

# Lecture 1: Firm Productivity & Trade-Industry Link

## A Selective Primer

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- Linked to estimation of Production Functions
- For example: Cobb Douglas Production Function for  $i^{th}$  firm at time  $t$  :

$$Y_{it} = A_{it} \cdot L_{it}^{\beta} \cdot K_{it}^{\gamma} \cdot M^{\delta}$$

- $Y$  : Measure of Output
- $L$  : Labour,  $K$  : Capital,  $M$ : Intermediate Input: Raw Material/Energy
- $A$  : Total Factor Productivity (TFP): Increases all factor's marginal product simultaneously

# Output or Value Added

- Often data is available for real value added ( $VA$ ): that is, Value of real output ( $Y$ ) minus cost of Intermediate Inputs ( $M$ ).
- Use of  $VA$  instead of  $Y$  in production function estimation assumes that the production function is additive separable in primary inputs ( $L, K$ ) and intermediate inputs (Materials, Intermediate energy inputs).
- If you use  $VA$ , you cannot look at the coefficients of intermediate inputs.

- Additive Separability may be a strong assumption because:
  - Choice of primary inputs is in general not independent of intermediate input prices
  - Marginal rate of substitution between primary inputs will vary in the amount of intermediate inputs.
- So use data on physical output whenever possible or deflated plant sales (though using aggregate product prices for deflation can lead to some bias when products across firms vary in quality).



$$Y_{it} = A_{it} \cdot L_{it}^{\beta} \cdot K_{it}^{\gamma} \cdot M^{\delta}$$

- Transform by taking logs.

$$y_{it} = \beta \cdot l_{it} + \gamma \cdot k_{it} + \delta m_{it} + u_{it}$$

where lower cases reflect logs of the variables.

- If coefficients of  $l$  and  $k$  and  $m$  are consistently estimated, then  $u$  is consistently estimated.
- Notice that  $u = \log A$

- Suppose we run an Ordinary Least Square Regression (OLS) on all the plant level data where our regressors are  $l$  and  $k$  and where the production function is as given before. Therefore

$$y_{it} = \beta \cdot l_{it} + \gamma \cdot k_{it} + u_{it}$$

- Assumption for consistency of estimators of  $\beta$  and  $\gamma$  :

$$\text{Cov}(l, u) = 0; \text{Cov}(k, u) = 0$$

# Problem 1:

- The choice of  $k$  and  $l$  depend on the unobserved  $m$ .
- So classic omitted variable problem: Any residual (TFP) from the OLS regression is inconsistent.
- Standard Solution: Use Instrumental Variables (i.e. variables that affect  $l$  and  $k$  choice but are not  $m$ .)
- A lot of early studies took this approach but instruments are difficult to find.

## Problem 2

- Suppose we have data for  $m$ .
- At least a part of TFP will be observed by the firm at a point in time early enough so as to allow the firm to change the factor input decision.

⇒

- The error term (TFP) of the production function is expected to influence the choice of factor inputs.
- To see this, let us split the error term  $u$  into two parts so that:

$$y_{it} = \beta \cdot l_{it} + \gamma \cdot k_{it} + \delta m_{it} + \omega_{it} + e_{it}$$

- So  $\omega_{it}$  is observed by the firm early enough to affect input choice. But since  $\omega_{it}$  is unobserved to the economist, the conditions for consistency are not met.



# Standard Remedy: Fixed Effects Estimation

- If the part of TFP that influences firm behaviour ( $\omega_{it}$ ) is specific to the firm and is invariant over time, then  $\omega_{it} = \omega_i$
- Then differencing (or demeaning) the observations over time for each firm is going to remove  $\omega_i$  and estimators will be consistent.

$$y_{it+1} - y_{it} = \beta \cdot (l_{it+1} - l_{it}) + \gamma \cdot (k_{it+1} - k_{it}) + \delta(m_{it+1} - m_{it}) + (e_{it+1} - e_{it})$$

- A Fixed Effects Estimation method uses only within firm variation over time, which tends to be lower than the between firm variation (cross sectional variation). This means you are losing a lot of variation in the dataset. Usually these make standard errors high. Therefore you are likely to see many insignificant coefficients.
- Assumption that  $\omega_{it} = \omega_i$  implies that the part of TPF that influences firm behaviour is fixed over time: May not be true; in which case we are back to the problem of inconsistency

## Problem 3

- Many firm (plant) level data sets contain missing values associated with firms dropping out of the sample.
- Standard practice when using panel data: keep firms who are always in the panel: *"Creating a balanced panel"*
- But what if the exit decision of firms that are not in the sample biases estimates? (More on this later)

# A Standard Approach to Address these Problems

- Olley Pakes (1996)
- A Modification to OP : Levinhson Petrin (1996)

- Dynamic Model of Firm Behaviour
- To tackle Simultaneity of Input Choice: Model must make explicit what is known at the time inputs are decided
- To deal with Firms exiting the Sample: Model must generate an exit rule.

- At the time a firm makes decisions what is known to it?:
- Information about itself (Firm State Variables)
  - Capital Stock  $k_t$
  - An index of the firm's efficiency:  $\omega_t$  :This is modelled as a random variable (a productivity shock) which is revealed to the firm at the start of every period. However, firms with higher  $\omega$  today are more likely to have a higher expected  $\omega$  in the future.
- Information about Input Prices (and some probability over future input prices): Assumed Same across all firms.

- .At the beginning of a period a firm makes 3 decisions
  - Exit or Continue in Operation
- If it exits it received a selling value of the firm  $\Phi$  (and never reappears)
- If it continues in operation: it chooses investment ( $i$ ) and variable input ( $l$ )

$$y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + e_{it}$$

- Endogeneity of Variable Input Choices: You expect more productive firms to hire more labour Therefore  $Corr(l_{it}, \omega_{it}) > 0$  : Coefficient of  $l$  : Upward bias
- Self Selection induced by plant shut down
  - Survival will depend in part on  $TFP (\omega)$  revealed every period. Higher the  $\omega$ , better the chance.
  - Firms which get a bad productivity shock in any period can withstand that if they have a larger capital stock. This is because they expect larger future returns.
- When we are looking at the effect of capital on output, we will underestimate it since higher capital stock allows firms with lower  $\omega$  to survive.



# Olley Pakes: Algorithm

- The trick is to be able to take care of the unknown  $\omega_{it}$ .
- Use the fact that

$$i_{it} = i_t(\omega_{it}, k_{it})$$

- If  $i_t > 0$ , it can be shown that the function can be inverted, so that:

$$\omega_{it} = h_t(i_t, k_t)$$

$$y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + h_t(i_{it}, k_{it}) + e_{it}$$

- Lets say we assume some polynomial approximation for the unknown function  $h$
- But now how to interpret the coefficient of  $k$ . Since it will not only be  $\beta_k$  but also the coefficient of linear term in the unknown function  $h$ .

- The way out is: Define some polynomial function (3rd/4th order) that approximates  $\beta_0 + \beta_k k_{it} + h_t(i_{it}, k_{it})$ . call it  $\phi(i_{it}, k_{it})$
- So now

$$y_{it} = \beta_l l_{it} + \phi_{it} + e_{it}$$

- We can estimate the coefficient of  $l$  consistently, since  $\phi$  takes into account  $\omega$ .

- Suppose that there is no attrition. Then

$$V = y_{it} - \beta_l l_{it} = \beta_k k_{it} + \omega_t + e_{it}.$$

- Since  $\omega_t$  follow a time series process, we assume first order. So  $\omega_t = g(\omega_{t-1})$
- But now, we can substitute for  $\omega_{t-1}$  by  $h_{t-1}(i_{it-1}, k_{it-1})$
- So we get  $\omega_t = g(h_{t-1}(i_{it-1}, k_{it-1}))$ . But this is just  $g(\phi_{t-1} - \beta_0 + \beta_k k_{it-1})$
- Note that now we have

$$V = y_{it} - \beta_l l_{it} = \beta_k k_{it} + g(\phi_{t-1} - \beta_0 + \beta_k k_{it-1}) + e_{it}.$$

- We don't know  $g$ . Again take an approximation. Estimate to get  $\beta_k$ .

- With Attrition: Estimate a probit model of survival. Predict probability of survival  $\hat{p}_t$ . A little more arithmetic and taking expectations condition on survival show that:

$$V = y_{it} - \beta_l l_{it} = \beta_k k_{it} + g(\phi_{t-1} - \beta_0 + \beta_k k_{it-1}, \hat{p}_{t-1}) + e_{it}.$$

- with the  $\hat{p}_t$ , we are correcting for attrition when we run the model on firms that survive.
- This is in spirit the same as a heckman type correction. As a matter of fact, if there is only attrition and no correlated productivity differences, then its an approximation to a heckman correction. [If you dont know heckman correction, ignore this line!]

- One of problems with OP is that it requires the inversion and this is true only when  $i > 0$
- In real life data, huge number of firms for many years report 0 investment. Then OP is not useful
- LP idea: the same idea as OP except:

$$\omega_t = h(m_t, k_t)$$

where  $m$  is intermediate input: materials.

- This induces some endogeneity issues but allows the inversion more since intermediate inputs are always non zero.

# Plant and Firm Level Analysis of Trade: Some Background

- In the literature, 3 static predictions of trade theory
  - Protection can change firm's pricing behavior
  - When trade policies affect prices, they generally also change the set of active producers (market share reallocation) and/or their output levels (scale). These affect productivity.
  - Changes in intensity of foreign competition and/or firm's opportunities to export can affect their technical efficiency.

- Static Optimization

$$\frac{p}{c} = \frac{\eta}{\eta - 1}$$

- The Right hand side is elasticity of demand. Trade liberalization increases  $\eta$ . Mark up therefore falls.
- Many ways to motivate this:

- Demand Elasticity for domestic goods rises as relative price of foreign goods fall (if they are substitutes)
- Removal of import quota has the same effect.
- Increase in product varieties increases demand elasticity of domestic firms.

# Price Cost Margin: Empirical

- Usually no data on prices and marginal cost (esp the latter)
- Let  $PCM$  be the price cost margin.

$$PCM = \frac{\Pi_{it}}{p_{it}q_{it}} + \frac{(r_t + \delta) k_{it}}{p_{it}q_{it}}$$

- Controlling for ratio of capital stocks to sales, variables that measure intensity of foreign competition should contribute nothing to explain  $PCM$  if  $\Pi_{it} = 0$  (perfect competition)
- On the other hand, if  $\Pi_{it} > 0$  and trade liberalization increases demand elasticity, then  $\Pi_{it}$  will fall.
- Therefore in the regression

$$PCM_{it} = \beta_0 + \beta_1 \left( \frac{k_{it}}{p_{it}q_{it}} \right) + \beta_2 Im_{it} + \dots + \varepsilon_{it}$$

we expect coefficient of  $Im$  (proxy for intensity of import competition) to be negative.



# Price Cost Margin: Alternate

- If  $q_{it} = A_{it}h(v_{it})$  where  $v_{it} = (v_{it}^1, v_{it}^2, \dots, v_{it}^J)$  is the vector of J factor inputs, then output growth can be decomposed as:

$$d \ln(q_i) = \left( \frac{\eta}{\eta - 1} \right) \sum \left( \frac{v_i^j w_j}{p_i q_i} \right) d \ln(v_i^j) + d \ln(A_i)$$

- Regression of output growth on the share weighted rate of input growth, treating  $d \ln(A_i)$  as the mean productivity growth plus noise. We get the *PCM* as the slope coefficient.
- let the slope coefficient vary through time (pre reform/ post reform): one can test if trade policy affects mark up.
- Problem: This requires instruments that are correlated with input growth but not with the transitory part of productivity.

# Firm Size Distribution/Market Share and its effect on productivity

- When demand elasticities rise with liberalization, price cost mark ups are squeezed. This should induce exit until the remaining firms can make up on volume what they lost on margin.
- Let  $B$  be industry productivity. It can be shown that

$$\frac{dB}{B} = \Delta Eff_{Scale Economies} + \Delta Eff_{Mkt Sh Real} + \Delta Eff_{intra firm TFP}$$

- There is evidence that the first two components have a big effect.

# Intra Firm Productivity Gains

- Fall in price of Imported Inputs prices
- Product Variety Changes: Increase in menu of available inputs (We discuss this more in detail)
- Technology Diffusion through imports (we won't take about it more in this lecture)

# Using Theory to Generate Testable Implications

- In the second lecture, we will explore an empirical fact: As input tariffs fall, the number of product varieties in an Industry rise.
- Usual empirical work can provide a correlation, sometimes even causality: But without a mechanism, it is difficult to interpret results.
- In this part of the lecture, we will provide an example of a theoretical structure that motivates empirical work on the empirical fact mentioned above

- Specify a Cobb Douglas Production Function:

$$Y_q = AL^{\alpha_{Lq}} S^{\alpha_{Sq}} \prod_{i=1}^I X_i^{\alpha_{iq}}$$

where  $Y_q$  denotes Output of product  $q$ ,  $A$ : TFP,  $L$ : Labour,  $S$ : Other non tradeable Inputs (electricity, water).  $X_i = fn(X_{iD}, X_{iF})$  are domestic and imported inputs. Let the coefficients add up to 1.

- There is a fixed cost for production of final good.

- The Minimum cost of manufacturing one unit of output is:

$$C_q = A^{-1} \left[ \prod P_i^{\alpha_{iq}} \right] \left( P_L^{\alpha_{Lq}} P_S^{\alpha_{Sq}} \right) \bullet \text{Coeffs}$$

- Each input sector  $i$  has a domestic and an imported component that are combined according to a CES aggregator:

$$X_i = \left( X_{iD}^{\frac{\gamma_i-1}{\gamma_i}} + X_{iF}^{\frac{\gamma_i-1}{\gamma_i}} \right)^{\frac{\gamma_i}{\gamma_i-1}}$$

- $\gamma_i$  is the elasticity of substitution between the two input bundles.

- The over all price index of input industry  $i$  is the weighted average of the price index for the domestic ( $\Pi_{iD}$ ) and foreign input bundles ( $\Pi_{iF}$ )

$$P_i = (\Pi_{iD})^{\omega_{iD}} (\Pi_{iF})^{\omega_{iF}}$$

- The imported input industry  $X_{iF}$  is itself a CES aggregator of imported varieties:

$$X_{iF} = \left[ \sum_{v \in I_{iF}} \alpha_{iv}^{\sigma_i} X_{iv}^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}}$$

- Cost of purchasing one unit of foreign input bundle:

$$c = \left[ \sum_{v \in I_{iF}} \alpha_{iv} p_{iv}^{\sigma_i-1} \right]^{\frac{1}{\sigma_i-1}}$$



- Feenstra (1994) showed that relative to period 1 the price index in period 2 if the set of imported varieties was constant was:

$$P_{iF}^{Conv} = \frac{c_2}{c_1} = \prod_{v \in I_{iF}} \left( \frac{p_{iv}^2}{p_{iv}^1} \right)^{w_{iF}}$$

- Feenstra (1994):

$$\Pi_{iF} = P_{iF}^{Conv} \Lambda_{iF}$$

Here  $\Lambda_{iF}$  is the variety index. : measures the expenditure of varieties that are available in both periods relative to the expenditure on the varieties that are available in the current period.

- The more important the new varieties are, the lower the  $\Lambda_{iF}$ . and smaller the exact price index will be relative to the conventional index  $P_{iF}^{Conv}$
- The more substitutable the varieties are, the lower the difference between exact and conventional price.

# Cost After Substitution of Price of Foreign Input

- Recall

$$C_q = A^{-1} \left[ \prod P_i^{\alpha_{iq}} \right] \left( P_L^{\alpha_{Lq}} P_S^{\alpha_{Sq}} \right) \bullet \text{Coeffs}$$

- Substitution of  $P_{iF}$  and taking logs yields:

$$\begin{aligned} \ln C_q = & \left\{ \sum \alpha_{iq} \omega_{iF} \ln P_{iF}^{Conv} + \alpha_{Lq} \ln P_L + \alpha_{sq} \ln P_S \right\} \\ & + \left\{ \sum \alpha_{iq} \omega_{iF} \ln \Lambda_{iF} \right\} + v \end{aligned}$$