\text{A48})$$

$$\tau_{H,t} = \bar{\tau}_H + \phi_{\tau b} (b_{P,t-1} - b_{P,t}) \quad (\text{A49})$$

Regime 2

Replace (A47) by (A47') and drop (A49).

$$\ln(1 + R_t) = \ln(1 + \bar{R}) + \phi_{r\pi} \ln\left(\frac{\Pi_t}{\bar{\Pi}}\right) + \phi_{ry} \ln\left(\frac{y_t}{\bar{y}}\right) + \phi_{rb} (b_{P,t-1} - b_{P,t}) + \ln \xi_{R,t} \quad (\text{A47}')$$

Shocks

$$\ln g_t = (1 - \rho_g) \ln \bar{g} + \rho_g \ln g_{t-1} + x_{g,t} \quad (\text{A50})$$

$$\ln \xi_{Y,t} = (1 - \rho_Y) \ln \bar{\xi}_Y + \rho_Y \xi_{Y,t-1} + x_{Y,t} \quad (\text{A51})$$

$$\ln \xi_{r,t} = (1 - \rho_r) \ln \bar{\xi}_r + \rho_r \xi_{r,t-1} + x_{r,t} \quad (\text{A52})$$

$$\ln p_{M,t}^* = (1 - \rho_{PM}) \ln \bar{p}_M^* + \rho_{PM} p_{M,t-1}^* + x_{pm,t} \quad (\text{A53})$$

Endogenous variables:

$$AC_{PD}, AC_{PX}, AC_{WH}, AC_{WL}, b_B, b_P, c_H, c_L, cr, cu_H, cu_L, dac_{PD}, dac1_{PD}, dac_{PX}, dac1_{PX}, \\ dac_{WH}, dac1_{WH}, dac_{WL}, dac1_{WL}, d_H, g, i, k, l_B, mb, mc, n_H, n_{HB}, n_{HY}, n_L, \\ r, r_D, r_L, p_D, p_M, p_M^*, p_X, p_X^*, re, q, R, w_H, w_L, y, s, \\ z, z_D, z_M, z_X, \tau_H, \Pi, \xi_y, \xi_r$$

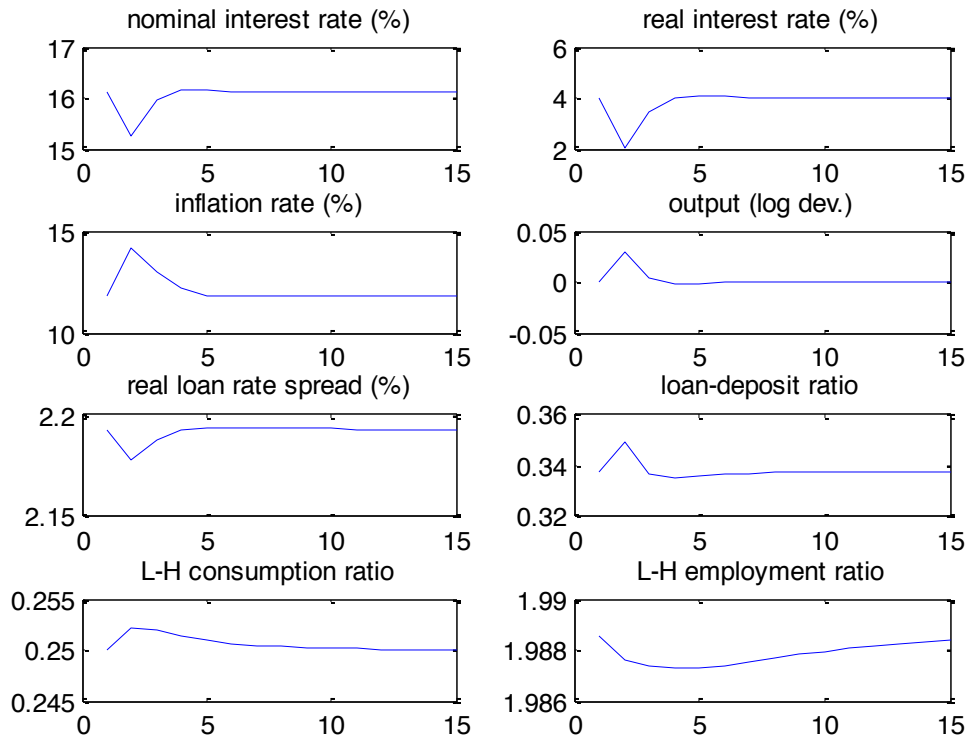
Exogenous variables:

$$\bar{b}_P, cf, g, \bar{l}, \bar{\Pi}, \tau_L, \tau_H, \bar{z}^* (= \psi^* z_t^*), x_g, x_Y, x_r, x_{pm}, x_{cf}$$

Parameters:

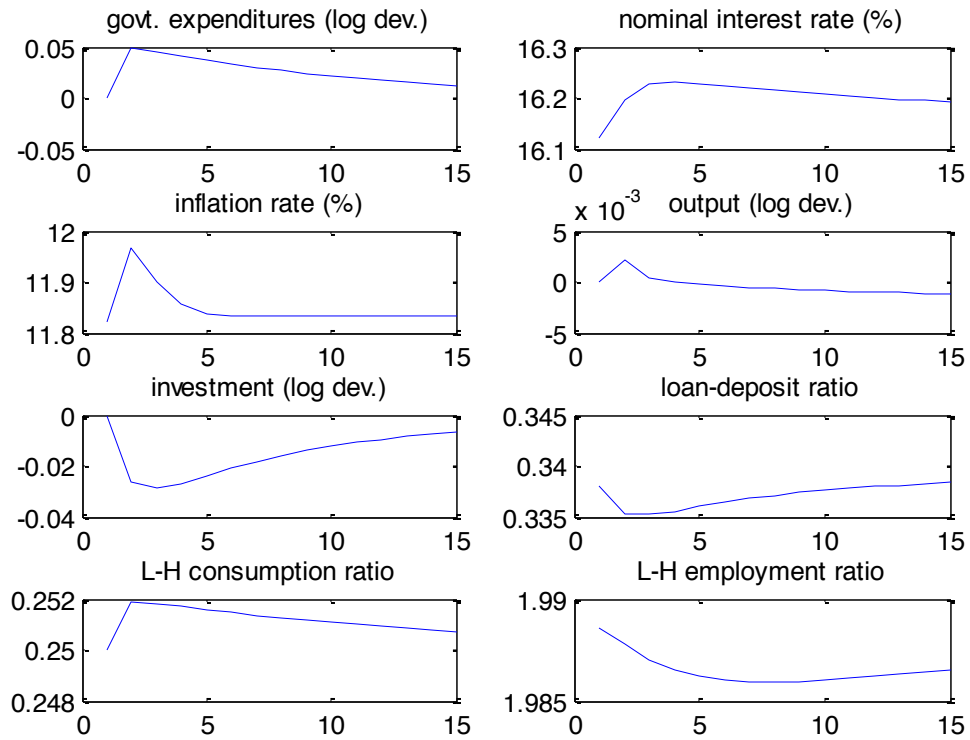
$$\alpha_H, \alpha_L, \beta, \phi_\pi, \phi_y, \phi_\tau, \chi, \delta, \gamma, \kappa, \nu, \theta, \sigma, \xi_{BD}, \xi_{BL}, \\ \xi_{HC}, \xi_{HD}, \xi_{HN}, \xi_F, \xi_{LC}, \omega_I, \omega_H, \omega_L, \omega_P, \eta, \psi$$

Figure 1. Dynamic Effects of a Temporary Reduction in the Interest Rate



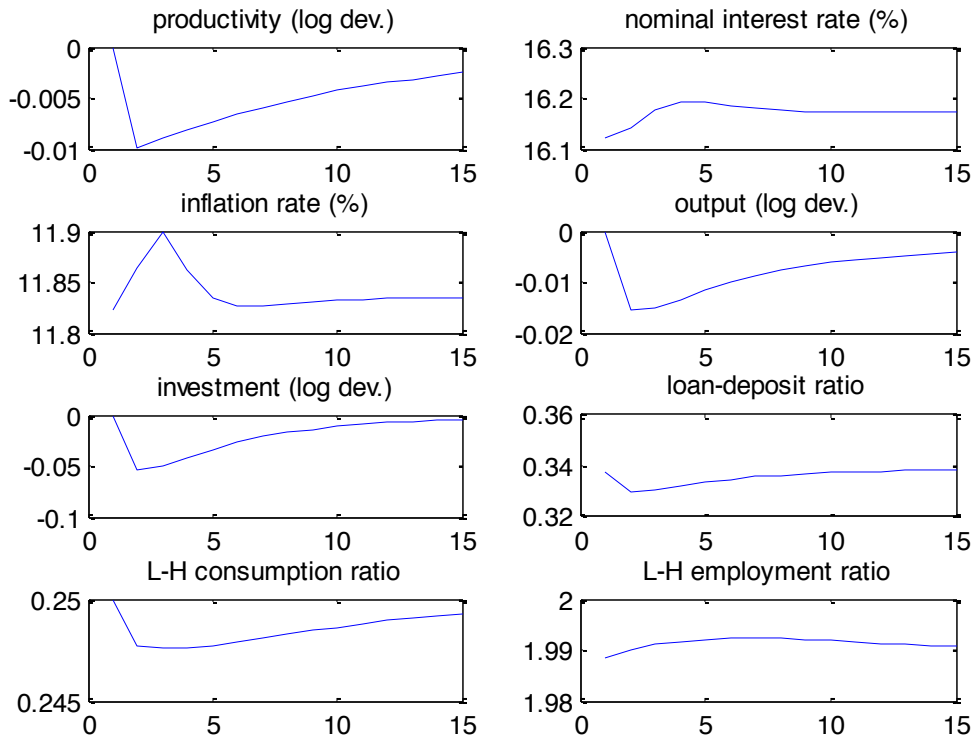
Note: The figure shows the response over 15 quarters (with quarter 1 showing values for initial steady state). The interest rates are converted to a percentage annual value and defined as: nominal interest rate = $R \times 400$, real interest rate = $r \times 400$, real loan rate spread = $(rl - r) \times 400$. Other variables are defined as: output = $\ln(y / \bar{y})$, loan-deposit ratio = lb / dh , L-H consumption ratio = cl / ch , and L-H employment ratio = nl / nh .

Figure 2. Dynamic Effects of a Temporary Increase in Government Expenditures



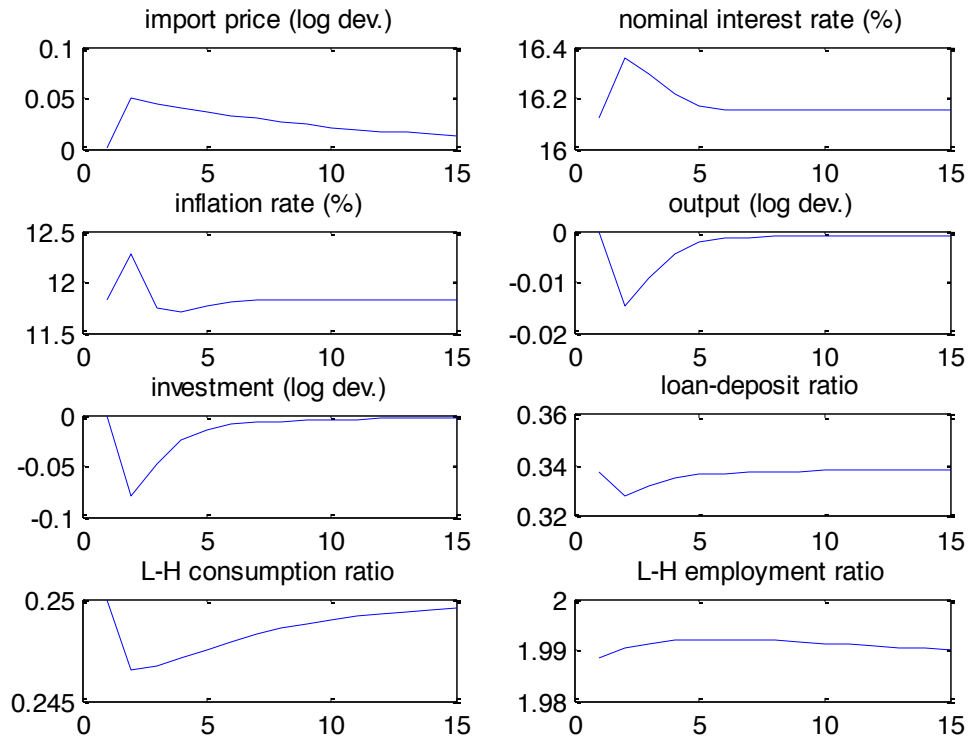
Note: Government expenditures and investment variables are defined as: $\text{govt. expenditures} = \ln(g / \bar{g})$, and $\text{investment} = \ln(i / \bar{i})$. Other variables are defined in the note to Figure 1.

Figure 3. Dynamic Effects of a Temporary Decline in Productivity



Note: The productivity variable is defined as $\ln(\xi_y / \bar{\xi}_y)$. Other variables are defined in the note to Figure 1.

Figure 4. Dynamic Effects of a Temporary Increase in Foreign Import Price Index



Note: The import price index is defined as $\ln(\xi_m / \bar{\xi}_m)$. Other variables are defined in the note to Figure 1.

Figure 5

Dynamic Effects of a Temporary Increase in Govt. Expenditures under Fiscal Dominance

(a) Interest rate responds only to debt ($\phi_{r\pi} = 0, \phi_{rb} = -0.1$)

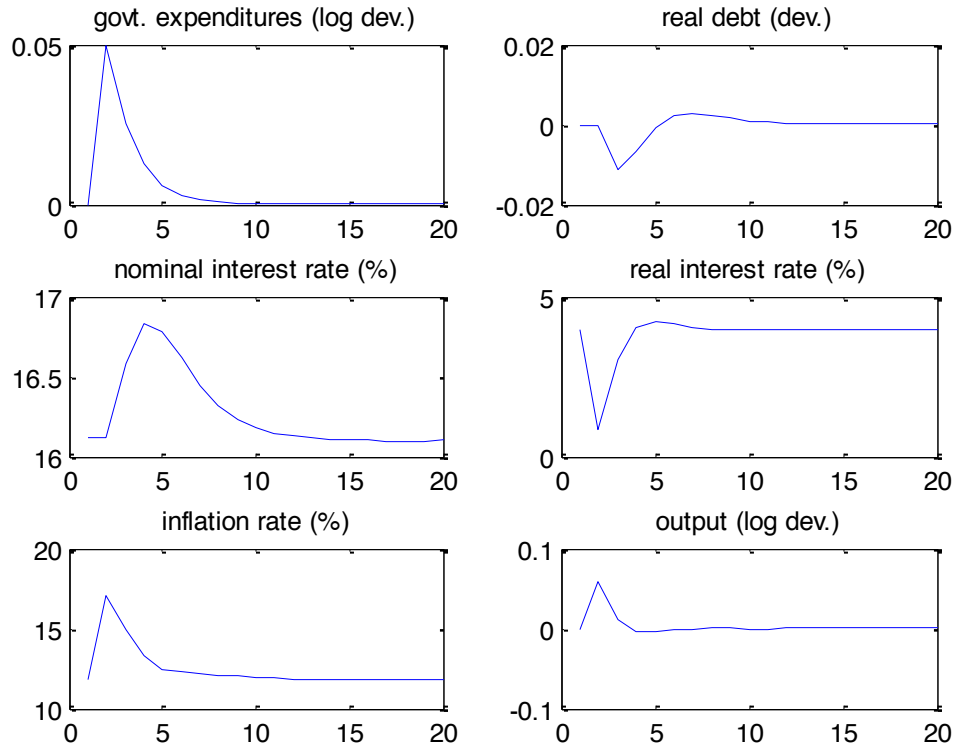


Figure 5 (Continued)

(b) Interest rate responds to both inflation and debt ($\phi_{r\pi} = .5, \phi_{rb} = -.1$)

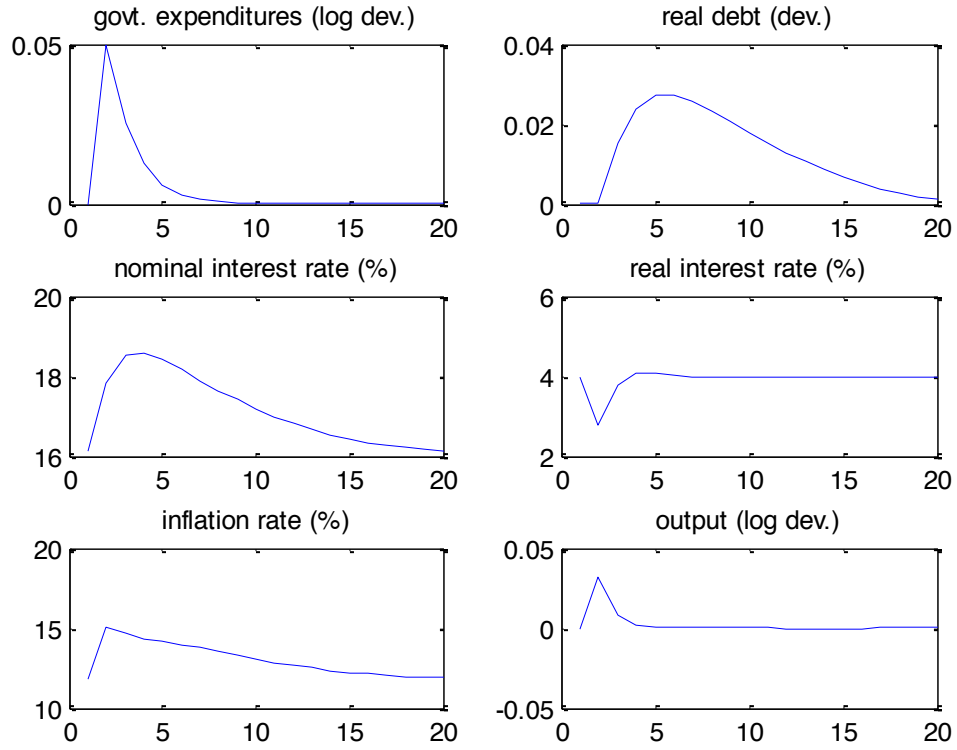


Figure 5 (Continued)

(c) Interest rate responds only to inflation ($\phi_{r\pi} = -.75, \phi_{rb} = 0$)

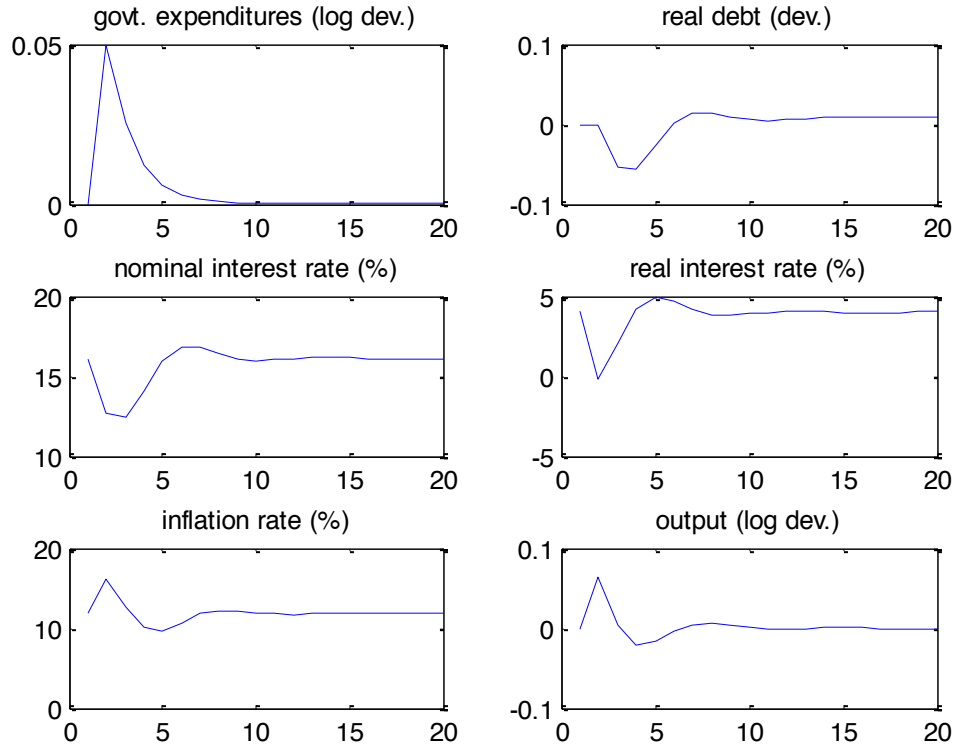


Table 1. Data for Calibration

Description	Average Annual Value
Bank Deposit to GDP Ratio	0.263
Currency to Deposit Ratio	0.389
Cash Reserves to Deposits Ratio	0.052
Government Securities to Deposit Ratio for Banks	0.610
Govt. Expenditures as Share of GDP	0.198
Investment Expenditures as a share of GDP	0.188
Rate of Capital Depreciation	0.084
Share of Imports in GDP	0.161

Table 2. Performance of Different Monetary Rules

	Inflation Gap (Std. Dev.)	Output Gap (Std. Dev.)	Expected Utility	
			<i>L</i> Household	<i>H</i> Household
Baseline Rule ($\rho_{rr} = 0.9, \phi_{r\pi} = 0.5$)	0.0058	0.0948	-91.6400	-39.5828
Less Anti-Inflation ($\rho_{rr} = 0.9, \phi_{r\pi} = 0.15$)	0.0185	0.0900	-91.6538	-39.6047
More Anti-Inflation ($\rho_{rr} = 0.9, \phi_{r\pi} = .85$)	0.0045	0.0954	-91.6375	-39.5700
Less Inertia ($\rho_{rr} = 0.6, \phi_{r\pi} = .5$)	0.0262	0.0920	-91.6502	-39.6311
React to Output ($\rho_{rr} = 0.9, \phi_{r\pi} = .5, \phi_{ry} = 0.25$)	0.1679	0.0703	-91.6898	-39.2842
Stronger Output Reaction ($\rho_{rr} = 0.9, \phi_{r\pi} = .5, \phi_{ry} = 0.5$)	0.2915	0.0640	-91.7403	-39.0868

Note: Inflation and output gaps are defined as $\ln(\Pi / \bar{\Pi}) \times 4$ and $\ln(y / \bar{y})$, respectively. Utility indexes for L and H households equal period utility levels given in (7) and (1).

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