Lecture 1
Empirical regularities and time-series methods

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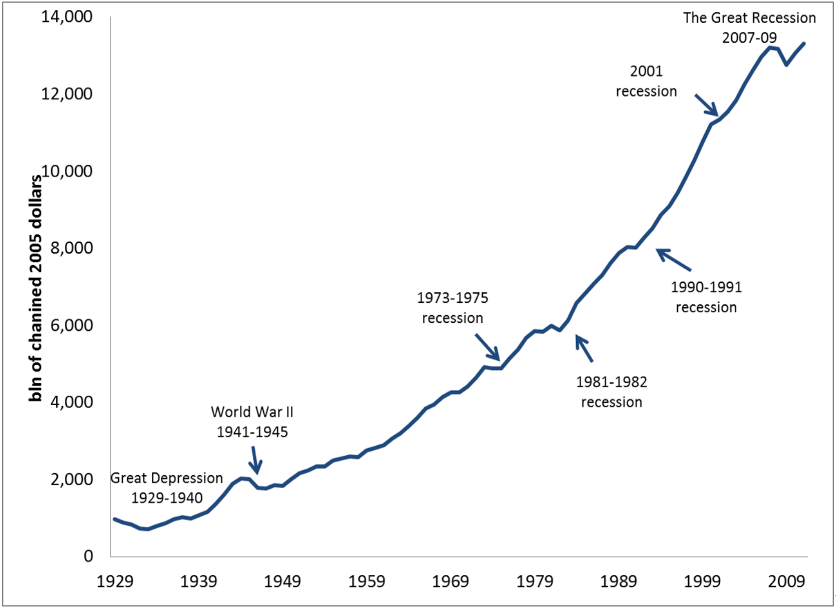
Outline

- Empirical regularities: developed vs developing countries
- Time-series methods: Vector Autoregression (VAR)
- Dynamic stochastic general equilibrium (DSGE) models

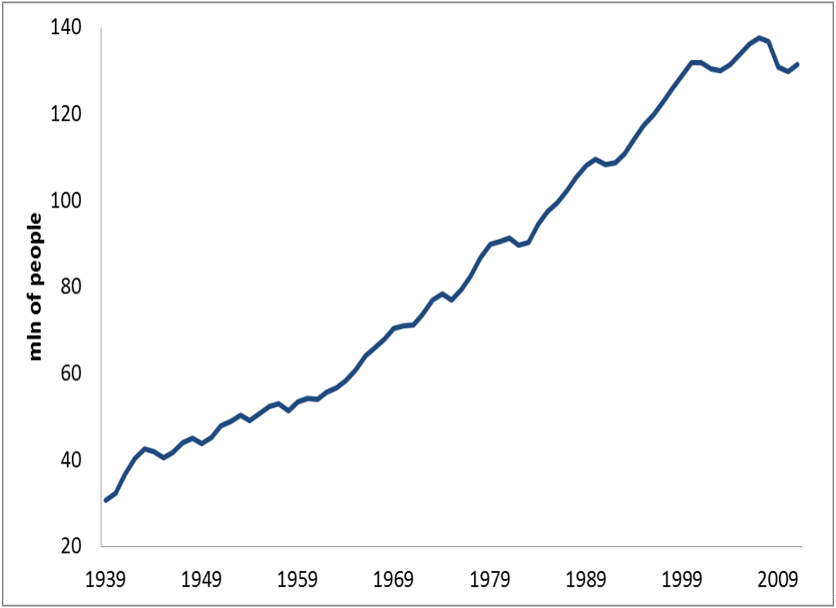
A first look at the data

- Long-run performance (economic growth): US and India
- Short-run fluctuations (business cycles): US and India
- How bad was the most recent recession?
- What is the role of the government and policy (fiscal)?

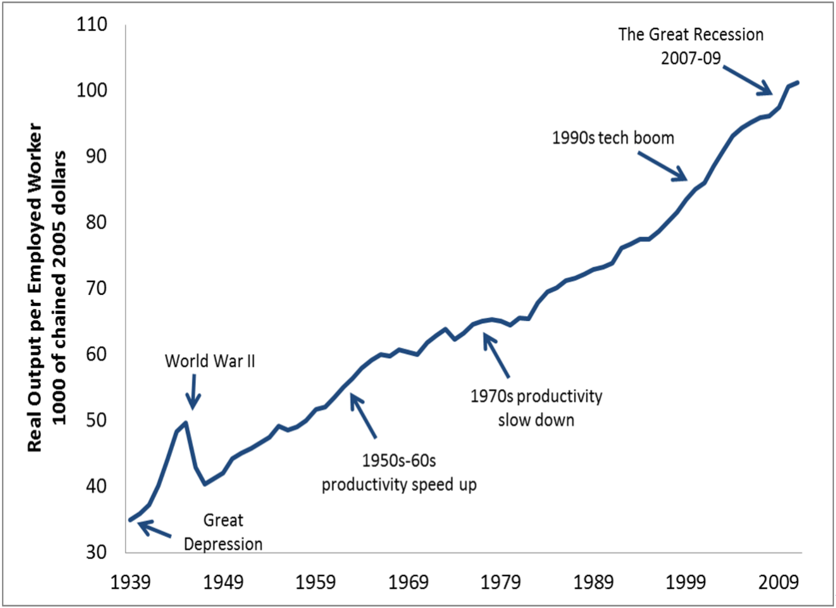
U.S. Real GDP, 1929-2011



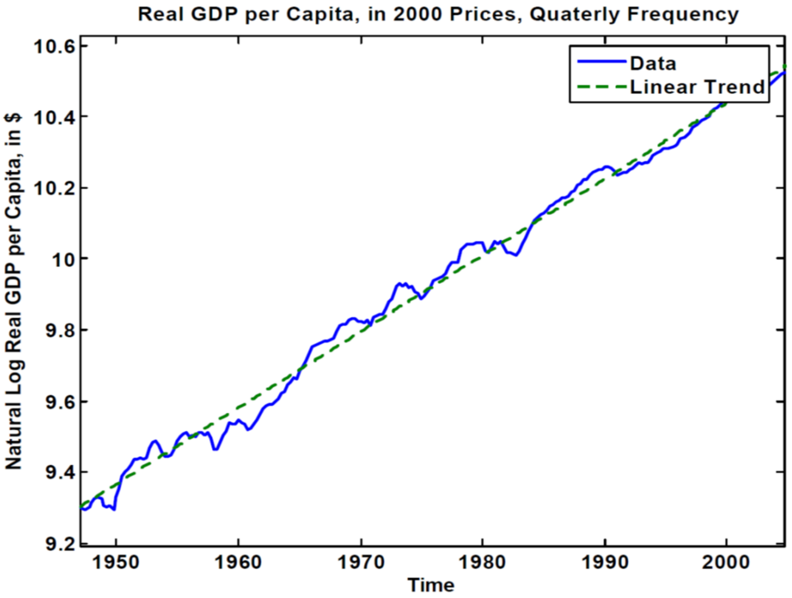
U.S. Employment, 1939-2011



U.S. Labor Productivity, 1939-2011



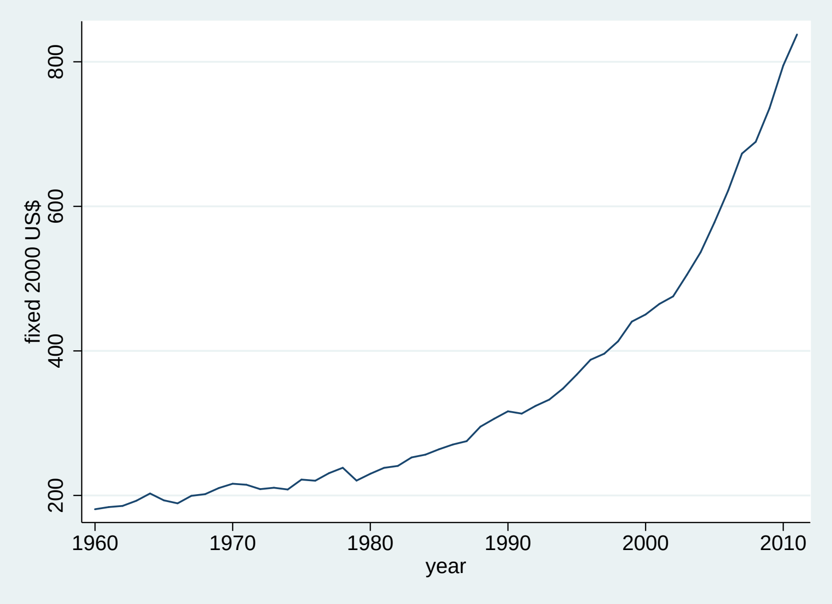
U.S. GDP per capita with trendline



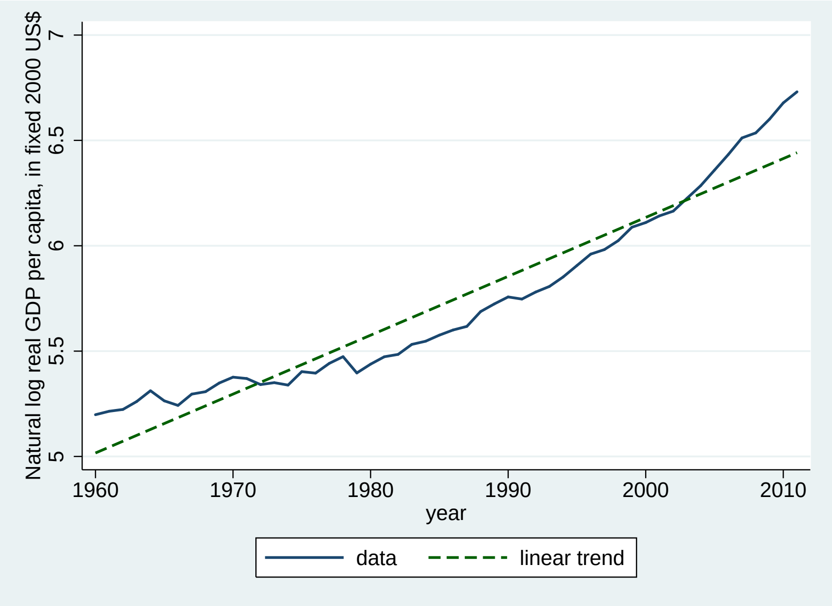
The U.S. growth experience

Period	GDP	GDP per capita
1929-1948	2.54%	1.50%
1948-1973	3.70%	2.22%
1973-1982	1.55%	0.49%
1982-2006	3.60%	2.60%
2006-2011	0.52%	1.71%
1929-2011	**3.18%**	**1.91%**

India GDP per capita, 1960-2011



India GDP per capita with trendline



India growth experience

Period	GDP	GDP per capita
1960-1980	3.43%	1.20%
1981-1990	5.41%	3.19%
1991-2011	6.31%	4.64%
1960-2011	**5.00%**	**3.00%**

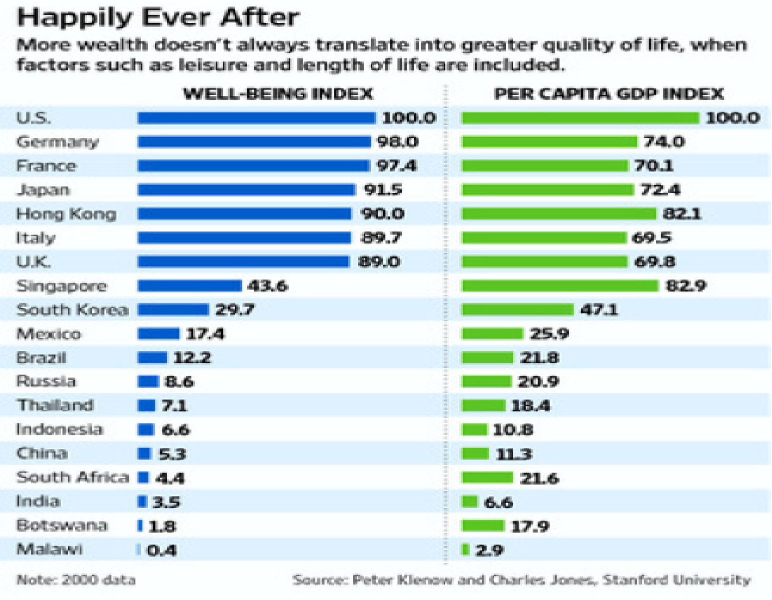
International growth experiences

- link Gapminder

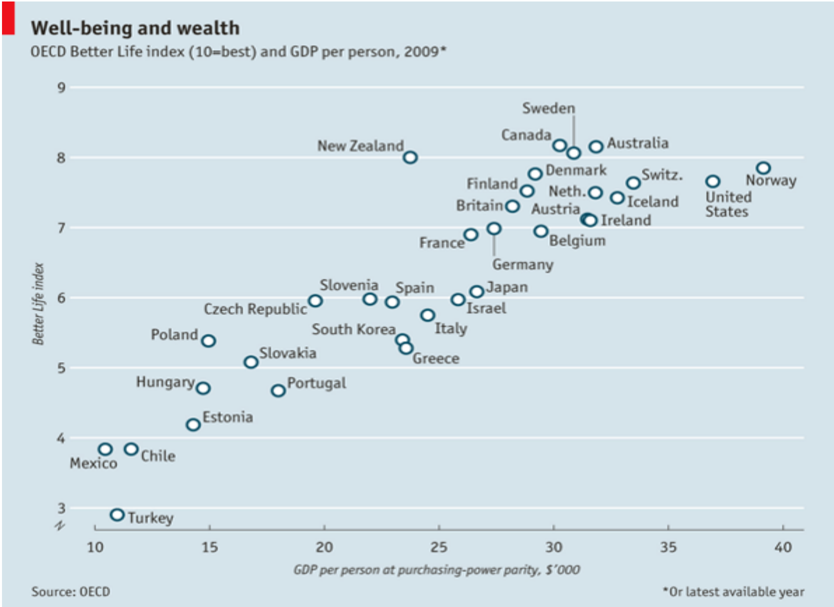
Money isn't everything

- GDP remains the most commonly used measure of country's well-being
- However, more wealth does not necessarily translate into greater well-being when leisure, length of life, pollution, etc. are taken into account
- Alternative measures of well-being have been constructed to account for other aspects:
  - OECD: link OECD
  - Klenow and Jones, 2011: link Jones and Klenow

Money isn't everything...



Money isn't everything, but matters!



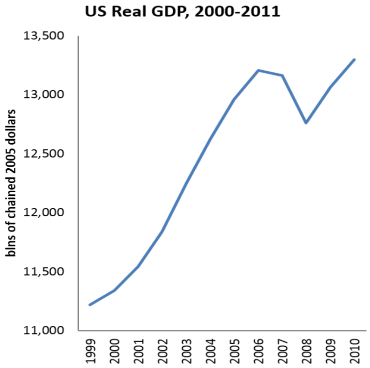
Business cycles: definition

- Fluctuations in aggregate economic activity
  - GDP, employment, trade, financial markets
- Recurrent expansions and contractions
  - Deviations from long run growth
- Persistent but of variable length (i.e. not periodic)
  - fluctuations between 1.5 to 8 years
- Recession
  - decline in output, employment, and trade, lasting at least 6 months
  - simple definition: 2 consecutive quarters of negative output growth

NBER business cycles

- National Bureau of Economic Research – Business Cycle Dating Committee
- Makes a determination of business cycle peaks and troughs (which month)
- Key monthly indicators: employment, personal income, industrial production, sales
- Key quarterly indicator: GDP
- Dating after the fact
- link NBER

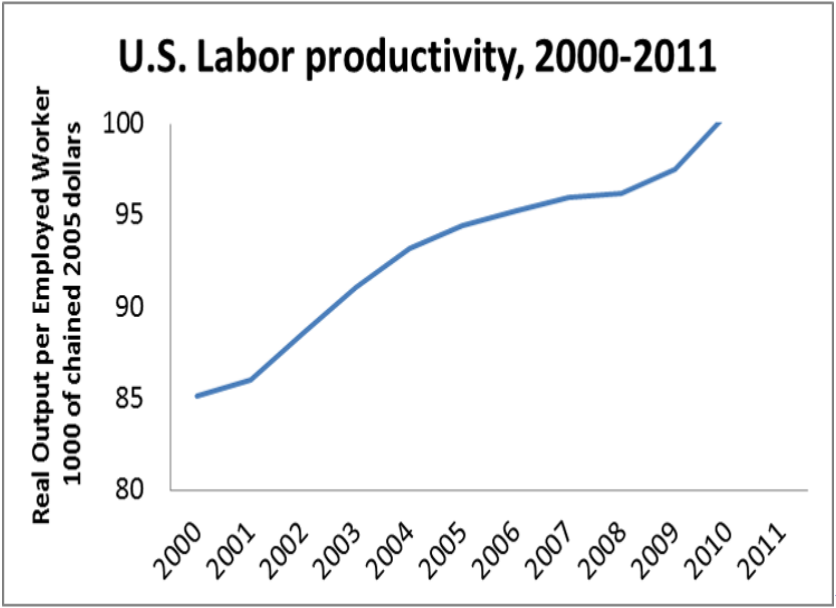
The Great Recession, 2007-2009



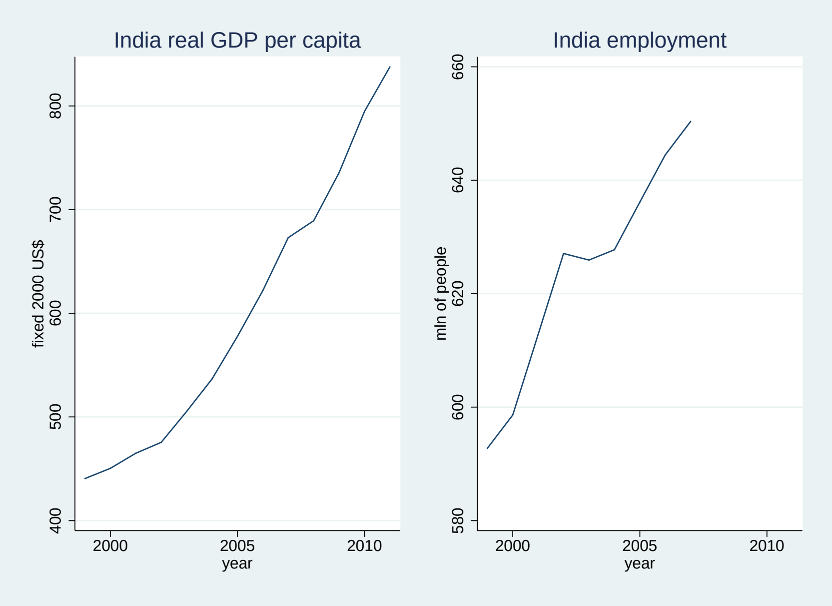
(a) US Real GDP

(b) US employment

The Great Recession, 2007-2009



India, 2007-2009



Trend-cycle decomposition

- NBER labels are helpful.
- Can we develop a formal procedure for defining business cycles?
- Once we have a formal "trend-cycle decomposition" procedure in place, can study cyclical properties of major macro variables.

Trend-cycle decomposition

Various ways of decomposing the series into a trend and cycle components:

- Linear trend
- Hodrick-Prescott (HP) filter
- Band-Pass (BP) filter

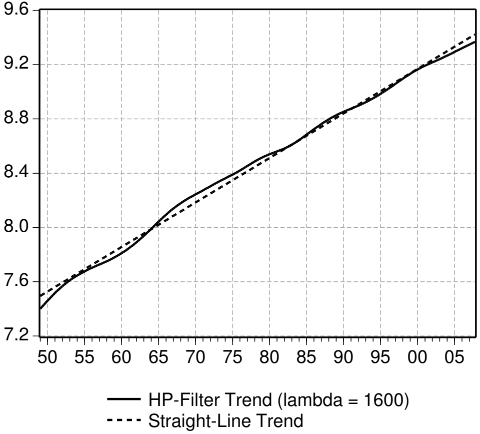
Linear trend

- One “trend-cycle decomposition” of, say, log real GDP plots a straight line through the series.
  - Straight line: trend
  - Deviations from straight line: cycle
- Denote trend real gdp as y_t^*. With *linear trend* for log real GDP, the growth rate of trend real GDP is a constant g:

$$\ln\left(y_t^*\right) - \ln\left(y_{t-1}^*\right) \approx \frac{y_t^* - y_{t-1}^*}{y_{t-1}^*} = g \ .$$

- Is a straight-line trend for log real GDP the “best-possible” trend?
  - Or should we allow the trend rate of growth of real GDP to change over time?
  - Next graph shows trend log real GDP computed 2 ways:
    - Straight Line.
    - HP-Filter.

Figure : Trend Log Real GDP, Trend Computed Using the HP-Filter and a Straight Line, 1949:1 - 2007:4



Cyclical component with linear trend

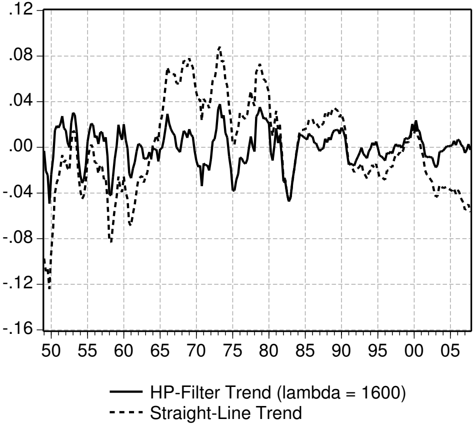
We can evaluate the desirability of the trend by examining the properties of the cycle.

The cycle is defined as the deviation from trend:

$$\begin{aligned}
\text{Cycle} &= \ln(y_t) - \ln(y_t^*) \\
&= \ln\left(\frac{y_t}{y_t^*}\right) \\
&= \ln\left(1 + \frac{y_t - y_t^*}{y_t^*}\right) \\
&= \frac{y_t - y_t^*}{y_t^*} .
\end{aligned}$$

In this case, the cycle is equal to the percent deviation of real GDP from its trend.

Figure : Log Real GDP less Trend, Trend Computed Using the HP-Filter and a Straight Line, 1949:1 - 2007:4



Cyclical component with linear trend

The cycle based on the straight-line trend has the feature that it is almost always below zero over the 1949-1965 and 1990-2007 periods; and above zero in the 1970s and 80s.

This is undesirable. A cyclical variable has two desired properties:

1. Avg. value of the variable is approx. zero in any sub-sample of the data of reasonable length.
2. Variable crosses the zero line at a relatively frequent pace.

Cycle based on the HP-Filter trend appears to have both properties.

Linear trend

- One reason straight-line trend for log real GDP has undesriable properties: Trend rate of growth of real GDP appears to change over time!
- Example:
  - average real GDP growth, 1947-1973: 3.8 percent per year
  - average real GDP growth, 1973-2007: 3.0 percent per year
- Straight-line trend "averages" through these different growth rates, producing cycles with average values different from zero for lengthy periods of time.

H-P filter

- Seems to make sense to allow trend rate of growth to change over time. But how?

- Procedure proposed by Hodrick and Prescott (HP-Filter): compute $\ln(y_t^*)$ in each period to minimize the following:

$$\sum_{t=1}^{T} [\ln(y_t) - \ln(y_t^*)]^2 \; + \; \lambda \sum_{t=2}^{T-1} \left[\Delta \ln\left(y_{t+1}^*\right) - \Delta \ln(y_t^*)\right]^2 \;,$$

where $\Delta \ln(y_t^*) \equiv \ln(y_t^*) - \ln\left(y_{t-1}^*\right)$.

H-P filter

$$\sum_{t=1}^{T} \left[\ln\left(y_t\right) - \ln\left(y_t^*\right)\right]^2 \; + \; \lambda \sum_{t=2}^{T-1} \left[\Delta \ln\left(y_{t+1}^*\right) - \Delta \ln\left(y_t^*\right)\right]^2$$

- λ is a "smoothing parameter"
- $\lambda = 0$ implies changes to trend are unimportant: $\ln\left(y_t\right) = \ln\left(y_t^*\right)$
- $\lambda \to \infty$ implies straight line trend: $\Delta \ln\left(y_{t+1}^*\right) = \Delta \ln\left(y_t^*\right) = g$

H-P filter

Typical business cycle studies using quarterly data set $\lambda = 1,600$.

As Hodrick and Prescott explain:

> *Our prior view is that a 5 percent cyclical component is moderately large, as is a one-eighth of 1 percent change in the growth rate in a quarter. This led us to select* $\sqrt{\lambda} = 5/(1/8) = 40$ *or* $\lambda = 1,600$ *as a value for the smoothing parameter.*

A moderately large quarterly change in the growth rate of trend is $1/8$ of one percent implies a moderately large *annual* change in the growth rate of trend could be $4/8 = 1/2$ of one percent.

$\sqrt{\lambda} = 5/(1/2) = 10$ or $\lambda = 100$ for annual data.

H-P filter

$$\sum_{t=1}^{T} \left[\ln\left(y_t\right) - \ln\left(y_t^*\right)\right]^2 \; + \; \lambda \sum_{t=2}^{T-1} \left[\Delta \ln\left(y_{t+1}^*\right) - \Delta \ln\left(y_t^*\right)\right]^2$$

The sequence of y_t^* that minimizes the above expression (given λ) can be found with a "Kalman Filter" or by a simple matrix inversion.

H-P filter

The link below has a Microsoft Excel add-in that allows you to HP-Filter any sequence of data.

$http://www.web-reg.de/hp_addin.html$

In STATA use

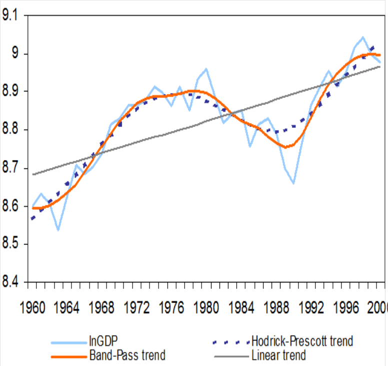
- **hprescott** add-on module
- **tsfilter hp** built-in module in stata 13

Band-Pass filter

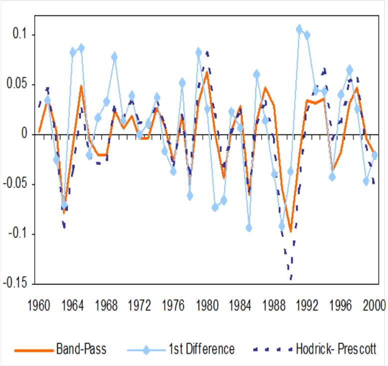
- Band-Pass (BP) filter is an alternative filtering technique developed by Baxter and King (1995)[1].
- It is based on the decomposition of the data into three components: low frequency, business cycle and high frequency components.
- It extracts the cyclical (business cycle) component by eliminating the components outside a range of frequencies, specified by a researcher and dependent on the characteristics of the business cycles in the particular economy under study.
- Baxter and King (1995) defined business cycle to be no less than 6 quarters in duration and last fewer than 32 quarters. For the annual data this corresponds to a cycle of 2 to 8 years.
- You can download an Excel add-in for the BP filter at the same link as above
- In stata 13 use tsfilter bk.

[1] Marianne Baxter and Robert G. King, 1999. "Measuring Business Cycles: Approximate Band-Pass Filters For Economic Time Series," The Review of Economics and Statistics, MIT Press, vol. 81(4), pages 575-593.

Comparing filters



(a) Trend components of real GDP, Argentina



(b) Cyclical components of real GDP, Argentina

Business cycles

- After you have extracted the cyclical component of macro series, you are ready to analyze their business cycle properties.

- Typical statistics of interest:
  - volatility: std.dev.
  - comovement with output and other variables: correlation coefficient
  - persistence: AR(k), k=1,... coefficients
  - conditional responses: Vector Autoregression (VAR)

Business cycles: Volatility

Table 6. *Standard deviations for 15 developing countries, HP, percentage*[a]

	GDP	CON	PRC	PUC	INV	PRI	PUI	IMP	EXP	SAV	TOT	RER	INF	CPI	M2	CRE	FDI	AID	ODA
Sub-Saharan Africa																			
Côte d'Ivoire	1.33	2.26	2.06	3.53	5.42	9.16	8.11	2.55	4.24	8.46	4.76	5.32	53.28	1.59	5.54	6.57	53.47	10.08	19.54
Malawi	2.14	2.71	4.43	3.95	12.41	28.90	10.36	6.43	5.15	41.07	3.58	na	na	na	3.59	5.02	346.81	8.86	18.60
Nigeria	2.09	3.89	4.00	7.87	7.29	na	na	4.84	6.03	8.79	9.14	7.11	31.54	3.67	5.99	8.33	46.60	11.49	20.12
S. Africa	1.13	0.58	0.70	0.84	6.27	8.96	2.63	3.80	1.17	3.30	1.98	3.78	4.91	0.43	3.42	3.74	232.94	na	na
Zimbabwe	1.68	2.12	4.37	8.22	7.33	na	na	3.28	3.50	10.78	2.43	2.82	24.36	1.65	4.13	6.92	204.17	21.40	23.84
Latin America																			
Chile	1.01	2.31	2.65	1.03	7.67	12.64	6.51	3.72	2.04	8.85	3.07	3.47	na	na	4.78	5.37	50.25	50.04	7.15
Colombia	0.40	0.40	0.81	2.49	3.47	5.50	4.20	2.78	2.64	3.06	3.89	1.75	9.59	1.05	1.98	9.39	16.04	13.79	22.46
Mexico	0.99	1.04	1.13	0.93	3.08	4.41	4.64	5.80	1.45	2.38	3.67	4.98	21.04	4.14	7.34	11.11	13.77	18.99	17.32
Peru	1.70	1.80	1.73	3.14	4.65	5.50	4.85	4.81	3.36	7.10	5.22	5.39	na	na	8.17	8.11	78.33	9.29	10.24
Uruguay	1.17	1.62	1.84	1.78	9.95	13.10	8.34	2.87	2.73	11.80	4.50	4.51	na	na	5.62	8.14	264.31	17.79	34.77
Asia and North Africa																			
India	1.12	0.82	0.91	1.08	3.34	5.59	3.22	3.32	1.96	3.04	4.19	2.44	44.17	1.70	2.82	2.21	212.26	10.86	12.35
Korea	1.50	1.13	1.42	1.30	5.56	7.05	4.90	3.94	5.22	6.22	2.43	2.84	34.84	2.72	4.55	4.44	na	91.68	56.14
Malaysia	0.88	1.08	1.27	1.50	3.28	5.78	4.40	3.48	2.06	5.05	2.86	1.54	31.03	0.77	1.56	5.97	11.08	19.38	33.13
Morocco	1.16	1.37	1.56	1.50	2.31	4.93	3.82	2.29	1.71	4.11	1.25	1.47	12.97	0.61	1.67	5.99	200.74	10.45	6.15
Pakistan	0.61	1.10	1.49	3.06	1.71	6.30	2.30	4.15	4.61	8.80	4.98	1.84	14.68	0.80	3.08	3.22	42.07	11.61	7.38

[a] GDP—real gross domestic product, CON—real total consumption, PRC—real private consumption, PUC—real general government consumption, INV—real gross domestic investment, PRI—real private investment, PUI—real public investment, IMP—real imports of goods and services, EXP—real exports of goods and services, SAV—real savings, TOT—terms of trade index, RER—real effective exchange rate index, INF—inflation (percentage change in consumer prices index), CPI—consumer price index (1995 = 100), M2—nominal money and quasi money (M2), CRE—private sector credit, FDI—foreign direct investment, AID—official development assistance (disbursements), ODA—official development assistance (commitments). Data sources include World Bank (2001), Global Development Finance (2001), OECD (2001), IMF (2001c) and Global Development Network database from www.worldbank.org/research/growth/.

Business cycles: Volatility

Table 10. *Standard deviations for five developed countries, HP, percentage*[a]

	GDP	CON	PRC	PUC	INV	PRI	PUI	IMP	EXP	SAV	TOT	RER	INF	CPI	M2	CRE	FDI	AID	ODA
Canada	1.56	0.85	1.22	0.88	6.40	na	na	4.34	3.85	6.03	2.58	3.09	46.22	1.37	4.05	5.00	71.88	na	na
France	0.96	0.63	0.82	0.75	5.21	na	na	3.83	2.28	3.92	3.10	7.60	17.80	1.31	na	25.14	na	na	na
Japan	1.39	1.12	1.26	1.23	4.29	na	na	5.93	3.97	3.02	6.76	8.13	57.60	1.92	8.36	8.22	na	na	na
UK	1.72	1.34	1.88	0.91	7.41	na	na	3.52	2.09	5.39	2.64	7.34	30.55	2.48	na	13.36	81.60	na	na
USA	1.73	1.09	1.41	0.66	6.68	na	na	5.37	3.90	5.28	3.11	na	25.38	1.61	1.15	2.66	32.55	na	na

[a]See Table 6. Following Ravn and Uhlig (2002) the smoothing parameter is 1,600/256.

Figure : Source: Rand and Tarp, 2002

Business cycles: Co-movement with output

Table 8. *Cross correlations with output for 15 developing countries, HP*

	CON	PRC	PUC	INV	PRI	PUI	IMP	EXP	SAV	TOT	RER	INF	CPI	M2	CRE	FDI	AID	ODA
Sub-Saharan Africa																		
Côte d'Ivoire	0.82	0.78	0.78	0.38	0.06	0.42	0.21	−0.53	0.11	0.45	−0.13	−0.11	−0.32	0.01	−0.14	0.06	−0.38	−0.11
Malawi	0.78	0.77	−0.42	−0.16	−0.04	−0.32	0.11	0.02	0.09	0.17	na	na	na	0.68	0.04	−0.22	−0.04	−0.09
Nigeria	−0.00	0.04	−0.14	−0.05	na	na	−0.08	0.63	0.67	0.35	0.11	−0.52	−0.31	0.22	0.01	0.05	−0.18	−0.09
S. Africa	0.76	0.78	0.15	0.88	0.92	0.08	0.93	−0.12	0.48	0.29	−0.04	−0.25	−0.35	0.15	0.16	0.17	na	na
Zimbabwe	0.70	0.45	0.02	0.64	na	na	0.05	0.58	0.70	−0.53	−0.66	−0.44	−0.75	0.29	−0.09	0.25	0.00	0.06
Latin America																		
Chile	0.90	0.90	0.61	0.88	0.86	0.44	0.92	0.54	0.66	0.45	0.43	na	na	0.34	0.53	0.35	−0.16	0.18
Colombia	0.66	0.39	−0.02	0.38	0.55	−0.51	0.50	0.23	0.39	0.05	−0.10	−0.52	−0.58	0.43	0.42	−0.13	0.21	0.16
Mexico	0.94	0.94	0.53	0.78	0.38	0.72	0.82	−0.47	0.01	0.67	0.68	−0.58	−0.52	0.83	0.82	0.47	−0.16	0.30
Peru	0.79	0.82	0.48	0.66	0.61	0.52	0.74	0.29	0.27	0.10	0.37	na	na	0.19	0.16	0.09	−0.19	0.02
Uruguay	0.91	0.89	0.28	0.21	0.19	0.21	0.65	0.17	0.14	0.14	0.35	na	na	0.34	0.12	0.22	0.16	0.17
Asia and North Africa																		
India	0.92	0.92	0.10	0.50	0.47	0.17	0.39	−0.34	0.41	0.12	0.38	−0.23	0.12	−0.07	0.20	−0.19	−0.04	0.23
Korea	0.63	0.72	−0.37	0.62	0.66	−0.18	0.83	0.47	0.88	0.67	0.46	−0.35	−0.66	0.70	0.36	na	−0.13	0.06
Malaysia	0.47	0.53	−0.07	0.62	0.69	−0.38	0.65	0.69	0.84	0.81	0.18	0.50	0.20	0.26	0.16	0.29	−0.13	0.23
Morocco	0.84	0.81	0.40	0.49	0.54	−0.10	0.18	0.18	0.47	−0.11	0.11	0.19	0.10	0.07	−0.21	−0.19	−0.00	−0.08
Pakistan	0.44	0.42	−0.06	0.04	0.08	−0.16	−0.14	0.23	0.42	−0.03	0.10	0.03	0.03	−0.30	−0.18	−0.03	0.04	0.00

Notes: See Table 6.

Business cycles: Co-movement with output

Table 11. *Cross correlations with output for five developed countries, HP*[a]

	CON	PRC	PUC	INV	PRI	PUI	IMP	EXP	SAV	TOT	RER	INF	CPI	M2	CRE	FDI	AID	ODA
Canada	0.84	0.94	−0.39	0.91	na	na	0.81	0.69	0.90	−0.34	−0.32	−0.11	−0.73	−0.19	0.21	0.38	na	na
France	0.59	0.72	−0.26	0.85	na	na	0.81	0.78	0.90	0.46	0.09	0.30	−0.44	na	0.21	na	na	na
Japan	0.85	0.87	0.08	0.86	na	na	0.68	−0.13	0.87	0.27	−0.02	0.27	−0.68	0.25	0.21	na	na	na
UK	0.82	0.87	−0.36	0.90	na	na	0.84	0.62	0.80	0.33	0.13	−0.08	−0.73	na	0.34	0.14	na	na
USA	0.88	0.90	−0.22	0.95	na	na	0.87	0.39	0.93	0.55	na	0.17	−0.62	0.39	0.81	0.43	na	na

[a] See Table 10.

Figure : Source: Rand and Tarp, 2002

Business cycles: Summary

- Volatility of macro aggregates is higher in developing countries than in developed economies.
- Consumption is more volatile than output in developing countries, while the opposite is true of developed economies.
- Consumption and investment are strongly pro-cyclical with output in both groups.
- Fiscal variables are pro-cyclical in developing countries, but counter-cyclical in advanced economies.
- Real interest rates are counter-cyclical in developing countries, but are a-cyclical in developed economies.

From unconditional to conditional correlations

- Correlations give us unconditional relationship between the two variables
- Not very useful if interested in analyzing the effects of policy on macroeconomy
- Can we isolate the effects of policy on macro variables?
- This is not straightforward because policy is endogenous to the economic conditions.
- Thus, in order to isolate the effects of unanticipated policy actions, one must extract the exogenous component of those actions, i.e. the one that is not responding to the state of the economy.
- How? One approach: Vector Autoregressions (VAR).

VAR: Introduction

- We will be working with time-series data, so few cautions are in order:
  - An assumption of random sampling (i.e. the variables and error terms were independent across observations) often made in cross-section data does not apply in time-series. In time series data there is a strong likelihood of some dependence over time.
  - Time-series variables typically are non-stationary, i.e. they have a distinct trend. This invalidates the usual asymptotic results for consistency of estimators.
- VAR methodology was pioneered by Sims (1980) "Macroeconomics and Reality", published in Econometrica.
- He emphasized that in a macroeconomy all variables are endogenous in some respect, and therefore the interaction between them needs to be taken into account and modelled.

VAR: A little bit of theory

- A VAR is an n equation, n variable model in which each variable is explained by its own lagged values, plus (current) and past values of the remaining $n-1$ variables.
- Effectively, it is a reduced form of a dynamic economic system of a vector of variables z_t.
- VAR's have three varieties: reduced form, recursive and structural. In each case, the objective is to solve the identification problem – i.e. recover estimates of all parameters.
- Consider a two-variable, 2-equation VAR model in output (Y_t) and government consumption (G_t).
- The so-called structural form is

$$\begin{aligned} Y_t &= -a_{yg} G_t + b_{yg} G_{t-1} + b_{yy} Y_{t-1} + \varepsilon_{yt} \\ G_t &= -a_{gy} Y_t + b_{gg} G_{t-1} + b_{gy} Y_{t-1} + \varepsilon_{gt} \end{aligned}$$

where

- a_{yg}, a_{gy}, b's are structural parameters to be estimated
- ε_{yt}, ε_{gt} are the uncorrelated structural shocks with standard deviation σ_y and σ_g, also to be estimated.

VAR: A little bit of theory

- Note that the two equation cannot be estimated with OLS because of correlation between the error term and regressors. Recall that OLS is inconsistent in this case.
- This is easy to see. Suppose we estimated the first equation above with OLS. G_t will be correlated with ε_{yt}:

$$\begin{aligned}
cov(G_t, \varepsilon_{yt}) &= cov(-a_{gy} Y_t + b_{gg} G_{t-1} + b_{gy} Y_{t-1} + \varepsilon_{gt}, \varepsilon_{yt}) \\
&= -a_{gy} cov(Y_t, \varepsilon_{yt}) \\
&= -a_{gy} cov(-a_{yg} G_t + b_{yg} G_{t-1} + b_{yy} Y_{t-1} + \varepsilon_{yt}, \varepsilon_{yt}) \\
&= a_{gy} a_{yg} cov(G_t, \varepsilon_{yt}) - a_{gy} cov(\varepsilon_{yt}, \varepsilon_{yt}),
\end{aligned}$$

  which implies

$$cov(G_t, \varepsilon_{yt}) = -\frac{a_{gy}\sigma_y^2}{1 - a_{gy} a_{yg}}$$

- Thus, OLS estimates of equation 1 will yield inconsistent estimates of the parameters unless we assume that $a_{gy} = 0$; that is the contemporaneous effect of output shocks on government consumption is zero.

VAR: A little bit of theory

- Suppose indeed we assumed that $a_{gy} = 0$. Then equation 1 can be estimated with OLS.
- Effectively, we are saying that a shock to output, $\varepsilon_{yt} = 1$ will have an effect on Y_t equal to 1; and on G_t equal to 0.
- A shock to G_t of $\varepsilon_{gt} = 1$ will have an effect on G_t equal to 1, and an effect on Y_t equal to $-a_{yg}$.
- This way of estimating the system is called recursive, because we are estimating equation after equation by excluding one variable from the second equation.

VAR: A little bit of theory

- Hence, our VAR in a structural form is

$$\begin{bmatrix} 1 & a_{yg} \\ a_{gy} & 1 \end{bmatrix} \begin{bmatrix} Y_t \\ G_t \end{bmatrix} = \begin{bmatrix} b_{yy} & b_{yg} \\ b_{gy} & b_{gg} \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ G_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{gt} \end{bmatrix}$$

- In a reduced form it is

$$\begin{bmatrix} Y_t \\ G_t \end{bmatrix} = \begin{bmatrix} 1 & a_{yg} \\ a_{gy} & 1 \end{bmatrix}^{-1} \begin{bmatrix} b_{yy} & b_{yg} \\ b_{gy} & b_{gg} \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ G_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & a_{yg} \\ a_{gy} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{gt} \end{bmatrix}$$

- The reduced form system can be consistently estimated equation by equation using OLS. The problem, now, however, is that error terms in these two equations are correlated with one another. That is, we can write the reduced form shocks as

$$\begin{aligned} e_{yt} &= \frac{1}{1 - a_{gy} a_{yg}} (\varepsilon_{yt} - a_{yg} \varepsilon_{gt}) \\ e_{gt} &= \frac{1}{1 - a_{gy} a_{yg}} (\varepsilon_{gt} - a_{gy} \varepsilon_{yt}) \end{aligned}$$

VAR: A little bit of theory

- Thus, the shocks do not represent independent structural shocks to the separate equations, as before, but instead are combinations of the two shocks. Hence, the reduced-form system does not have a structural or theoretical explanation.
- Not all is lost, however. If $a_{gy} = 0$, then

$$
\begin{aligned}
e_{yt} &= \varepsilon_{yt} - a_{yg}\varepsilon_{gt} \\
e_{gt} &= \varepsilon_{gt}
\end{aligned}
$$

- Which implies
  1. We can first estimate the reduced form equation for G_t, i.e. G_t on G_{t-1} and Y_{t-1} using OLS. Then use it to recover the structural shock ε_{gt} as e_{gt}.
  2. Estimate structural equation for Y_t, i.e. Y_t on G_t, G_{t-1} and Y_{t-1} using OLS. Note that OLS is consistent since $a_{gy} = 0$. Use it to recover residual ε_{yt}.
  3. Thus, all structural parameters are estimated.

VAR: A little bit of theory

With more than two variables, the estimation can be streamlined as

- Estimate the reduced form for all equations
- Compute the Choleski decomposition of the variance-covariance matrix of the residuals.
  - To show this, let's write the reduced form VAR in vector notation as

$$z_t = A^{-1}Bz_t + A^{-1}\varepsilon_t$$

  - The var-cov matrix of $e_t = A^{-1}\varepsilon_t$ becomes

$$\Sigma_e = A^{-1}\Sigma_\varepsilon^{1/2}\Sigma_\varepsilon^{1/2}A^{-1\prime}$$

  - The Choleski decomposition of Σ_e is

$$chol(\Sigma_e) = \Sigma_\varepsilon^{1/2}A^{-1\prime}$$

  - Now A^{-1} can be recovered as $chol(\Sigma_e)'\Sigma_\varepsilon^{1/2}$

VAR: Impulse response functions

- Impulse responses give the effects of current and future values of a variable of interest to a 1 unit increase (or to a 1 standard deviation increase) in the current value of one of the VAR errors.
- Since A and Σ_ε are now known, we can just use the definition:

$$\begin{bmatrix} e_{yt} \\ e_{gt} \end{bmatrix} = \begin{bmatrix} 1 & -a_{yg} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{gt} \end{bmatrix}$$

- A 1-unit positive shock to government consumption implies:

$$\begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{gt} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

  and leads to

$$\begin{bmatrix} Y_0 \\ G_0 \end{bmatrix} = \begin{bmatrix} 1 & -a_{yg} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- For every $s > 0$:

$$\begin{bmatrix} Y_s \\ G_s \end{bmatrix} = A^{-1} \begin{bmatrix} b_{yy} & b_{yg} \\ b_{gy} & b_{gg} \end{bmatrix} \begin{bmatrix} Y_{s-1} \\ G_{s-1} \end{bmatrix}$$

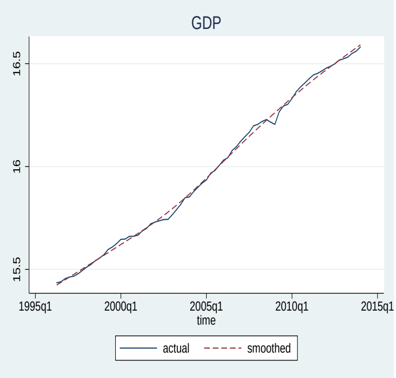
VAR: Forecast error variance decomposition

- Another useful tool in the VAR analysis is variance decomposition
- It separates the variance of a variable into the components attributable to the shocks.
- It allows us to access the relative importance of innovations to different variables.

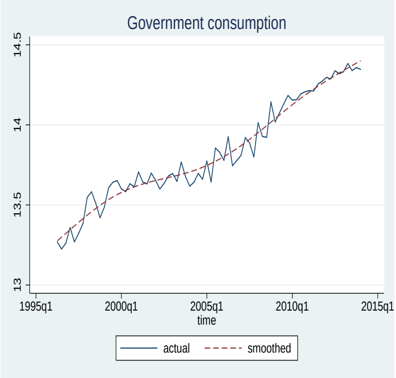
VAR: Application to fiscal shocks in India

- Let's estimate the effects of fiscal policy shocks on output in India during 1996Q1-2014:Q1 period.
- The use of quarterly data is imperative due to the identification assumption that fiscal authorities require a quarter to respond to output shocks (Blanchard and Perotti (2002), Ilzetzki et al. (2013)).
- This implies that the contemporaneous effect of Y on G is zero, or $a_{gy} = 0$ in our model notation.
- In terms of variables ordering in the VAR, this assumption implies: G_t, Y_t. we may also add trade balance as in Ilzetzki et al. (2013).

VAR: Application to fiscal shocks in India

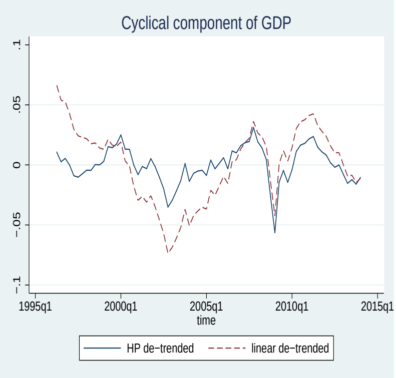


(a) Real GDP, India

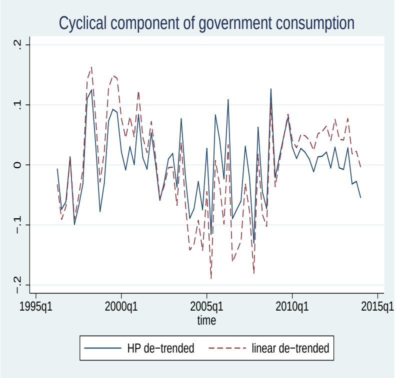


(b) Real G, India

VAR: Application to fiscal shocks in India

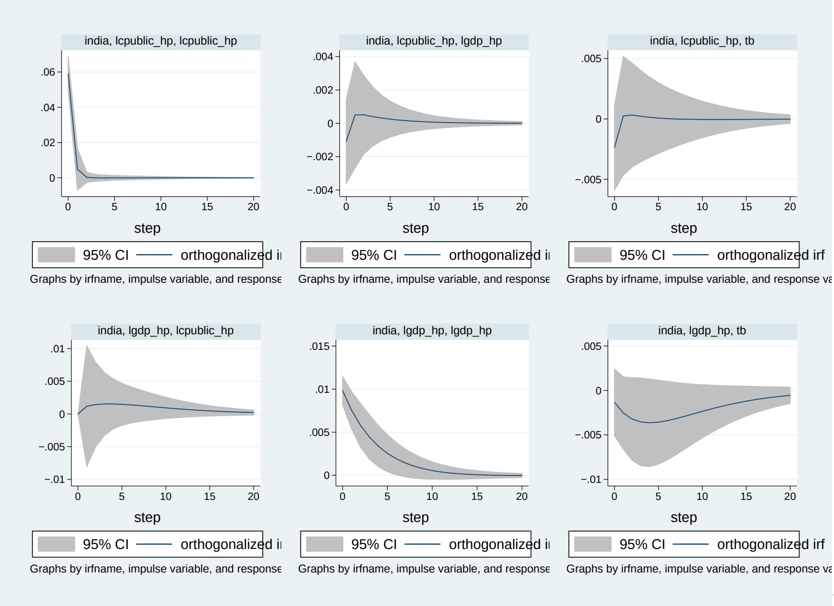


(c) Cycical component of real GDP, India



(d) Cyclical components of real G, India

VAR: Results


india, lcpublic_hp, lcpublic_hp
india, lcpublic_hp, lgdp_hp
india, lcpublic_hp, tb
india, lgdp_hp, lcpublic_hp
india, lgdp_hp, lgdp_hp
india, lgdp_hp, tb
step
95% CI
orthogonalized irf
Graphs by irfname, impulse variable, and response va

Implications

- VARs are often used to motivate more structural models
- Macroeconomists try to build dynamic micro-founded models that are close to the evidence derived from the data, including VARs.
- The advantages of a micro-founded model are
  - it helps develop intuition for the structural relationship between variables
  - it can be used to conduct welfare evaluations of different policies
  - it can be used to conduct counter-factual experiments that are robust to Lucas' critique