

Working paper



International
Growth Centre

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for commodity
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March 2019

When citing this paper, please
use the title and the following
reference number:
F-33416-GHA-1

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From Bilateral Trade to Centralized Markets: A Search Model for Commodity Exchanges in Africa*

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March 19, 2019

Abstract

Several African countries have recently centralized their agricultural markets by launching a commodity exchange. What would be the impact of such a move? Who will be the winners and the losers? This paper develops a simple search model for understanding the impact of a commodity exchange in a market where traders and farmers search and bargain to trade. We study the efficiency gains from moving from the status quo trading regime to trading under a commodity exchange system. We use the model to describe how the gains from trade are distributed between farmers, traders and the commodity exchange itself. We show that the gains from a commodity exchange depend on search costs and the degree of mismatch between farmers and traders. We begin our analysis with a description of the trading regimes currently found in many rural areas in Sub-Sahara by presenting a case study. We use this case study to motivate the search model and its conclusions.

*We thank the International Growth Center and Anonymous donors for the funding that made this research possible. We would like to thank Victor Arshavskiy for several comments on the model. We would like to thank Karolina Wilczynska for research assistance. All errors are those of the authors. Email: yaw.nyarko@nyu.edu, heitor.pellegrina@nyu.edu

1 Introduction

The absence of modern trading institutions is perceived as an important cause of the large costs associated with agricultural trade in Africa (Mezui, Rutten, Sekioua, Zhang, N’Diaye, Kabanyane, Arvanitis, Duru, and Nekati, 2013; Rashid, Winter-Nelson, Garcia, et al., 2010). In most African countries, agricultural markets are still decentralized: farmers and traders search for a trading partner in local markets or at the farm gate to trade on a bilateral basis. However, this trading environment is expected to change in the near future. As shown in Figure 1,¹ a few African countries have recently launched a commodity exchange and many are planning to follow in the next decade (Mezui, Rutten, Sekioua, Zhang, N’Diaye, Kabanyane, Arvanitis, Duru, and Nekati, 2013). In contrast to the decentralized system, in a market governed by a commodity exchange transactions between farmers and traders occur in a predetermined location and are typically mediated by market makers who could be thought of as the Walrasian auctioneer as used in standard economics discourses.

Figure 1: Commodity Exchanges around the Globe



In this paper, we contribute to the debate about the expansion of commodity exchange markets in Africa by providing a simple theoretical framework to understand their potential effects on a decentralized economy (Hernandez, Lemma, Rashid, et al., 2015; Andersson,

¹In 2018, Ghana effectively started the implementation of a commodity exchange.

Bezabih, and Mannberg, 2017; Minten, Tamru, Kuma, and Nyarko, 2014; Minten, Assefa, and Hirvonen, 2017; Gelaw, Speelman, and Van Huylenbroeck, 2017; Meijerink, Bulte, and Alemu, 2014; Sehgal, Rajput, and Dua, 2012; Tenderere and Gumbo, 2013). In the context of our model we address the following questions: How does a commodity exchange affect market efficiency? How are the efficiency gains distributed between farmers and traders, and within these two groups?

We start our analysis by making several observations about the operation of decentralized markets in Africa. Based on these observations, we model the status quo of a typical African agricultural market using the structure of search and bargaining models (Mortensen and Pissarides, 1994; Rubinstein and Wolinsky, 1987). In our model, the economy is dynamic. Farmers have heterogeneous trade costs and traders have a homogeneous price at which they resell agricultural goods in retail markets. In every period, traders have the choice of paying to search for a farmer in a network or not paying the cost and therefore not engaging in trade. If a trader chooses to search, she is randomly matched to a farmer and the transportation cost of the farmer is revealed to the trader. A trader that is matched to a farmer has the option to negotiate the price at which she trades one unit of an agricultural good. If they reach an agreement, they form a production relationship that is carried over to future periods until an exogenous shock breaks their connection. If there is no agreement on the price, the trader has to pay the search cost again and wait an additional period to search for another potential trading partner. In this environment, the trader may choose to strategically reject a negotiation with a farmer in order to wait for another farmer with lower transportation costs in a new round of search.

We characterize the equilibrium of this search and bargaining environment and show that there exist two sources of inefficiency in this economy. First, there is one source that stems from the randomness of the search process. In every period a mass of farmers who could potentially generate positive surplus – i.e., transportation costs are below the price of a unit of the agricultural good in the local market – are not matched to any trader due to the randomness of the search process. Second, a source of inefficiency that comes from the strategic rejection by traders. There is a mass of farmers that, despite finding a match and having transportation costs that would generate positive surplus for the economy, are strategically rejected by traders who instead choose to wait to find a better match.

The paper will begin by modeling this *status quo* economy, which we refer to as the bilateral trading model. We then proceed in the following section to discuss the impact of introducing a commodity exchange. We start by considering a fully mandated system where all traders are required to operate through the commodity exchange, as is the case of Ethiopia’s recently established commodity exchange, whose principal commodity is coffee.

In the fully mandated system, parallel markets outside the commodity exchange for commodities traded in the exchange are explicitly prohibited by law. This system is typically implemented to achieve the minimum scale of operations necessary to cover the cost of operating a commodity exchange. We model this system by introducing a Walrasian auctioneer who chooses a single price that makes the supply of farmers equal to the demand from the members of the commodity exchange. Under this fully mandated system, a commodity exchange brings full efficiency to the economy and the outcome is equivalent to the perfect competition benchmark.

We also examine the partial mandate system where the commodity exchange coexists with a bilateral trade market, which is the case of the commodity exchange in Malawi and the one currently being established in Ghana. In this system, there are no restrictions on trade outside of the exchange. We model this system by introducing a fee that creates a wedge between the price obtained by the commodity exchange and the one obtained by the farmer. We find that with a high enough fee, farmers with high transportation costs still choose to operate in the bilateral trade market. Therefore, in equilibrium with a certain parameter on the fee, we have the coexistence of the commodity exchange with the bilateral trade environment.²

We find important distributional gains associated with the implementation of a commodity exchange. First, traders lose with the commodity exchange, since now they have to operate in the bilateral trade market where farmers have higher transportation costs – that is because the ones with low transportation costs have left the bilateral trading environment and are now operating in the commodity exchange. Second, farmers with low transportation costs are better off since now they can operate in the commodity exchange which is better for them than the bilateral trading environment they used to trade in. Third, farmers with high transportation costs are also better off, since they remain in the bilateral trading environment and now they are less likely to be strategically rejected by traders, and they also have a higher bargaining power when negotiating prices – that is because traders have fewer farmers, all with high cost, competing for their attention.

We close our analysis by discussing the aggregate implications of the commodity exchange

²A common risk that commodity exchanges face is the lack of sufficient transaction volumes. In that case, the capacity that the commodity exchange has to guarantee the delivery of a product is compromised. Furthermore, commodity exchanges have large fixed costs of running storage, but low marginal costs of individual transactions. If there is not enough volume transacted on the floor, commodity exchanges may not generate sufficient revenues to pay for their fixed costs. The risks of insufficient scale are higher when a commodity exchange coexists with a decentralized market, which is the more common system. In this case, farmers and traders can choose whether they operate on the commodity exchange or through bilateral transactions. In some cases, to minimize this risk, governments opt for a fully mandated system, where the government bans some types of market transactions from taking place beyond the commodity exchange floor.

for the economy. With the implementation of the commodity exchange, the aggregate quantity supplied in the economy expands. This expansion leads to a drop in aggregate prices that mitigates part of the gains accrued to farmers. However, we argue that the expansion in supply tends to attract international or larger traders who require a minimum scale of operation to enter a market. This last effect tends to minimize the drop in aggregate prices and could reverse the drop and cause an increase in prices for farmers. There is no firm data on this; however, this is part of the expectation and the planning of the senior management of the commodity exchange in Ghana.

This paper relates to three strands of research. First, it adds to an emerging literature on the impact of commodity exchanges in developing countries ([Hernandez, Lemma, Rashid, et al., 2015](#); [Andersson, Bezabih, and Mannberg, 2017](#); [Minten, Tamru, Kuma, and Nyarko, 2014](#); [Minten, Assefa, and Hirvonen, 2017](#); [Gelaw, Speelman, and Van Huylenbroeck, 2017](#); [Meijerink, Bulte, and Alemu, 2014](#); [Sehgal, Rajput, and Dua, 2012](#); [Tenderere and Gumbo, 2013](#)). This literature has studied the impact of commodity exchanges on the co-movement of prices, either between different regions of a country or the national versus the international market. There are a number of reports and studies describing the experience of different countries with the introduction of commodity exchanges ([Mezui, Rutten, Sekioua, Zhang, N'Diaye, Kabanyane, Arvanitis, Duru, and Nekati, 2013](#); [Rashid, Winter-Nelson, Garcia, et al., 2010](#)). We add to this literature by providing a framework to examine the effects of the implementation of a commodity exchange.

Second, our analytical framework borrows from existing research on market microstructure ([Spulber, 2002, 1996](#); [Rubinstein and Wolinsky, 1987](#); [Gehrig, 1993](#)). A subset of this literature studies the coexistence of centralized and decentralized markets ([Rust and Hall, 2003](#); [Miao, 2006](#)). Closer to our framework is the analysis in [Gehrig \(1993\)](#) and [Miao \(2006\)](#). Different from [Gehrig \(1993\)](#), our model is dynamic, which incorporates the influence of intertemporal tradeoffs on the bargaining of farmers and traders. [Miao \(2006\)](#) studies a model where one side of the market is heterogeneous and farmers and traders leave the market after trading. Besides bringing the insights from these models to examine the agricultural sector in an African country, this paper complements this literature by examining how the strategic rejection of traders affects the operation of the market.

This paper also relates to existing studies in trade and economic development on the role of search costs ([Allen, 2014](#); [Startz, 2016](#); [Antras and Costinot, 2011](#)) and studies using search and bargaining frameworks in trade ([Bickwit, Ornelas, and Turner, 2016](#)). Different from these papers, we evaluate the coexistence of different markets within a country.

The rest of this paper is organized as follows. Section 2 provides several observations about a commodity exchange in Africa. Section 3 formulates a simple model of the status quo

bilateral trading environment which will be used to understand the effect of the introduction of a commodity exchange. Section 4 introduces the commodity exchange to the model. Section 5 provides concluding remarks.

2 Case Study in a Rural African Market

In this section, we describe the *status quo* farming environment and market structure in our rural African study area. The market structure we describe here is based on research from 2015 through 2019.³ What we describe here will form the basis of the theoretical model we develop in Section 3 and will be the motivation for the particular structure we impose.

2.1 Our study area

For one crop, maize, a commodity exchange is slowly being introduced into our study area. Several other crops are being added slowly to the commodity exchange at the time of this writing. Our focus will be on smallholder farmers in Ghana. They form the bulk of the farming in the country, both in terms of the numbers of people involved and in terms of the output or quantities.

We begin with some very broad and general observations about the market microstructure in these areas. What we describe here is the *status quo* situation before any commodity exchange is introduced. Our study area is a portion of the central part of Ghana, in the Kumawu Traditional area (the Sekyere Kumawu and Sekyere Afram Plains parliamentary districts plus small amounts of 2 or 3 others surrounding these districts). This area covers around 5000 square kilometers, approximately 2% of the land mass of Ghana. As of the most recent publicly available census, our study area has around 120,000 inhabitants, making it a relatively sparsely populated area.

2.2 Main observations

Below we list some key general observations about the study area. The next section describes in greater detail the key agents in the market.

1. **Land.** Land issues are not currently a major constraint on existing production by the farmers who are primarily smallholder farmers, but could become an issue for large-scale farming, and as farming expands and becomes more profitable due to the later introduction of the commodity exchange.

³Nyarko is grateful to the International Growth Center and Anonymous donors for the research grants that enabled this research to take place.

2. **Labor.** Farmers use their own time and labor on their farms and also hire laborers. The labor is required to clear (or “weed”) the farms and also to carry produce from the interior of the farm to the farm gate. As of the time of writing it costs around GHS 20 (around US \$2) per day for these laborers, who in the local parlance are called “by day laborers.” These by day laborers help with cutting the weeds, harvesting or spraying. An alternative method of contracting labor is by acreage. As of the time of writing the charge is GHS 150 (around US \$30) per acre. The laborer given that contract will be required to work on that area to get paid and will be paid proportionately to the total acreage worked on.
3. **Transport cost.** The transport sector involves high fees for moving produce for farmers, relative to their revenues. Yet those with the produce can still transport the goods to the markets for the most part. These fees are commonly perceived by farmers as surmountable so long as they find customers to sell their produce to.
4. **Agricultural inputs.** Fertilizer use is extremely low. Farmers indicate to us that they know that fertilizer use is important, however for the farmers it does not make economic sense to invest in fertilizers. Some farmers are afraid of spending money on the fertilizer, perhaps with borrowed money, only to see the markets collapse on them at harvest time. Other farmers complain that they have liquidity or cash constraints which prevent them from purchasing fertilizers. Those farmers also do not go to the banks for loans because, again, they fear the consequences of a market collapse at harvest time when they have no money to repay their loans.
5. **Technology.** Advanced technology is non-existent and given current market structures and current technologies as well as the prices, the use of such technologies is probably not optimal at this time and at the scale of production of the farmers. There are no irrigation schemes among the smallholder farmers we worked with. Only one group, producing maize, hired the services of a tractor. The vast majority of farmers use only one implement in their farming, the cutlass.
6. **Finance.** Many of the farmers indicated that financing is a major issue. Most of them faced liquidity constraints with an almost hand-to-mouth existence. They indicated that with more capital they could expand their farms. When asked why they did not go to the bank for a loan, they said that this is because of fear of not getting a good price for their output and then falling into debt. Farmers said that they often take loans from traders in exchange for selling their goods to the trader at harvest time. However, the farmers said they did not like this arrangement. This is because the

traders would dictate a price to them when the harvest came, thereby extracting an exceptionally high implicit interest rate on the loan.

7. **Demand.** Lack of sustainable demands for farmers' crops seems to be the biggest constraint to the development of the smallholder agricultural sector. Farmers complain a lot about not being able to get buyers for their produce. When farmers are asked why they do not use fertilizer or advanced technology or take bank loans, the answer almost always seems to involve the lack of sustained markets for their goods.
8. **Traders.** Due to the low opportunity cost of labor, there is a large number of people in the trading sector, each extracting small amounts of income in the food value chain. Many different levels of traders operate the markets – from the very big who operate large lorries and have storage facilities to the very small who sell small tins or baskets of goods on the side of the streets, and very many in between.
9. **Price.** The prices of commodities are erratic and farmers do not always know what the prices are going to be for their goods.
10. **Storage.** Warehouses and storage facilities are non-existent for many crops of many farmers. There are a variety of techniques that farmers employ which amount to implicit storage. For example, yam farmers keep the yams in the ground until they are ready to sell. Other crops are left unmatured and treated with chemicals to make them flower quickly when there is a need to sell these.

In the face of the imperfections in the agricultural market in our study area, market participants have developed very creative trading processes and market structures for dealing with the myriad problems related to the lack of storage, lack of credit and poor transportation facilities.

2.3 Key agents in the market

Farmers

The main crops grown by farmers in our area are yam, plantain, cassava and maize. There are people who grow cocoa in this area too. Cocoa is a cash crop and is managed by the government. Some of the less important crops, by volumes and revenues, include garden eggs (eggplant), tomatoes and other vegetables, cocoyam, groundnut and very small amounts of rice. Very few of the farmers we interacted with use tractors or any kind of mechanization that we could discern. The main implement used by farmers is the cutlass and nothing

else. The cutlass is used to clear weeds, make holes in the ground to insert seeds, etc. Farmers do pay attention to the seeds and the methods of planting. They obtain seeds from the previous harvest or through nurseries in neighboring communities. They get a lot of advice from the government Ministry of Food and Agriculture (MoFA) extension agents. The farmers use chemicals, namely weedicide, to keep out unwanted grass and shrub. Some farmers complained about pests affecting their crops. They also complained that because of insufficient funds they are unable to engage in pest control and use herbicides. Farmers indicate that they are cash-constrained and almost never purchase the required amount of fertilizers as instructed by the government extension agents. When they “get some good money they will invest in fertilizers”, they told us, otherwise, they take their chances on the over-worked soil on their farms. Some farmers indicated that animals destroy their farms. None of the farms we studied had any fences or barriers demarcating and sealing off their farms. The region is currently being invaded by cows roaming the bush led by itinerant or nomadic pastoralists. These pastoralists travel from the Sahel areas, particularly those affected by climate change. They head south to less affected areas. This has resulted in many violent clashes recently between the owners of these roaming cowherds and the indigenous people farming on the same contested lands. These issues have been documented in our study area and in surrounding areas.

Traders

We did not document in our research buyers who buy for their consumption (like the poultry farmers who purchase maize). Almost all of the buyers in our study area are buyers who then resell what they have bought, therefore we refer to them as traders. There are many different types of traders who buy the produce from the farmers. Most of these were women. They are intermediaries of various sizes. Some are very small. A few are much larger. Many of the smaller traders take goods from the farms or farmers and send them to the local markets which are around 1 hour to 2 hours away by car. These small traders are the majority of those who live in our study area. Our farmers also occasionally interact with big traders who collect goods from farmers for sale in Accra. One farmer mentioned, “I trade with 2 people from Accra and I sell to them on Thursdays.” There is one very interesting feature of the traders’ activities which caught our attention. Sometimes the traders “buy the farm,” as they say in the local parlance. What this means is that the farmer and trader negotiate for a certain amount of the farmer’s farm – for example, 2 acres of a farmer’s plantain farm. The trader will pay the farmer a price and then the trader will be responsible for hiring the laborers to harvest the produce (the plantain in this example) and pay for the transportation of the produce from the farm gate to the city or wherever the trader will resell the produce.

This is an interesting way in which the farmers deal with their lack of liquidity or their lack of ability to pay upfront for their labor and transportation costs. The traders who buy from farmers in this manner represent the principal method farmers in our study area use to sell their goods. In the theoretical sections below, these are the traders we have in mind when we do our modeling. They are the ones who will search for the farmers and then engage in bargaining with them.

Markets, local monopolies and local competition

There is one interesting feature of markets in these areas which caught our attention and may be a local response to the various constraints faced by market participants in these areas. In any one market town, the markets operate once a week. The days of the week differ in different towns. For example, the market town Bodomase operates on Fridays and the market in Juaben on Wednesdays. By having markets open once a week, traders are able to aggregate produce from many different farmers and get the volume needed to make their operations scale up and be profitable. All traders would, for example, converge on Bodomase on Fridays. The farmers in that area will farm most of the week, then collect all their produce on Thursday night at their house or in a local storage area and have them ready for traders to inspect and hopefully purchase early in the morning on Friday. In some markets, we did hear of price fixing by traders in the local markets. We were told of some instances where the traders agree in advance on what the price of a particular crop should be, and the traders all try to pay no more than that agreed amount. We did not hear of this for all crops, and this effect seemed to be dying down. One farmer, a woman, said to us “they used have fixed prices for tomatoes, however that process has died.” We did not hear of much of this price fixing occurring in the big cities and towns.

Information asymmetries and search

First, in our research we recorded the fact that many farmers did not know the prices of goods in the major markets. One question we posed was this: why don't the farmers just call a friend in the market in the main city to ask for the current prices? We found out in research that the farmers did not have friends who had access to the market prices. Since prices moved around so much, even if they knew someone in the city, it would probably be hard to ask that person to go to the market each week just to check prices for them.

Farmer-trader matches and post-harvest losses

We also recorded from our farmers that they were constantly looking for traders to buy their goods at a good price. They told us that often negotiations with traders break down. They also told us often that they would be at their farm gate looking for or waiting for traders but would not have any visit them or meet them. These farmers often live in faraway and remote areas where transport costs are high. Similarly, we spoke to many traders who told us that it is often hard to find farmers to trade with. In particular, from our research, it seemed as if there could be viable matches if only the traders wanting goods and farmers with the goods to sell could locate each other. This lack of potential matches informs the search model we posit below. We also remark here that we found many situations where the farmers would negotiate with traders and not reach an agreement on the appropriate price to sell their goods at. Situations where either the farmer does not find a trader or is strategically rejected are the situations where produce may go bad as many of these crops are perishable. In other words, these situations present a form of post-harvest losses created by the search costs in this market.

2.4 From our observations to the model

Our observations about the study area of Kumawu motivate the formulation of a model where traders search for farmers to establish commercial relationships. The observations from our study suggest that this search process is costly and generates substantial post-harvest losses, as farmers are often not able to find a trader who is willing to purchase their produce. We also find that these large costs induce traders to be selective about whom they choose to start and maintain a commercial relationship with. In the next section, we formulate a search model that incorporates these features.

Based on our observations, we adopt the following simplifying assumptions in our model. First, our study observations indicate that the scale of production is extremely small, with a low application of inputs such as fertilizers. We thus abstract from scale economies to focus on the implications of search costs. Our results should, therefore, be interpreted with caution as there are many ways in which they may be underestimating the impact of the commodity exchange. While in the short run technologies may be fixed at the current low levels of adoption, in the long run the implementation of a commodity exchange might encourage the adoption of new technologies. Second, we did not explicitly model credit markets. Our study indicates that, at least in part, the low uptake of loans to finance production is itself a result of the uncertainty generated by the bilateral trade market. With the implementation of a commodity exchange, farmers might increase their loan uptake. Adding these additional

features in the model would magnify the effects of the commodity exchange.

3 The Model

This section develops a simple search and bargaining model to describe the agricultural market existing in our study area. This model will help us understand the impact of the introduction of a commodity exchange which is beginning to take place in this community. We start our analysis by characterizing the economy at the *status quo* when there is no commodity exchange. We then examine the impact of introducing a commodity exchange under a partial and a full mandate.

3.1 Search and bargaining with no commodity exchange

Consider an economy with two types of agents, farmers (F) and traders (T). This economy operates over time. The time dimension is discrete. Farmers and traders live forever and are risk-neutral. In each period, farmers produce one unit of a non-storable agricultural good. They can sell their agricultural good to a trader who then transports their produce to a local market. We denote by c the transport costs that farmers incur when taking the agricultural good from their farms (which are usually in remote areas) to the farm gate (roadside) or to local markets. We suppose that these are the only costs although it is easy to think of the cost c as incorporating other costs of production. These transport costs come from a distribution $g(c)$. In our numerical examples, we assume that they are uniformly distributed between 0 and c^{max} , so that $g(c) = \frac{1}{c^{max}}$. We normalize to 0 the value to the farmer of not selling his or her produce. We are thinking of a scenario where the farmer has an abundance of food for subsistence and the crops we are studying are primarily for sale, with those for consumption coming from their gardens or easily obtained on an almost daily basis from their farms.

We model this economy as a network of traders and farmers who do not observe where each market actor is located within the network. To enter in this network, traders have to pay an upfront cost κ in every period. Once in the network, traders can search for farmers, but since they do not observe the exact location of farmers in the network, they cannot direct their search towards farmers with specific transport costs. Farmers focus on production and do not search for traders in the network. They wait in their farms for traders to call or to appear at their farm gate. Farmers in the network are matched with a trader with an exogenous probability μ^F and traders also in the network are matched with a farmer with

an exogenous probability μ^T .⁴ When they are matched with each other, the transport cost of the farmer c is revealed to the trader who then decides whether to enter a negotiation over the price of the produce $p(c)$. If they reach an agreement, they leave the network and start a trading partnership that is exogenously broken with a probability⁵ β . Once the farmer leaves the network, she is replaced with another farmer with the same transport cost. This replacement guarantees that the decisions of traders and farmers are replicated in every period.

In this environment, search is costly for two reasons: first, because traders have to pay an upfront cost κ to enter in the network and search; second, because if traders do not find a farmer during the search process, or if they do find a farmer but choose to reject negotiation, they have to wait until the next period to search again, but they discount the future by a factor of δ .

To characterize the decision of traders and farmers, we have to account for the fact that agents consider the outside option of waiting to obtain a better trading partner. In particular, the gains from trade for them is the value of establishing a partnership and being matched against the value of being unmatched and waiting for a potentially better trading partner.

Let V^{FM} and V^{FU} be the values of being matched and unmatched for a farmer, and V^{TM} and V^{TU} the respective values for a trader. We argue that the value functions of farmers and traders are given by the following Bellman equations:

$$V^{FM}(c) = p(c) - c + \delta \{ \beta V^{FU}(c) + (1 - \beta) V^{FM}(c) \} \quad (1)$$

$$V^{TM}(c) = \max \{ \bar{p} - p(c) + \delta \{ (1 - \beta) V^{TM}(c) + \beta V^{TU} \}, V^{TU} \} \quad (2)$$

$$V^{FU}(c) = \delta \{ \mu^F V^{FM}(c) + (1 - \mu^F) V^{FU}(c) \} \quad (3)$$

$$V^{TU} = \max \left\{ \delta \left\{ \mu^T \int V^{TM}(c) g(c) dc + (1 - \mu^T) V^{TU} \right\} - \kappa, 0 \right\}, \quad (4)$$

The first equation describes a farmer with cost c who happens to have just matched

⁴In a typical search model, the probabilities μ^F and μ^T are determined by a matching function - which generally adopts a constant returns to scale form - that depends on the mass of unemployed farmers and traders in the market. In particular, if we define u^F and u^T as the mass of farmers and traders searching in the market, a model could stipulate that $m = \kappa (u^T)^\theta (u^F)^{1-\theta}$, where m is the number of matches and κ and θ are parameters of the function. In this case, the probability of a farmer being matched would be $\mu^F = \frac{m}{u^F} = \left(\frac{u^T}{u^F} \right)^\theta$ and the probability of a trader being matched would be $\mu^T = \frac{m}{u^T} = \left(\frac{u^F}{u^T} \right)^{1-\theta}$. Alternatively, one could use a linear matching function where $\mu^F = \frac{u^F}{u^F + u^T}$ and $\mu^T = \frac{u^T}{u^F + u^T}$, or use simple ratios $\mu^T = \min\{\frac{u^F}{u^T}, 1\}$ and $\mu^F = \min\{\frac{u^T}{u^F}, 1\}$. In the appendix, we discuss the additional conditions that we need to close the model.

⁵The goal here is to suppose for analytical tractability that we do not have to worry about “search without replacement” of matched trader-farmer pairs.

with a trader. It says that the value function of the matched farmer with cost c is equal to what the farmer gets in the first or current period, the price $p(c)$ minus the cost c , plus the discounted value of the future value. The future value is determined by whether the farmer is unmatched or matched in the next period, $V^{FU}(c)$ or $V^{FM}(c)$, events that occur with probabilities β and $(1 - \beta)$ respectively. Since the farmer receives 0 if the trader does not agree to a trade and we will always look at solutions where $p(c) \geq 0$, whenever there is a trade with the farmer it will be characterized by (1). As we shall show later, there will be some values of c for which the farmer with that value of cost never gets a trader who will be willing to trade. We shall not insist on (1) holding in that case, and we will set $V^{FM}(c) = V^{FU}(c) = 0$.

The second equation (2) pertains to a trader who has just been matched with a farmer with cost level c . That trader has to decide whether to trade with that farmer (the left hand term in the bracket after the max) or else to walk away (we call this strategic rejection) and become unmatched, with a value function of V^{TU} . If the trader does trade with the matched farmer, then the trader makes in the current period a profit equal to the difference between the big city price \bar{p} and the bargained price $p(c)$ and makes in the future the discounted value of the expected return to being matched with the same trader in the next period $V^{TM}(c)$ or being unmatched (i.e., having the match broken) with value V^{TU} , these two events occurring with probabilities $(1 - \beta)$ and β respectively.

The third and fourth equations, (3) and (4), pertain to the unmatched farmer and unmatched trader. In each case they receive 0 in the current period, their future returns are discounted by δ and they receive the expected return to being matched and unmatched in the next period, events that take place with probabilities μ^F and $1 - \mu^F$ for the farmer and μ^T and $1 - \mu^T$ for the trader. The trader pays a search cost of κ when unmatched and beginning a search.

Let η denote the bargaining power⁶ of the trader and define $\phi = (1 - \eta)/\eta$ as the power of a farmer relative to a trader. In particular, at each point (or value of c) where there is trade between the farmer and the trader we have the relation that the surplus going to the farmer is ϕ times the surplus going to the trader:

$$V^{FM}(c) - V^{FU}(c) = \phi \{V^{TM}(c) - V^{TU}\}. \quad (5)$$

We stress here that the equation above will be required to hold only for those values of c such that both parties, the farmer and the trader, want to trade. In particular, the equation

⁶ η is the Nash bargaining weight so that any surplus (less outside options) is shared in the proportions η for the trader and $(1 - \eta)$ for the farmer.

above will only be required to hold when the maximum on the right hand side of (2) and also (4) each occurs in the first term and not in the second term. If we consider as parameters $\delta, \beta, \mu^F, \mu^T, c$ and \bar{p} , the equations (1)-(5) are a system of 5 equations in the 5 unknowns $V^{FM}, V^{TM}, V^{FU}, V^{TU}$, and $p(c)$. It is easy to see that we can therefore in principle get solutions for the 5 unknowns in terms of the parameters.

We solve the equilibrium of this system of equations defined by (1)-(5), in the process defining the conditions for the existence of an equilibrium, as follows. Below, we show the key equations that we use to prove the existence. See the appendix for more details.

First, we use equations (1) and (3) to get expressions for $V^{FU}(c)$ and $V^{FM}(c)$ as a function of $p(c)$

$$V^{FM}(c) - V^{FU}(c) = \frac{p(c) - c}{1 - \delta(1 - \beta) + \delta\mu^F}.$$

We can insert the expression above into (5) to obtain

$$p(c) = \rho\phi(V^{TM}(c) - V^{TU}) + c.$$

where we defined $\rho \equiv 1 - \delta(1 - \beta) + \delta\mu^F$. One can think of this term as an adjustment associated with the perpetual gain from flows of $p(c) - c$.

We use the expression for $p(c)$ to construct (2)

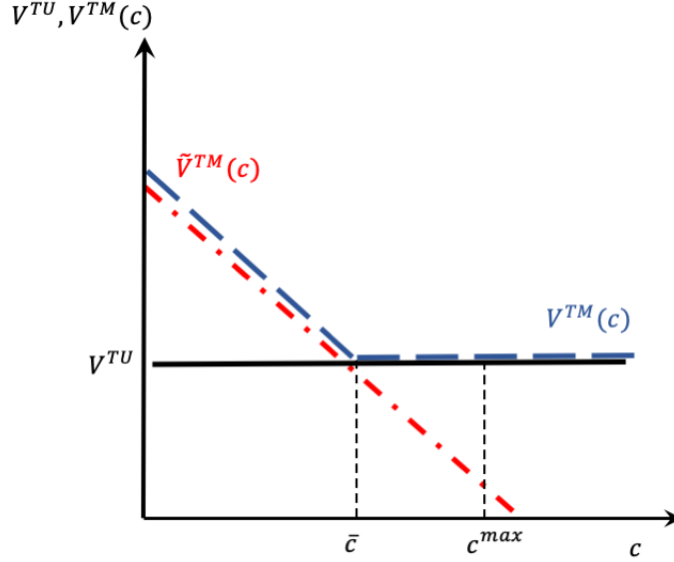
$$V^{TM}(c) \equiv \max\{\tilde{V}^{TM}(c), V^{TU}\},$$

where $\tilde{V}^{TM}(c) \equiv \bar{p} - c + (\delta(1 - \beta) - \rho\phi)V^{TM}(c) + (\rho\phi + \delta\beta)V^{TU}$. This expression allows us to define the region of strategic rejection. In the appendix, we show that $\tilde{V}^{TM}(c)$ is decreasing in c . In Figure 2, we graph $V^{TM}(c)$ as a function of c , assuming fixed V^{TU} (and of course fixed parameters $\delta, \beta, \mu^F, \mu^T$, and \bar{p}). There is a linear downward sloping function $\tilde{V}^{TM}(c)$ which is the value of assuming that the max in (2) occurs in the first term and not the second term (V^{TU}). The graph also shows the level of V^{TU} which of course does not depend upon c . As the figure shows, this implies that there is a cutoff point \bar{c} above which traders prefer to strategically reject a negotiation with farmers. In particular, for small values of c ,

$$\bar{p} - p(c) + \delta\{(1 - \beta)V^{TM}(c) + \beta V^{TU}\} > V^{TU},$$

the trader will trade with the farmer while at high values of c the trader will decide not to bargain with the trader. The cutoff point or point of indifference is some value \bar{c} indicated below. In particular for all values of c in $[0, \bar{c}]$ there will be trading, while for those above the cutoff there will be strategic rejection.

Figure 2: Region of Strategic Rejection



The intuition for this strategic rejection is straightforward. It occurs with farmers of a high cost. If the trader sticks with this farmer, then the trader has a probability of being stuck with that farmer in the next period. This may be because of loans given, friendships made, other farmers not wanting to interfere with an existing relationship, etc. That trader who has met a high-cost farmer would, therefore, like to not be stuck with that farmer but instead to be free to sample another farmer.

This phenomenon is similar to what happens in worker-firm matching in job search models. In those models where search on the job is hard and search off the job is much easier, workers may decide to break the match with their firms even when there is positive value in the match so that they can have the ability to search freely or much more easily while unemployed. In those models, such search of course only makes sense when the value of the worker-firm match is low. We are seeing the same thing here in our farmer-trader model. Traders are like the workers who have a better search environment when they are not attached to an existing farmer.

The economics here are interesting. Matches between traders and farmers where there are gains from trade may happen to not take place due to two reasons. The first is that there is no trader-farmer match. Indeed, $1 - \mu^F$ is the fraction of farmers who do not get matched due to simply not meeting traders. Let us call this a lack of matching due to geographic

reasons. We now argue that there are, in addition, strategic reasons for a lack of a match. Equation (2) indicates that there is a possibility that an otherwise profitable match results in no trade. In particular, there could be a match between a trader and farmer where the cost c is less than the big city price \bar{p} such that the gains from trade are positive, and yet the trader decides not to trade. The reason for this is that the trader has an opportunity cost of time and that one trade, when completed, will force the trader to stay with that farmer. We are implicitly assuming that trading takes time and that the trader cannot search while trading.

Theorem 1. *(Strategic rejection) As β falls to zero (i.e., the stronger the bond between a matched farmer and trader), the larger is the set of values of c which are rejected for strategic reasons.*

By equalizing $\tilde{V}^{TM}(c) = V^{TU}$, we can find a simple expression for the cutoff value \bar{c} :

$$\bar{c} = \bar{p} - (1 - \delta)V^{TU}.$$

Interestingly, the cutoff value depends only on prices and the value of being unmatched. Of course, since V^{TU} is itself a function of the endogenous variables in the model, \bar{c} depends on all the other parameters in equilibrium. The expression above shows that, when traders do not discount the future, then the cutoff is equal to \bar{p} . This is the case when traders can wait for an indefinite period of time to obtain the best match. When V^{TU} is higher, then the cutoff is lower, since the returns from waiting are large.

Instead of proving the existence and uniqueness of equilibrium, we focus on the relevant cases where we have strategic rejection in equilibrium. This occurs when $\bar{c} > 0$ so that at least some farmers are accepted by traders. In the appendix, we use the expressions above to prove the following theorem:

Theorem 2. *(Existence and Uniqueness) There exists a unique solution to the system of equations defined by (1) to (5) if*

$$\frac{\delta\mu^T}{1 - \delta + \rho\phi + \delta\beta} \times \frac{\bar{p}^2}{2c^{max}} \geq \kappa,$$

where $\bar{c} \in (0, \bar{p}]$. If this condition holds as an inequality, we have $\bar{c} \in (0, \bar{p})$ and traders strategically reject a non-null mass of farmers. If this condition holds as an equality, traders do not strategically reject any farmer with $c \leq \bar{p}$.

This theorem shows that, as long as the fixed costs of search are not too large, there is an equilibrium where farmers trade with traders. If the fixed costs are too large, then traders

choose not to trade and collect $V^{TU} = 0$.

3.2 Equilibrium

We now describe the characteristics of the equilibrium in our model and the distributional gains from trade in the bilateral trade market.

Figure 3 Panel (a) shows how we can derive the supply curve of the bilateral trade market using familiar supply and demand figures. In dashed blue line we have what would be the supply curve in the absence of search frictions. In this case, when $p = \bar{p}$, the supply of agricultural goods would be exactly at the point where $c = \bar{p}$, which is the farmer with the highest transportation cost who can still make positive surplus by selling her good at the local market. The decentralized market reduces the quantity in two ways. First, quantities fall because only a proportion μ^F of all the farmers can actually find a trader to sell their goods. As a consequence, the effective supply of goods falls from p to $\mu^F p$. We highlight this first drop in quantity with the dashed blue line in the figure. One can imagine that the actual supply curve contains holes due to the unmatched farmers and that we squeeze out these holes to obtain the new supply curve. Second, quantities also fall because among the farmers who are matched with traders, those with high transportation costs are strategically rejected. This leads to an additional drop in the supply of $(p - \bar{c}(p))\mu^F$. The effective supply curve is therefore given by the solid black line in Figure 3. The expression that describes this supply curve is given by

$$Q^S(p) = \mu^F \bar{c}(p), \quad (6)$$

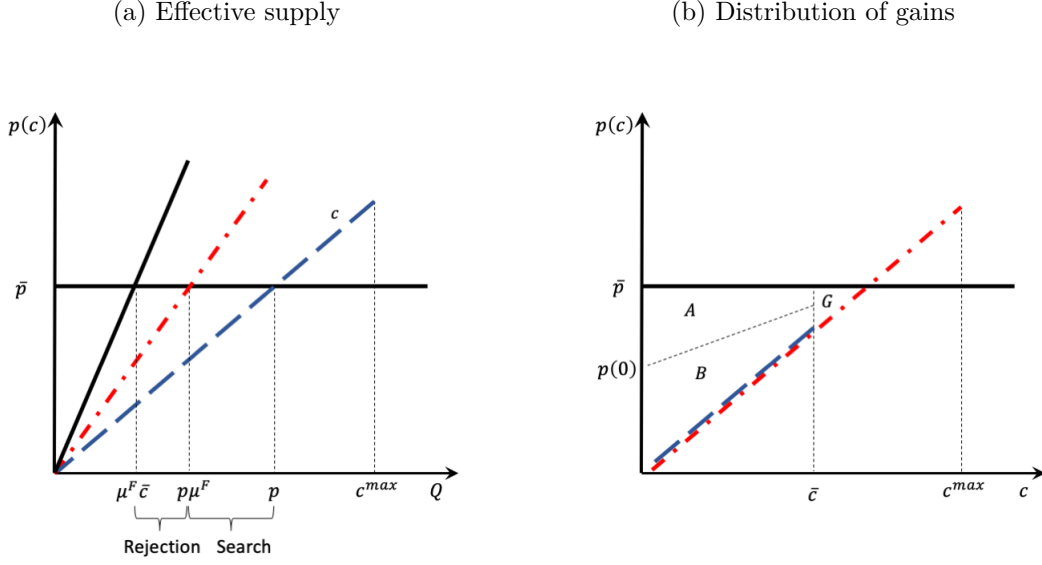
where \bar{c} is a function of prices and the exogenous parameters of the model.

Figure 3 Panel (b) describes the relationship between bargained prices and the transportation costs of farmers. Farmers above \bar{c} and below p are rejected by traders, which generates a deadweight loss coming from the strategic rejection. All farmers below \bar{c} obtain a price $p(c)$ for their produce. Area A represents the profits obtained by traders when matched, area B the surplus generated by farmers when matched and area G the surplus that would be generated if there were no strategic rejections.

4 Commodity exchange in a bilateral trade market

This section examines the effects of the introduction of a commodity exchange in an African economy where transactions occur in a bilateral trade market. We introduce a commodity exchange as follows. The commodity exchange has members who can purchase a license to operate in the commodity exchange floor. For simplicity, we assume that traders in the

Figure 3: Supply of Agricultural Goods in a Decentralized Market



bilateral trade market cannot obtain this license. In reality, some traders obtain licenses to operate in the commodity exchange. Farmers can bring their produce and sell it to the members of the commodity exchange. The commodity exchange has an auctioneer whose task is to equalize the demand from its members with the supply of farmers who brought their produce. The auctioneer operates as a Walrasian auctioneer.

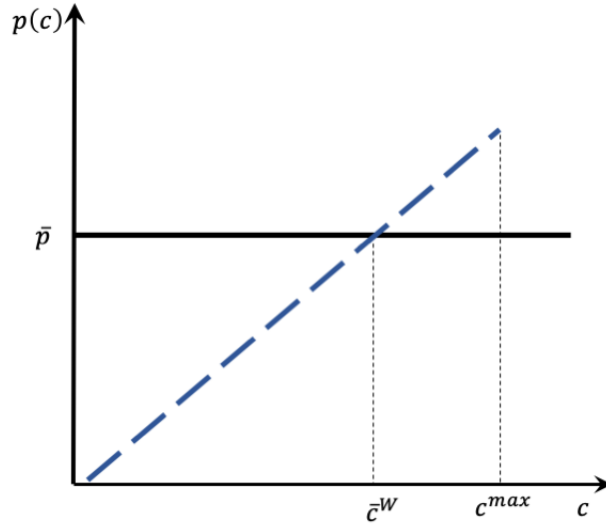
We examine two forms of implementation of the commodity exchange. First, we present the case where we have a full mandate system where all transactions must happen on the commodity exchange floor. This case represents the perfect competition benchmark. Second, we discuss the partial mandate system where we allow for the coexistence of the commodity exchange with the bilateral trade market.

4.1 Full mandate system

In the full mandate commodity exchange with a Walrasian auctioneer, in every period farmers may sell their produce to the members of the commodity exchange. These members can resell the produce at a price \bar{p} . The transactions are mediated by a Walrasian auctioneer. This auctioneer sets up a price p^W , then purchases agricultural goods from farmers who are willing to sell at this price and sells these goods to traders who are willing to buy at this price. To find the equilibrium price, the auctioneer announces different prices p^W until the supply of farmers equals the demand from traders. If the announced price is $p^W > \bar{p}$, then no member

buys agricultural goods since they would make negative profits by reselling them. If the announced price is $p^W < \bar{p}$, then all members would buy agricultural goods but only farmers with costs below p^W would sell them. Therefore, the final announced price p^W must be equal to \bar{p} . At this price, members are indifferent between purchasing or not, and farmers with costs below \bar{p} sell their goods to the auctioneer. We define c^W as the farmer who obtains no profit in this auction ($c^W = \bar{p}$). Figure 4 illustrates the Walrasian market. The area below the demand curve represents the surplus from farmers. Here, traders obtain no gains from trade. The supply curve is given by $Q^W(p) = p$.

Figure 4: Commodity Exchange as a Walrasian auctioneer in a Full Mandate system



4.2 Partial mandate system and dual markets

What happens if the commodity exchange, due to its own internal costs, imposes a high transaction fee for participation in the commodity exchange? Would it be possible to have dual markets in this case? That is to say, is it possible to have some farmers trading in the commodity exchange while others trade in the usual bilateral trade manner in the remote villages?

We now show that there could be dual markets taking place. There could be some farmers who trade in the commodity exchange while others, at the same time, continue to trade in

the bilateral trading situation that is the case in the *status quo*.

Suppose that if a farmer goes to the commodity exchange, she has to pay a tax or a fee. Let τ be the tax at the commodity exchange, so that the final price received by farmers is $\bar{p}(1 - \tau)$. To build intuition, let us abstract from the strategic rejection by assuming that $\beta = 1$ and $\kappa = 0$. In this particular case, traders have to search and bargain in every period to purchase a good since they are always unmatched in the beginning of the period.

If you are an unmatched farmer in the bilateral trade market, the price you may receive is $ER = \mu^F \cdot p(c) + (1 - \mu^F) \cdot 0$. Suppose you have equal weight Nash bargaining, which implies $\phi = 1$. Then $p(c) = c + \frac{1}{2}(\bar{p} - c) = \frac{1}{2}(\bar{p} + c)$. Putting this into the earlier equation results in

$$ER = \mu^F \cdot p(c) + (1 - \mu^F) \cdot 0 = \mu^F \left\{ \frac{1}{2} (\bar{p} + c) \right\}.$$

We ask whether we can obtain a situation where it is beneficial for some farmers to be outside of the commodity exchange. For this, we require that the price after the tax at the commodity exchange is less than what the farmer will receive under bilateral trade. This requires that

$$\bar{p}(1 - \tau) \leq \frac{\mu^F p(c)}{2}.$$

Substituting the value for $p(c)$ and isolating c in the equation above gives

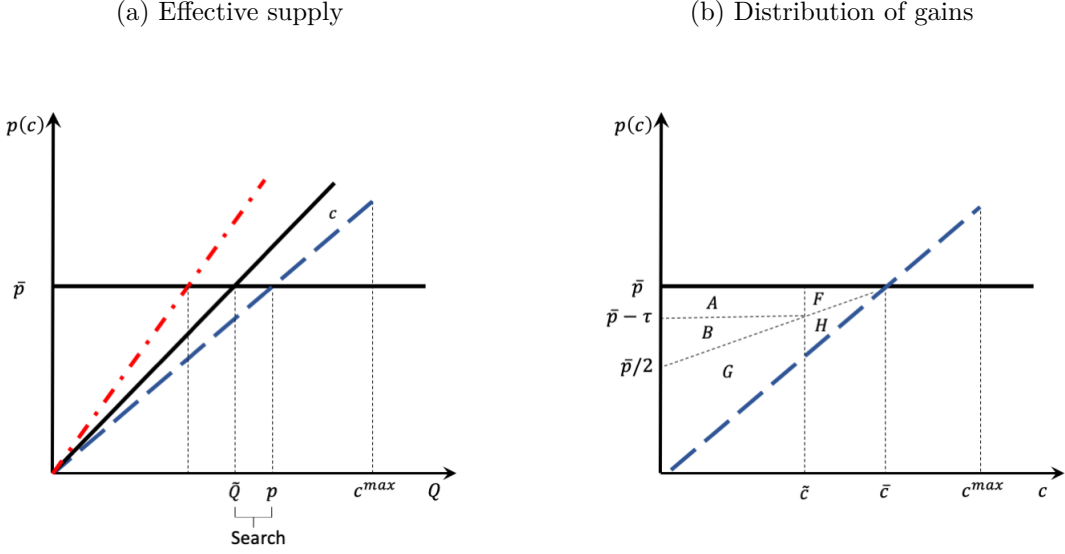
$$c \geq \left(\frac{2(1 - \tau)}{\mu^F} - 1 \right) \bar{p}.$$

From the expression above, we get the result that it is the farmers with high transportation costs who stay outside of the commodity exchange. When this equation holds as an equality, we can compute the cutoff value \tilde{c} associated with the farmer with the lowest transportation cost who would still choose to sell at the commodity exchange. As a quick and easy example, suppose that $\tau = 0.5$ and $\mu^F = 2/3$. Then the right hand side of the equation above becomes $c \geq 0.5\bar{p}$. Note that when prices are higher, we have a larger fraction of farmers who would choose to sell their produce in the commodity exchange market.

Figure 5 Panel (a) shows the effective supply curve in the market. The red short dashed line shows the supply curve in the absence of the commodity exchange. In this case, the supply would be equal to the probability of a farmer with costs below p being matched to a trader $p\mu^F$. With the introduction of the commodity exchange, farmers with transportation costs below \tilde{c} always sell their produce, and the randomness of the market only affects farmers with costs above \tilde{c} and below \bar{c} . Therefore, the total supply equals $\tilde{Q}(p) = \tilde{c}(p) + \mu^F(p - \tilde{c}(p))$.

Figure 5 Panel (b) shows the distributional consequences of the implementation of a

Figure 5: Commodity Exchange in a Dual Market without Strategic Rejection



commodity exchange for the particular case discussed above. Area A captures the revenues obtained by the commodity exchange, area $B + G$ is the surplus that farmers with costs below \tilde{c} gain with the implementation of the commodity exchange, area H is the surplus of matched farmers in the bilateral trade and F is the surplus of traders. The area $A + B$ would be captured by traders if matched. The gains for farmers are associated with area B .

In the application above, we do not consider the profit motives of the commodity exchange. In several countries, commodity exchanges are implemented by a market maker, a company that obtains a license to set up a commodity exchange and charge a fee for each transaction on the commodity exchange floor. In principle, one can think of the fee that this company chooses to maximize its profit as equivalent to a commodity exchange with a high operational cost. The market maker would choose the fee that maximizes its profit constrained by the participation constraint of farmers.

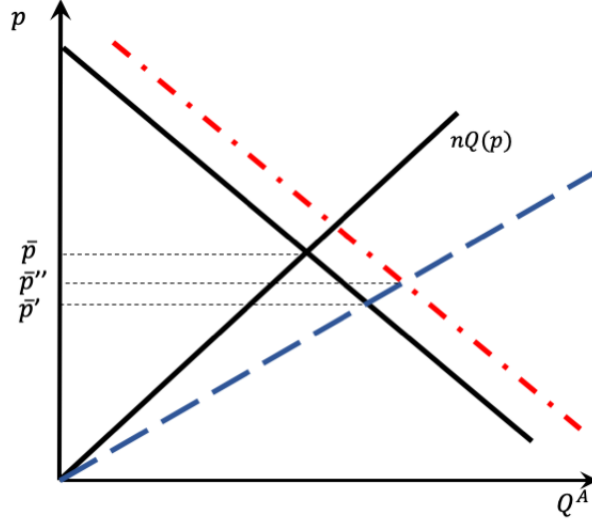
4.3 Aggregate Equilibrium

In previous sections, we analyzed the implementation of the commodity exchange taking the big city price \bar{p} as exogenous. We now examine how the implementation of the commodity exchange affects this price. We consider an economy that is composed of n regional bilateral trade markets and one large metropolitan area. Regional bilateral trade markets operate as

defined in previous sections. The metropolitan area hosts the bulk of final consumers and firms such as chicken farms that use the produce from regional markets as inputs. In the case of Ghana, for example, the metropolitan area would be the city of Accra and one of the regional markets would be our study area of Kumawu. We assume that the commodity exchange is implemented with regional branches, that regional markets are symmetric and that farmers and traders only operate in a single regional market at a time. The aggregate supply in the economy is thus the summation of the supply in the n regional markets: $Q^A(p) = nQ(p)$.

Figure 6 depicts this supply curve together with an aggregate demand curve in the economy. With the implementation of the commodity exchange, the supply curve shifts to the right in direction of the blue dashed line, and the price moves from \bar{p} to \bar{p}' . However, agricultural market specialists in our study area indicate that the conventional analysis of the demand and supply might be incomplete. A sufficient expansion in the supply of agricultural goods tends to shift the demand upwards as well. This occurs because international commodity traders require a minimum scale of operation to enter a market. The conjecture of these specialists in the field is that the implementation of the commodity exchange might attract some of these international traders. We illustrate this effect with the short dashed line in red. With the additional shift in demand, the aggregate price in the economy would increase from \bar{p}' to \bar{p}'' , restoring part of the gains accrued to farmers and potentially reverting this drop altogether.

Figure 6: Aggregate Equilibrium



5 Conclusion

In this paper, we discussed the market structure of smallholder farmers in some parts of sub-Saharan Africa. We modeled the status quo bilateral trading market microstructure and showed that there are some disadvantages to the bilateral trading model. This comes in two ways. First, there will be some farmer-trader matches which do not occur simply because farmers and traders do not find each other. Second, there are matches that do not take place because traders strategically reject the farmers that they are matched to because their costs are too high and it is in the trader's interest to re-sample the market. We then modeled the introduction of the commodity exchange. We showed that the commodity exchange eliminates many of the disadvantages of the bilateral trading model. Curiously, we showed that despite the advantages of the commodity exchange, there could still be dual markets where the commodity exchange co-exists with the bilateral trade environment. This occurs when the commodity exchange charges high transaction fees and high-cost farmers find it profitable to stay in the bilateral trading environment.

One of the implications of the theoretical model is that many of the traders who were in existence in the bilateral trading environment will go out of business with the introduction

of the commodity exchange. This is because the commodity exchange is able to provide intermediation much better than the traders. The commodity market, by creating a centralized market, is able to eliminate the lost farmer-trader matches. This was seen upon the introduction of the Ethiopian Commodity Exchange about a decade ago. In Ethiopia, these traders in the bilateral model were called Akribis. For the most part, the Akribis were wiped out. We suspect the same will be true in Ghana’s case. If the commodity market fees are too high, however, there may be high-cost farmers still in the bilateral trading environment.

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Table 1: Commodity Exchanges in Africa

| | Country | Founded | Status | Commodities | Contracts |
|--------------------|--------------|---------|----------------|--|------------------|
| JSE | South Africa | 1995 | Operational | Maize, wheat, soy and sunflower | Futures |
| ACE | Malawi | 2004 | Operational | Rice, wheat, beans, ground nuts and peas | Forward |
| ECX | Ethiopia | 2008 | Operational | Coffee, sesame, beans, wheat, maize | Spot |
| ZIMACE | Zimbabwe | 1994 | Closed in 2001 | Maize, wheat, soy | Spot and forward |
| ZAMACE | Zambia | 2007 | Closed in 2012 | Maize, wheat, soy, sunflower | Spot and forward |
| GCX | Ghana | 2018 | Implementation | Maize | Spot and forward |
| KCX | Kenya | 1997 | Operational | Main niche is perishable commodities | - |
| Bourse Africa Ltd. | Mauritius | 2010 | Operational | Gold, silver and crude oil | Futures and CFDs |
| TCX | Tanzania | 2015 | Project | Cashew nuts, coffee, cotton and rice | - |

Notes: Bourse Africa Limited was previously named GBOT.

A Characterizing μ^T and μ^F

For the steady state. At any time there will be two populations of farmers and traders. There are those who are currently matched and therefore not part of the bilateral trading in a given period. Let us call the mass of those matched populations as T^M and F^M . We will similarly use the notation T^U and F^U to denote the populations of traders and farmers who are unmatched and therefore market participants in the bilateral trading.

During any given period a fraction β of the partnerships between farmers and traders will break. In particular, the mass βT^M of traders and the mass βF^M of farmers enter the pool of market participants in the bilateral trading. At the same time, at the end of the same period, the mass $\mu^T T^U$ of traders and $\mu^F F^U$ of farmers leave the bilateral trading system with their matches. In a steady state, we want the inflows into bilateral trading to equal the outflows. In particular, the full equilibrium requires that

$$\beta T^M = \mu^T T^U$$

and

$$\beta F^M = \mu^F F^U.$$

Note that in our comparative statics, when we change β , we also change μ^T and μ^F in the equations above. Therefore, when we change β in our counterfactuals, we are implicitly considering a change in the search structure such that the conditions above hold. Since the focus of this paper is not in understanding how the search process comes about but rather the consequences of it, we choose to present the analysis with exogenous values of μ^T and μ^F .

B Existence of a Solution to the Bilateral Search Model

As stated in the main body of the article, we have five equations that determine the value of five endogenous variables: V^{TU} , $V^{TM}(c)$, $V^{FU}(c)$, $V^{FM}(c)$ and $p(c)$. As we solve for these five variables, we also obtain the value of \bar{c} .

Let us first use equations (1) and (3) to get expressions for $V^{FU}(c)$ and $V^{FM}(c)$ as a function of $p(c)$:

$$V^{FM}(c) = \frac{(p(c) - c)(\delta\mu^F + 1 - \delta)}{(1 - \delta)(1 - \delta(1 - \beta) + \delta\mu^F)}, \quad (7)$$

and

$$V^{FU}(c) = \frac{(p(c) - c)\delta\mu^F}{(1 - \delta)(1 - \delta(1 - \beta) + \delta\mu^F)}. \quad (8)$$

These two equations gives

$$V^{FM}(c) - V^{FU}(c) = \frac{p(c) - c}{1 - \delta(1 - \beta) + \delta\mu^F}.$$

From the equation above, we can see that a necessary condition for farmers to obtain positive gains from trade is

$$\rho \equiv 1 - \delta(1 - \beta) + \delta\mu^F > 0.$$

This condition is always satisfied since $\delta \in [0, 1]$ and $\beta \in [0, 1]$. Therefore, farmers are always interested in trading as long as $p(c) > c$. One can think of this denominator as an adjustment that gives the perpetual gain from flows of $p(c) - c$.

We can use the expression above to eliminate $V^{FM}(c)$ and $V^{FU}(c)$ and in the Nash bargaining expression (5), and so be able to obtain an expression for $p(c)$ as a function of c , $V^{TM}(c)$ and V^{TU} :

$$p(c) = \rho\phi(V^{TM}(c) - V^{TU}) + c.$$

Substitute this expression into $V^{TM}(c)$

$$V^{TM}(c) = \max\{\bar{p} - c - \rho\phi(V^{TM}(c) - V^{TU}) + \delta\{(1 - \beta)V^{TM}(c) + \beta V^{TU}\}, V^{TU}\}$$

we redefine this term as

$$V^{TM}(c) = \max\{\tilde{V}^{TM}(c), V^{TU}\},$$

where

$$\tilde{V}^{TM}(c) \equiv \bar{p} - c + (\delta(1 - \beta) - \rho\phi)V^{TM}(c) + (\rho\phi + \delta\beta)V^{TU}.$$

Let us now define the function $V^{TM}(c)$.

When $\tilde{V}^{TM}(c) > V^{TU}$, we have $V^{TM}(c) = \tilde{V}^{TM}(c)$. This gives

$$V^{TM}(c) = \frac{\bar{p} - c + (\rho\phi + \delta\beta)V^{TU}}{1 - \delta + \rho\phi + \delta\beta}.$$

For a given V^{TU} , the value that a trader obtains is linearly decreasing in c . Clearly, $\tilde{V}^{TM}(c)$, which is a function of c and $\tilde{V}^{TM}(c)$, is thus also decreasing in c . To find the cutoff point of c such that $\tilde{V}^{TM}(c) > V^{TU}$ no longer holds, we compute the point where $\tilde{V}^{TM}(c) = V^{TU}$.

In this case, we have

$$V^{TU} = \frac{\bar{p} - \bar{c} + (\rho\phi + \delta\beta)V^{TU}}{1 - \delta + \rho\phi + \delta\beta}.$$

Isolating \bar{c} gives

$$\bar{c} = \bar{p} - (1 - \delta)V^{TU}.$$

Interestingly, the cutoff value depends only on prices and the value of being unmatched. Of course, since V^{TU} is itself a function of the endogenous variables in the model, \bar{c} depends on all the other parameters in equilibrium. The expression above shows that, when traders do not discount the future, then the cutoff is equal to \bar{p} . This is the case when traders can wait for an indefinite period of time to obtain the best match. When V^{TU} is higher, then the cutoff is lower, since the returns from waiting are large. For later derivations, it will be useful to obtain $V^{TM}(0) - V^{TU}$. In this case, we get

$$V^{TM}(0) - V^{TU} = \frac{\bar{p} + (\rho\phi + \delta\beta)V^{TU}}{1 - \delta + \rho\phi + \delta\beta} - V^{TU}.$$

After manipulating the expression above, we get

$$V^{TM}(0) - V^{TU} = \frac{\bar{p} - (1 - \delta)V^{TU}}{1 - \delta + \rho\phi + \delta\beta}.$$

Substituting the value of \bar{c}

$$V^{TM}(0) - V^{TU} = \frac{\bar{c}}{1 - \delta + \rho\phi + \delta\beta}.$$

Figure 2 shows the whole derivation of $V^{TM}(c)$. Since the denominator is always positive, as long as $\bar{c} > 0$, the gains from trade are positive when a trader matches with a farmer with cost $c = 0$.

We have now characterized $V^{TM}(c)$ as a function of c and V^{TU} . We $V^{TM}(c)$ into the final equation for V^{TU} . We use, however, a different version of the equation

$$V^{TU} = \max \left\{ \delta \left\{ \mu^T \int_0^{\bar{c}} (V^{TM}(c) - V^{TU}) g(c) dc + \mu^T \int_0^{c^{max}} V^{TU} g(c) dc + (1 - \mu^T) V^{TU} \right\} - \kappa, 0 \right\}.$$

To keep our analysis interesting, we first solve for the case where $V^{TU} > 0$. Later we define the range of parameters that guarantee that this condition is valide.

Let us focus on the first integral. This is given by the area of the triangle in Figure 2

between 0 and \bar{c} for the region of $\tilde{V}^{TM}(c)$ above V^{TU} . Therefore, we have

$$\int_0^{\bar{c}} (V^{TM}(c) - V^{TU}) g(c) dc = \frac{\bar{c}}{1 - \delta + \rho\phi + \delta\beta} \times \frac{\bar{c}}{c^{max}} \times \frac{1}{2},$$

where we used the fact that $g(c) = 1/c^{max}$ and uniform. Therefore, we have

$$V^{TU} = \delta\mu^T \left(\frac{\bar{c}}{1 - \delta + \rho\phi + \delta\beta} \times \frac{\bar{c}}{2c^{max}} \right) + \delta V^{TU} - \kappa.$$

Instead of solving for the values of V^{TU} , we instead search for the values of \bar{c} that solve the equation above. We examine under which we have an interior solution for our problem where \bar{c} is inbetween 0 and \bar{p} . Substituting V^{TU} gives

$$0 = \frac{\delta\mu^T}{2c^{max} (1 - \delta + \rho\phi + \delta\beta)} \times \bar{c}^2 + \bar{c} - \bar{p} - \kappa.$$

Therefore, we have a quadratic form for \bar{c} :

$$f(c) = a\bar{c}^2 + b\bar{c} + cons \tag{9}$$

where

$$a \equiv -\frac{\delta\mu^T}{2c^{max} (1 - \delta + \rho\phi + \delta\beta)},$$

$$b \equiv 1$$

and

$$cons \equiv -\bar{p} - \kappa.$$

We know that $a < 0$. Therefore, $f(\bar{c})$ is a concave function. An immediate evaluation of the constant shows that it is negative. Therefore, we have two solutions for (9) above 0. Now, we show that only one of them is below \bar{p} . To do so, we substitute \bar{p} into expression (9) and show that we obtain $f(\bar{p}) > 0$, which indicates that $f(\bar{c})$ must cross zero at some point between 0 and \bar{p} according to the intermediate value theorem.

$$f(\bar{p}) = -\frac{\delta\mu^T}{2c^{max} (1 - \delta + \rho\phi + \delta\beta)} \bar{p}^2 + \bar{p} - \bar{p} - \kappa.$$

Simplifying the expression above gives

$$f(\bar{p}) = \frac{\delta\mu^T \bar{p}^2}{1 - \delta + \rho\phi + \delta\beta} \frac{1}{2c^{max}} - \kappa.$$

This expression shows that, if

$$\frac{\delta\mu^T}{1-\delta+\rho\phi+\delta\beta} \times \frac{\bar{p}^2}{2c^{max}} \geq \kappa,$$

then we have a solution for \bar{c} that is between 0 and \bar{p} . When we have $\bar{c} > \bar{p}$, then traders do not strategically reject any farmer. Now, we check whether we have the conditions for $V^{TU} > 0$. Since $\bar{c} = \bar{p} - (1-\delta)V^{TU}$, then we need to show that $\bar{p} > \bar{c}$. The conditions for this to be true are equivalent to the conditions that we need to get an interior solution to the problem.

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