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Off-grid lighting business models to serve the poor

Evidence from a structural model and field experiments in Rwanda

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Off-Grid Lighting Business Models to Serve the Poor: Evidence From a Structural Model and Field Experiments in Rwanda

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A significant proportion of the world's population does not have access to grid-based electricity and so relies on off-grid lighting solutions. Rechargeable lamp technology is becoming prominent as an alternative off-grid lighting model in developing countries. In this paper, we explore the consumer behavior and the operational inefficiencies that result under this model. Specifically, we are interested in (i) measuring the impact of inconvenience (of travel to recharge the lamp) – which is a peculiar feature of this model – along with the impact of liquidity constraints (due to poverty of consumers) on lamp usage, and (ii) evaluating the efficacy of strategies that address these factors. We build a structural model of consumers' recharge decisions that incorporates several operational features of the impoverished regions. We conduct large-scale field experiments in Rwanda in collaboration with a company that operates a rechargeable lamp business and use the resultant data to estimate and test our model.

Using the structural model, we find that completely removing inconvenience and liquidity constraints from the current business model results in a 79% and 123% increase in both recharges and revenue, thereby suggesting that they are major sources of inefficiency. By implementing simple *operations-based strategies* – such as starting more recharge centers per village, visiting consumers periodically to collect their lamps for recharge, and allowing consumers to partially recharge their lamps and pay flexibly for the recharge – that change only the recharge and payment processes but not the recharge price and lamp capacity (i.e., the amount paid per recharge and the amount of light obtained in return), firm can reap up to half the benefits from completely removing the inefficiencies. In contrast, the *price/capacity-based strategies* that vary the economic variables (price and capacity) without affecting the operational model perform much worse than the aforementioned strategies. Overall, our analysis emphasizes the importance of managing operations effectively even in markets with cash-constrained consumers, wherein firms may have a natural tendency to focus more on reducing price. Moreover, the template – combining a structural model and field experiments – used for policy evaluation in our paper can be applied more broadly to generate hypotheses for experimentation and to arrive at appropriate business models that deliver life-improving goods and services to poor consumers.

1. Introduction

One fifth of humankind still does not have access to electricity (IEA 2015). More than 95% of this population inhabits countries in Sub-Saharan Africa and developing Asia. Not surprisingly, countries with low electrification rates are those in which most citizens live on less than $2 (US) per day (often referred to as the *bottom of the pyramid*, or BoP for short). Grid-based models of electricity supply have been unsuccessful in these countries because they require substantial capital investment, and in many cases it may neither be technically feasible nor economical to extend grid electricity to these regions. Hence, there is a huge market for off-grid energy in these countries. For example, Rwanda's current national electrification rate is 11% off-grid and 30% on-grid, and its 2024 objective is to electrify 48% off-grid and 52% on-grid (USAID 2018).

Currently, the predominant sources of lighting for poor households are either flame-based solutions (e.g., kerosene, candles) or battery-based solutions (e.g., flashlights), mainly because they are easily accessible in local retail stores. However, these solutions are expensive to consumers in the long run, and pose a threat to health and the environment due to either harmful smoke generation or the improper disposal of replaced batteries. Solar-based off-grid solutions (such as basic solar home systems) are cleaner and cheaper in the long run, but they require a high upfront investment that places them well beyond the reach of these liquidity-constrained consumers in countries like Rwanda.

An alternative off-grid lighting model that is becoming prominent in impoverished countries is rechargeable lamp technology. Instead of selling lamps to consumers at full price, under this model, firms either rent them or sell them at a subsidized price. Continued use of such lamps requires that they be recharged at a (usually village-level) recharge center for a small recharging fee. The revenue stream from repeated recharges makes it possible for the firm to subsidize the upfront price by financing it through ongoing payments. Sunlabob in Laos, Shidhulai in Bangladesh, and Nuru Energy in Rwanda are some companies that operate based on this model. In this paper, we explore, in close collaboration with Nuru Energy, the consumer behavior and the operational inefficiencies that result under the rechargeable lamp-based off-grid lighting models.

Because the consumers in our study are poor, their liquidity constraints naturally play a role in determining the usage of lamps. Moreover, the rechargeable lamp-based model requires that the consumers travel to a dedicated recharge center to get their lamps recharged. The villages in East African countries, for example, are spread over hills and typically have neither efficient public transportation nor even well-laid roads for walking. From our surveys in Rwanda, we know that most consumers walk to the recharge center, and for some consumers, a round trip can take up to an hour. Therefore, the time required to recharge lamps is a significant inconvenience, which impacts lamp usage.[1] We are specifically interested in examining, both theoretically and empirically, the impact of the above two factors – liquidity constraints and recharge inconvenience – on the usage of lamps.

Furthermore, our broader objective is to evaluate the efficacy – in terms of improving the number of recharges – of different strategies that address those two factors and thereby identify better business models to serve the poor. (We use the terms strategy, business model, and policy interchangeably.) Such an undertaking has implications for both firm-level operational decisions and government-level policy decisions. Firms such as Nuru aim to serve the poor population while making profits. Therefore, better business models enable higher usage rates of cleaner lighting sources and less use of harmful sources (such as kerosene and candles), which in turn would benefit firms, consumers, and the environment. Countries such as Rwanda, whose objective is to provide off-grid lighting to a large proportion of their population, are currently collaborating with multiple off-grid companies operating under different business models (World Bank 2017). Understanding the benefits

[1] This feature is peculiar to rechargeable lamp-based lighting model. The grid-based and solar-based lighting solutions do not require any travel. The consumers purchase lighting sources such as candles and flashlight batteries at local retail stores, where they also purchase several other retail goods (e.g., food-related items, cigarettes, toothpaste); thus, the overall inconvenience in making the purchase is split across several goods that are purchased. In contrast, when a consumer visits a recharge center, all that she gets in return is lamp's light.

and limitations for rechargeable lamp technologies enables policymakers to weigh them against the benefits and limitations of other technologies (such as solar home systems).

However, the task of identifying better business models to serve the poor is nontrivial. The lean startup philosophy (Ries 2011) taught in entrepreneurship classes advocates constant experimentation for rapid improvements in the business models. Yet, there are two issues with such an approach in a context like ours. First, the direct implementation of the candidate policies in the field to assess their performance may not be practical because of the remote location of the villages and the budget constraints of firms. Second, even if one has sufficient resources for constant experimentation, as there are multiple ways one could address inconvenience and liquidity constraints, it is often not clear which strategies deserve most attention (and hence should be tested first). Therefore, we need a method to predict the performance of policies *before* they are implemented. Since a policy change could vary from a simple change in a parameter to a sophisticated change in a process within the firm, the prediction framework must be flexible enough to generate a wide variety of *counterfactual* policies to facilitate their evaluation. We also need data to build such a framework, but because there is a dearth of reliable datasets in the BoP markets, the required data may not even be readily available. Our methodological approach in this paper responds to these concerns.

Methodological approach. We build a structural model of consumers' recharge decisions and estimate that model using the field data. A primary benefit of estimating a structural model of behavior is the ability to calculate outcomes under economic environments not observed in the data, and hence it acts as a framework to predict counterfactual policies. The consumer in our model is forward looking, and her decision process is represented by a stochastic dynamic program over a finite horizon. In a time period in which the consumer's lamp is discharged and she has sufficient money for recharge, if she chooses not to recharge her lamp, then she incurs a blackout cost for not having the lamp's light in that period; however, if she chooses to recharge, then she experiences no blackout cost in the upcoming periods as long as the lamp lasts but incurs an inconvenience cost of traveling to the recharge center and pays the recharge price in that period.[2] In contrast, in a period when the lamp is discharged but the consumer does not have money for the recharge, she simply incurs blackout cost. The liquidity constraints, which determine whether or not the consumer has money for recharge in a period, are captured in our setting through a model for disposable income for the lamp's light that follows a Markov process. The assumed structure can generate a variety of counterfactuals.

The policy evaluation using the structural model requires us to estimate consumers' sensitivities to inconvenience and blackouts, as well as the parameters of the consumption and disposable income processes. We argue that, under some mild structural assumptions, the intertemporal variation in consumers' recharge decisions, along with exogenous variations in recharge price, lamp capacity (hours of light obtained per recharge), and consumer inconveniences (proxied by distances from the recharge center), are sufficient to separately identify the model components. Consequently, we need the actual data on lamp usage at different prices,

[2] In our paper, we use the word "lamp" to refer to the whole package consisting of a light-emitting diode (LED) as the light source, a plastic housing for the LED, a strap, a battery, and a switch to turn the light on and off. The terms "lamp recharge" and "lamp capacity" actually refer to recharge of the battery inside the lamp and the capacity of that battery, respectively.

capacities, and inconvenience levels. Due to the lack of any such data source, we conduct field experiments in collaboration with Nuru in 29 villages of Rwanda, wherein we randomly assign the recharge price and lamp capacity to consumers. We implement an automated system that records recharge timestamps along with the identifiers of lamps getting recharged. We also record the global positioning system (GPS) coordinates of households and recharge centers to calculate the distances between them.

To rely on the predictions made by a structural model in a counterfactual setting, we must establish that the model is empirically consistent. For this purpose, we first conduct a simple reduced-form regression analysis and find that the theoretical predictions made by the model are directionally consistent with the experimental data. Thereafter, we test the ability of the model and its variants (e.g., including village- and individual-level heterogeneities, discounting) to predict the number of recharges both in-sample and out-of-sample. We observe that the best-fitting model predicts the number of recharges both in- and out-of-sample reasonably well. We also find that this model performs significantly better than some atheoretical approaches (namely, a regression model and a model that assumes random consumer behavior).

Results. We examine the performance of business model changes that target inconvenience, liquidity constraints, recharge price, and lamp capacity. We find that completely removing inconvenience and liquidity constraints from the current business model results in up to a 79% and 123% increase in recharges respectively, suggesting that they are major sources of inefficiency. Although these benchmark cases may not be achievable in practice, significant improvements – and sometimes half the benefits from completely eliminating those inefficiencies – can be achieved by implementing some simple strategies that (i) alleviate inconvenience, e.g., starting 2–3 more recharge centers per village (29% increase), visiting households door-to-door once a week to collect the lamps for recharge (37% increase), and visiting just five locations per village twice a week to collect the lamps in those localities (39% increase); and (ii) alleviate liquidity constraints, e.g., allowing consumers to partially recharge their lamps (19% increase), prepay for the recharge (43% increase), and recharge on credit 1–2 times (76% increase).

The above-mentioned strategies only vary the operational model of the firm by addressing the sources of inefficiencies and by changing the recharge and payment processes within the firm. They do not affect the recharge price or lamp capacity, and hence they increase both recharges and revenue simultaneously. We also examine strategies that vary price and capacity without affecting the current operational model. We find that the recharges are relatively inelastic with respect to price; thus, reducing price, although increases recharges, simply decreases revenue. Consequently, scaling up by offering subsidies to consumers may not be a sustainable strategy for the firm in the long run. If the firm also varies lamp capacity along with price, then we find that both recharges and revenue improve and that it is optimal for the firm to reduce both price and capacity; however, (i) the resultant improvements are quite limited in magnitude (15% increase in recharges and 3% increase in revenue), and (ii) the resultant ratio of capacity to price (i.e., bang for the buck) is lower than that at the status quo; in other words, the consumers pay a poverty premium when the light is provided in a smaller, affordable package. Overall, we find that operations-based strategies perform better than price/capacity-based strategies.

The firms and policymakers in poor countries generally lean toward price-based strategies because poverty, by definition, relates to lack of money. However, our analysis reveals that even when operating under poverty, wherein monetary constraints are overpowering and may limit technology adoption, the operational inefficiencies embedded in the business model (e.g., constraints such as making the consumer travel to a single village-level recharge center and making her pay only when she recharges her lamp) may also be major hindrances to adoption, and addressing them results in significantly more benefits.

In addition to shedding light on rechargeable lamp-based business models, our paper presents a template for policy evaluation combining structural modeling and field experiments; this template can be applied to settings that go beyond our context to generate hypotheses – grounded in both theory and data – for experimentation, and thereby to arrive at appropriate business models that deliver life-improving goods and services to poor consumers.

Organization of the paper. Section 2 provides an overview of our undertaking: it describes the relationship between our research objectives, our structural model, and the design of our field experiments. Thereafter, Section 3 gives the details of the field experiments, Section 4 describes the model and the estimation procedure, Section 5 presents the performance of the counterfactuals that we examine, and Section 6 makes some concluding remarks. After reading Section 2, Sections 3–5 can be read in any order depending on the interests and the priorities of the reader. For example, an entrepreneur or a policy official interested mainly in our findings and an outline of the methodology used to arrive at those findings can read only Sections 2 and 5 without becoming burdened by the technicalities, whereas an academic or a practitioner interested in the implementation details can additionally read Sections 3 and 4. We briefly review the related literature in the remainder of this section.

Related literature. Our paper is positioned at the intersection of two streams of literature: sustainable operations and the economics of poverty.

We broadly relate to two growing streams of literature studying sustainability issues in operations management (OM). First is the field of energy operations, wherein researchers have studied various operational aspects of the electricity markets: the effect of energy policies on supply and demand in electricity markets (Daniels and Lobel 2014, Sunar and Birge 2019), pricing of renewable energy technologies (Alizamir et al. 2016, Kok et al. 2018), strategic investment in renewable energy sources (Aflaki and Netessine 2017, Kok et al. 2020), and impact of operational flexibility on power supply intermittency and market competition (Wu and Kapuscinski 2013, Al-Gwaiz et al. 2017). Much of this work focuses on grid-based models in developed economies. Our work differs from those just cited by focusing on the operational aspects of an off-grid model at the bottom of the pyramid.

The second related stream of literature in OM studies the operational issues that arise in the business models that serve the BoP markets. Balasubramanian et al. (2017) study the inventory issues arising in the context of mobile money agents; Jonasson et al. (2017) develop models to improve the capacity allocation of laboratories for the early diagnosis of the human immunodeficiency virus (HIV) among infants; Gui et al. (2019) examine the efficacy of purchasing cooperatives and non-profit wholesalers in terms of replenishing goods for microretailers; de Zegher et al. (2018) propose payment strategies to curb illegal deforestation by smallholder farmers; Guajardo (2019) explores the relationship between consumer usage and payment behaviors in a rent-to-own business model for the distribution of solar lamps; and Kundu and Ramdas (2019) investigate the impact of timely after-sales service on the adoption of solar home systems by first-time users in developing countries.

Our paper is closely related to Uppari et al. (2019), who study, using an analytical model, why some consumers may prefer using kerosene to rechargeable lamps even when the latter costs less money than the former. They find that the consumers who face either high inconvenience costs or high blackout costs tend to prefer kerosene to lamps because the former's flexibility, with regard to quantity, helps reduce whichever cost is dominating. That paper also discusses some strategies (e.g., allowing partial recharges) to improve the adoption of lamps. The empirical work in this paper complements the theoretical work of Uppari et al. (2019). Our structural model includes inconvenience and blackout costs along with liquidity constraints much in the spirit of Uppari et al. (2019), and our field data allows us to estimate those costs and the magnitude of liquidity constraints. Furthermore, our paper quantifies the impact of the strategies discussed in Uppari et al. (2019), while also examining several additional strategies.

The method of deploying field experiments to analyze consumer behavior in the context of poverty has been used in the development economics literature. This literature mainly investigates various behavioral impacts of prices on the adoption of a technology. For example, Cohen and Dupas (2010) examine whether or not consumers waste goods that are distributed freely to them; Ashraf et al. (2010) analyze the role of higher prices in increasing product use through screening and sunk-cost effects; Dupas (2014b) studies the extent to which the consumers' anchoring on subsidized prices impacts the long-run adoption after those subsidies are removed; and Duflo et al. (2011) study the impact of seasonal income on the purchase of fertilizers. In most of these papers, consumers make no more than two purchase decisions at different price levels (e.g., subsidized and non-subsidized). In contrast, consumers in our setting make multiple recharge decisions over time at a given price level, but those price levels differ across consumers. In such a setting with *repeated purchases*, inconvenience cost – which is incurred with every purchase – plays an important role in the long-run usage of the technology. Therefore, our interest in this paper lies in examining the impact of price as well as the impact of inconvenience on the usage of lamps.

Another emerging field in development economics experimentally examines the demand for rural electrification. Using randomized prices for connection to the grid, Bernard and Torero (2015) assess the importance of social interactions in determining a consumer's choice to connect to an electrical grid in rural Ethiopia; Barron and Torero (2017) study the welfare improvements resulting from electrification via reductions in indoor air pollution in El Salvador; and Lee et al. (2020) conduct a detailed analysis of economic and non-economic (health, education, etc.) impacts of electrcity rollout in rural Kenya. Our study differs in that it focuses on the other end of the spectrum from full-scale government grid electrification: for-profit business models for low-cost off-grid lighting solutions in regions such as rural Rwanda, wherein the hilly terrain makes a country-wide grid extension uneconomical (at least in the medium-term).

Banerjee et al. (2017) advocate the addition of a structural model to experimental research as it facilitates "structured speculation"; that is, it can lead to a fully specified set of falsifiable predictions in external environments. We use a dynamic programming (DP)-based structural model in our paper. The introduction of the framework to estimate discrete-choice DP models is associated with independent contributions by Miller (1984), Pakes (1986), Rust (1987), and Wolpin (1984, 1987). For the methodology of this framework, we refer the reader to Rust (1994) and Aguirregabiria and Mira (2010). The framework has been applied in several contexts to evaluate economic policies that are particularly relevant to developing countries; see Todd and Wolpin (2010b) for a review of such applications. However, to the best of our knowledge, we are the first to use that framework in conjunction with field experiments for an *extensive business model analysis* in the context of poverty. Moreover, since the DP model in our paper incorporates several operational features of the impoverished regions that are not part of the existing DP models in the structural estimation literature, it presents a unique set of identification challenges that are resolved through field experiments. Finally, to rely on the predictions made by a structural model in a counterfactual scenario, Keane (2010) and Todd and Wolpin (2010b) emphasize the necessity of validating its predictive ability. In this study, we examine the predictive ability of our DP model – estimated using the data from treatment conditions – on the data from the control condition as well as a test treatment condition; such validation methods are also applied in Todd and Wolpin (2006) and Duflo et al. (2012).

2. Overview of Our Approach to the Problem

Our research objectives, our structural model of consumer behavior, and the design of our field experiments are intimately related to each other. Therefore, instead of delving into a deeper discussion on these three topics sequentially, we first provide an overview in this section to shed light on their relationship with one another. The technical details of our model and the implementation details of our field experiments are provided in the later sections.

2.1. Experimental Context and Research Objectives

The research in this paper has been conducted in collaboration with Nuru Energy in Rwanda. Nuru is a for-profit social enterprise, with operations in Rwanda, Burundi, and Kenya, that aims to address the issue of energy poverty through the provision of rechargeable lamps and lamp-recharging centers to off-grid poor rural communities. The lamps are sold below cost to make them affordable. (Each lamp costs Nuru 6 USD to manufacture but is sold at 1–1.5 USD to consumers. The lamps are made to last for 250 recharges.) Continued use of lamps requires that they be recharged at a centralized pedal-and-solar-powered recharge center operated by village-level entrepreneurs (VLEs) who charge lamps. The recharge centers are usually same as the houses of those VLEs. Under the current business model, the lamps are recharged at a price of 100 Rwandan Francs (RWF), and the lamp capacity is 18 hours. The VLEs earn 50 RWF per recharge. The revenue stream from recharges makes it possible for Nuru to subsidize the upfront price by financing it through ongoing payments; thus, it is important for Nuru to have a steady stream of recharges to make profits.

The quantity of interest to Nuru and other firms operating the rechargeable lamp business is the expected number of recharges if they adopt a particular policy $\mathcal{P}$. We use the term *policy* quite broadly here, and it may encompass several business-related decisions such as the price and the capacity of the lamps, the payment schemes, and the location and the number of recharge centers in a village. For instance, under the status quo policy, Nuru charges 100 RWF per recharge for the 18-hour capacity lamps, and the consumer travels to a village-level recharge center to recharge her lamp and to pay the recharge fee to the VLE.

We are interested in assessing how the number of recharges varies across different policies.[3] Given our broad definition of policy, it is important to note that we are not interested in the *optimal* policy of any particular type, but we focus on evaluating the efficacy of some implementable policies that address the inconvenience and liquidity constraints of the consumers. Such policy changes may vary from the simple ones such as changing a parameter (e.g., price and capacity) to more sophisticated ones such as decoupling payments from recharges through mobile payment schemes.

One way to evaluate candidate policies is by directly implementing them in the field to see how they perform. However, this approach may not always be feasible, especially in a context like ours, wherein the firms operate under tight budget constraints and the policies must be implemented in remote villages requiring nontrivial investments in both time and money. Therefore, we need to be able to assess the effectiveness of a policy *before* it is implemented in the field, i.e., we need to perform *ex-ante policy evaluation* (Todd and Wolpin 2010a). Such evaluation necessitates a formal framework for the consumers' recharge decisions.

Formally, assuming a discrete time domain, we denote the recharge decision of consumer j in period t as r_{jt}, where $r_{jt}=1$ if that consumer recharges her lamp in period t, and it is 0 otherwise. In an arbitrarily given duration $\{1,\dots,T\}$, we denote the sequence of consumer's recharge decisions as $\mathfrak{r}_j=(r_{j1},\dots,r_{jT})$, which we assume is a realization of the random variable $\tilde{\mathfrak{r}}_j=(\tilde{r}_{j1},\dots,\tilde{r}_{jT})$. Correspondingly, the total number of recharges are denoted by $R_j=\sum_{t=1}^{T} r_{jt}$ and $\tilde{R}_j=\sum_{t=1}^{T}\tilde{r}_{jt}$. Then, under a policy $\mathcal{P}$, the expected number of recharges by consumer j is

$$\mathbb{E}\tilde{R}_j(\mathcal{P})=\sum_{\mathfrak{r}_j\in\mathfrak{R}} R_j\times \Pr\left(\tilde{\mathfrak{r}}_j=\mathfrak{r}_j\mid\mathcal{P}\right), \tag{1}$$

where $\Pr\left(\tilde{\mathfrak{r}}_j=\mathfrak{r}_j\mid\mathcal{P}\right)$ is the probability of observing the recharge sequence $\mathfrak{r}_j$ under policy $\mathcal{P}$, and $\mathfrak{R}$ is the set of all 2^T possible recharges sequences.

To evaluate different policies, we need to know what the aforementioned probability would be under those policies. We take the following approach to estimate that probability in *counterfactual* settings. First, we build a model of consumer behavior wherein its structural components interact to generate the distribution of recharge decisions. Second, we estimate the parameters of the model components; this requires data with the necessary set of variations that are obtained through our field experiments. Finally, under a counterfactual policy, the model components would interact in a different manner, and because we know the parameter estimates of the components, we can also estimate the distribution of recharges under that policy. In the next two subsections, we give an overview of our structural model and discuss the relationship between its components and the variations required in the data to be able to identify those components.

[3] Because rechargeable lamps are sold at a low (subsidized) price, upfront purchase cost is unlikely to be a barrier to adoption; therefore, we do not consider its impact in this paper. Accordingly, we set the purchase price to zero in our field experiments. Since 2019, Nuru also set the purchase price to zero. (For a closer examination of the impact of upfront price in our context, we refer the reader to the parallel work by Clarke et al. 2020).

2.2. Structure of the Decision Process

Let P be the recharge price of the lamp, Q be the lamp's capacity, and I be the inconvenience experienced by the consumer in recharging the lamp. Figure 1 pictorially represents the decision process that we assume for our focal consumer. (For notational simplicity, we suppress the subscript j representing consumers.) The consumer recharges her lamp in period t if and only if the following three conditions are satisfied: (i) consumer's lamp is *discharged* in period t, (ii) she has sufficient *money* for the recharge in period t, and (iii) it is better to recharge *sooner* (i.e., in period t) than later (i.e., in a period $t' > t$). Conditions (i), (ii), and (iii) are, respectively, represented by the indicator variables $\mathbb{D}_t$, $\mathbb{M}_t$, and $\mathbb{S}_t$ in Figure 1.

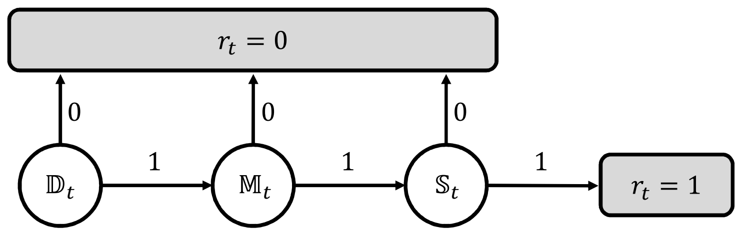

Figure 1 Decision process of the consumer.

Condition (i) is motivated by both empirical and anecdotal evidence: from the data recorded at the recharge centers of Nuru, we find that there is no charge remaining in the lamp when it is plugged in to be recharged. This finding is further supported by the survey data wherein consumers mentioned that they do not recharge their lamps before they are completely discharged.[4] Condition (ii) represents the liquidity constraints of the consumers; even if a consumer wants to recharge her lamp, she may not have enough money to do so, and hence she does not have the option to recharge.

Condition (iii) captures the trade-off between consumer's *inconvenience cost* and *blackout cost*. Assuming conditions (i) and (ii) are satisfied in period t, if the consumer chooses not to recharge her lamp in that period, then she incurs a disutility called blackout cost for not having the lamp's light; the magnitude of this disutility could vary over time depending on consumer's valuation of lamp's light (e.g., consumer might value it less when she has a stock of alternative lighting sources and more when her children have exams). Alternatively, if the consumer chooses to recharge her lamp in period t, then she incurs an inconvenience cost of traveling to the recharge center in that period *and* experiences no blackout cost in the upcoming periods as long as the lamp lasts. Consequently, the consumer may choose to (strategically) delay her recharge in period t if the blackout costs avoided in the current and the next few periods by recharging the lamp are relatively lower than the inconvenience cost of recharging.

The indicator variables $\mathbb{D}_t$, $\mathbb{M}_t$, and $\mathbb{S}_t$ may be random, and their collections over time are modeled as (nonstationary) stochastic processes in our paper. (The model specifications are presented in Section 4.) The three processes together determine the probability $\Pr\left(\tilde{\mathfrak{r}} = \mathfrak{r} \mid \mathcal{P}_0\right)$; here, $\mathcal{P}_0$ is the status quo policy of Nuru.

[4] This could be because from the current design of the lamps, consumers cannot directly know the charge remaining in their lamp, and so they consider recharging it only when it is completely discharged.

The structure in Figure 1 is simple, yet it allows the evaluation of the counterfactual policies that are of interest to us. For instance, any consumption-related policy interventions affect the decision process through $\mathbb{D}_t$, the policies that alleviate liquidity constraints affect the decision process through $\mathbb{M}_t$, and the strategies that target inconvenience affect through $\mathbb{S}_t$. Such policy evaluation first requires us to separately identify and estimate each of the component processes $\mathbb{D}_t$, $\mathbb{M}_t$, and $\mathbb{S}_t$. We next discuss what is further required to identify them.

2.3. Motivation for the Field Experiments

Assuming the decision process in Figure 1, the events between recharges are illustrated in Figure 2. The time between two successive recharges, hereafter called the *interarrival time* (IAT), consists of two components: (i) the *consumption time* (CT) – when the lamp's light is consumed, and (ii) the *blackout time* (BT) – when the consumer waits to get her lamp recharged. After the consumer recharges her lamp, she consumes the lamp's light in the next few periods, and $\mathbb{D}_t = 0$ in those periods. After the lamp is discharged ($\mathbb{D}_t = 1$), the consumer may not recharge her lamp immediately because she may not have sufficient money for the recharge. Even if she has sufficient money for the recharge, she may choose not to recharge and instead experience a few days of blackouts in order to minimize frequent visits, and thereby balance the recharge inconvenience against the experienced blackouts. Thus, the blackout time may arise either due to the consumer's liquidity constraints ($\mathbb{M}_t = 0$) or due to her strategic behavior to balance the inconvenience and blackout costs ($\mathbb{S}_t = 0$).

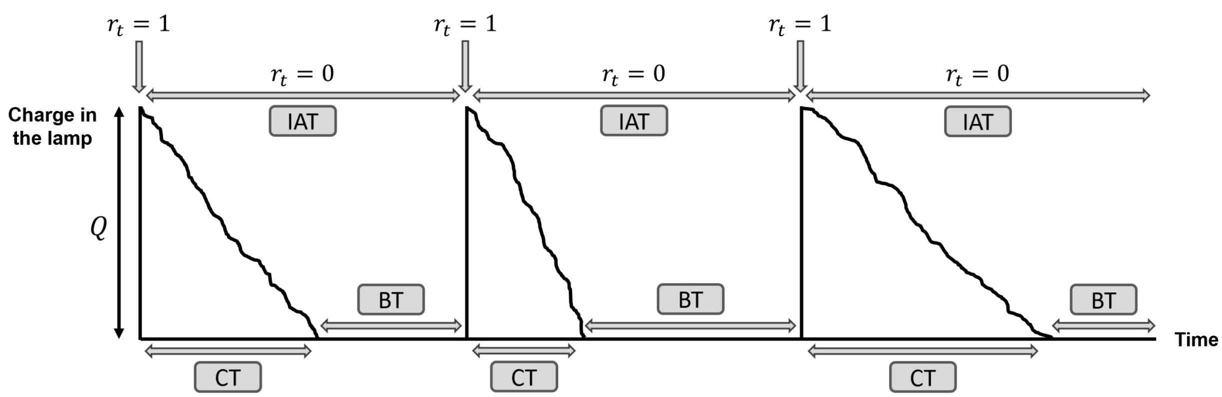

Figure 2 Consumption cycles, showing interarrival times (IATs), consumption times (CTs) and blackout times (BTs). The event $\{r_t = 1\} = \{\mathbb{D}_t = 1 \wedge \mathbb{M}_t = 1 \wedge \mathbb{S}_t = 1\}$, and CT and BT, respectively, consist of the events $\{\mathbb{D}_t = 0\}$ and $\{\mathbb{D}_t = 1 \wedge (\mathbb{M}_t = 0 \vee \mathbb{S}_t = 0)\}$.

Since consumers visit recharge centers to recharge their lamps, the firm can record the corresponding timestamps and thereby keep track of IATs. Unfortunately, it should be evident from Figure 2 that the data on IATs alone is not sufficient to separately identify the underlying stochastic processes $\mathbb{D}_t$, $\mathbb{M}_t$, and $\mathbb{S}_t$. A longer IAT could be because of either a longer CT or a longer BT, but if we do not separately observe CT and BT, then we cannot purely attribute the IAT's length either to $\mathbb{D}_t$ or to $\mathbb{M}_t$ and $\mathbb{S}_t$. Similarly, a longer BT could be either because of lack of sufficient money to recharge the lamp or because of the willingness to face a few extra days of blackouts. If we do not observe the consumer's disposable income for the lamp's light, then we cannot attribute the length of BT purely either to $\mathbb{M}_t$ or to $\mathbb{S}_t$.

Yet, if we perfectly observe (i) the instances when a consumer's lamp is discharged, (ii) the instances when she has sufficient disposable income to recharge, and (iii) the blackout costs experienced by her in every period when the lamp is discharged, then we know the CTs, BTs, and the liquidity constraints of that consumer, thereby allowing the identification of the components $\mathbb{D}_t$, $\mathbb{M}_t$, and $\mathbb{S}_t$. However, observing (iii) is almost infeasible, whereas observing (i) and (ii) requires recording the events at the consumer's household *between* their successive recharges (in contrast to the recharge data, which is recorded at the recharge center). Given the rural nature of our context and the lack of advanced technologies that closely monitor consumers' behavior, automated recording of such detailed instances is extremely difficult. Asking a consumer to regularly self-report (i) and (ii) may be costly to her, and the resultant data may not be reliable. Therefore, we need to resort to alternative means to disentangle the components of IATs.

If there exists a variable that affects only one of the component processes without affecting the other ones, then *all else equal*, when we vary this variable exogenously, the resultant variation in IATs can be purely attributed to the corresponding process, which consequently identifies that process.

Accordingly, we make the following structural assumptions: (A1) lamp capacity Q affects only $\mathbb{D}_t$, (A2) recharge price P affects only $\mathbb{M}_t$, and (A3) inconvenience I affects only $\mathbb{S}_t$. Then, under A1–A3, exogenous variations in Q, P, and I, respectively, identify $\mathbb{D}_t$, $\mathbb{M}_t$, and $\mathbb{S}_t$. That is, we need the recharge data at different exogenously assigned values of Q, P, and I for the estimation of our structural model. Since it would be difficult to create these variations at the individual level (and hence we may not be able to identify the processes at the individual level), we resort to field experiments, wherein we create those variations across individuals and use the recharge data of those individuals to estimate the model. We return to the discussion on identification in Section 4 in relation to the parameters of $\mathbb{D}_t$, $\mathbb{M}_t$, and $\mathbb{S}_t$ after we describe the models for these processes.

Remark 1. The model presented in Figure 1 is in the spirit of a *partial equilibrium model*, i.e., we model optimal behavior only with respect to recharge decisions. Although consumers could be making several monetary allocation decisions across their various needs and consumption decisions across different lighting sources, we do not explicitly incorporate those decisions.

Moreover, the structural assumptions A1–A3 are necessary to separately identify $\mathbb{D}_t$, $\mathbb{M}_t$, and $\mathbb{S}_t$ processes. Although these assumptions seem reasonable, they may break down if the consumer's decision process is more sophisticated. For example, the consumer may start saving for the recharges as the lamp is closer to the discharge point, or her consumption and blackout times may be intertwined (in contrast to Figure 2, wherein BT appears only after CT) based on some of her strategic decisions; these possibilities introduce direct dependencies between Q and $\mathbb{M}_t$ or $\mathbb{S}_t$. A1–A3 rule out such dependencies.

Introducing any of the aforementioned features into the decision process comes at the cost of requiring richer data (e.g., consumption, income, and expenditure decisions made across several needs, risks faced by and precautionary behavior exhibited by the consumers) for cleaner identification and model estimation. As we mentioned earlier, such data is difficult to record in our rural context. Moreover, as we will see in Section 4, the assumed decision model displays good out-of-sample predictive ability, suggesting that it captures the most important factors that determine consumers' recharge decisions.

3. Field Experiments

In collaboration with Nuru, we developed a purpose-designed data collection technology to record lamp recharges remotely via Global System for Mobile (GSM) communications technology. The lamps were programed to be able to communicate with the recharging device, and the recharging device was engineered to communicate via GSM to our cloud-based database. When a lamp is attached to the recharging device, it records the lamp's serial number and the date-time stamp of the recharge (accurate to a minute) and transmits this information via GSM to our database.[5] We use this data-recording mechanism in our experiments, which are described next.

3.1. Experimentally Varying the Recharge Price and the Lamp Capacity

The field experiments focused on randomly varying the recharge price and lamp capacity faced by consumers. There were ten treatment conditions in total: (i) seven conditions with seven different price levels (0, 50, 60, 70, 80, 100, and 120 RWF) and lamp capacity of 18 hours, (ii) two conditions with two price levels (80 and 100 RWF) and lamp capacity of 14 hours, and (iii) a test treatment condition where every fourth recharge was free (with a regular recharge price of 100 RWF and lamp capacity of 18 hours).

We were constrained to using only two lamp capacity conditions because of budget considerations and technological restrictions from Nuru (varying the capacity requires changes at the hardware level). The aforementioned price levels were chosen for 14-hour lamps because $14/80 \approx 18/100$. The test treatment condition was included in our experiments with the main purpose of testing the predictive ability – in a counterfactual setting – of the models that are estimated using the data from the first nine conditions (see Section 4.4 for details). The purchase price of the lamps in all treatment conditions was set to zero so that there were no selection effects with regard to consumers' purchase decisions. The VLEs remained unaffected by these treatment conditions as they were always reimbursed 50 RWF after a recharge.

The experiments were conducted in 29 villages of the Ruhango district of Rwanda. These villages are representative of rural Rwanda (and East Africa in general).[6] They had no grid connection, there were no plans to extend the grid to those villages in the near future, and Nuru had no prior operations in those villages. Before the experiments began, a list of all households in each village was obtained. Thereafter, a total of 2500 households, with around 80–90 households per village, were randomly selected and assigned to one of the above ten conditions. The random assignment of households to the treatment conditions was stratified at the village level in order to achieve balance (with around 8–9 households in a village per treatment condition).[7]

[5] The inbuilt hardware mechanisms ensure that Nuru's lamps can be charged only by Nuru's proprietary charging devices. Enabling the lamps to be charged using alternative sources (called unlocking the lamps) requires tampering with the hardware by cutting through the plastic covering of the lamp, which is usually not possible for typical consumers. None of the lamps used in our experiments were unlocked. This was verified through a survey conducted at the end of the experiments wherein the consumers were asked to show their lamps to examine if there was any tampering.

[6] All of our experiments and surveys were conducted by the organization Innovations for Poverty Action, which uses standard protocols for data sampling and collection.

[7] The design with multiple treatment arms per village is common in the development economics literature; see e.g., Ashraf et al. (2010), Cohen and Dupas (2010), Meredith et al. (2013), Dupas (2014a,b), Barron and Torero (2017). Alternatively, the design with only one treatment arm per village requires hundreds of villages to detect treatment effects with reasonable statistical power, thereby resulting in enormous costs of experimentation.

The treatments ran for a total of three months from the beginning of December 2016 to the end of February 2017. (The business, thereafter, continued with Nuru's regular business model.) The experimental conditions in those three months were operationalized through a coupon system. When the lamps were handed over to the consumers, they were also given a coupon card containing 15 perforated coupons, with each coupon having its own identifier (ID).[8] The coupon card explicitly mentioned the ID of the lamp assigned to the household, the recharge price and lamp capacity assigned to that household, and the names and birthdates of up to two household heads. The consumers were aware that the coupon cards they received were the result of a lottery and that the coupons would expire after three months.

To recharge during the experiments, households had to bring their lamp, coupon card, and their government-issued personal ID to the recharge center. The VLE recharged the lamp only if the ID printed on the lamp matched the lamp ID on the coupon card *and* the name and date of birth on the personal ID presented to the VLE matched the ones on the coupon card. If the details matched, then the VLE recharged the lamp, tore a coupon from the coupon card, collected the price written on the coupon, and sent the coupon ID through a mobile message to Nuru's reimbursement system. When the lamp was recharged, the lamp's ID and the recharge timestamp were automatically recorded by the recharging device and sent to Nuru's database. The VLE was then reimbursed only if the coupon ID sent by the VLE belonged to the lamp that was actually recharged (whose ID was automatically recorded).

This experimental design ensured that the households and VLEs did not deviate much from the protocol. As long as the VLE performed the required checks, it would not be possible for a consumer to bring some other consumer's lamp or coupon and get the recharge done unless they also brought the personal ID of that consumer. However, people in Rwanda are usually uncomfortable sharing their personal IDs with others, and thus this is an unlikely event. The VLE was also incentivized to perform the checks because an inconsistent pairing of coupon and lamp ID would not result in any reimbursement.

It is important to note that the aforementioned checks can maintain the consistency between the lamp and its assignee at the time of recharge but not after recharge. For example, a consumer could lend her lamp to her neighbors for a short period of time between two successive recharges. We can neither detect nor prevent such cases. However, the qualitative evidence from our field visits and surveys suggests that the households are usually possessive of their lamps and tend not to share them with others, mainly because each household had only one lamp, and its light (when available) was used on a daily basis. Accordingly, we ignore the possibility of lamp sharing and attribute the observed usage of a lamp only to its owner.

3.2. Variation in Inconvenience

To measure recharge inconvenience, we recorded the (three dimensional) GPS coordinates of recharge centers and of all households in the sample. We quantify the inconvenience faced by a consumer as the distance between the GPS coordinates of her house and the recharge center. Unlike the variation in recharge price

[8] Fifteen coupons are sufficient if the consumers recharge once every week in the three months of experimental duration. In reality, however, the average number of recharges per household in those three months was only 2.93 (3.02), and only four households in our dataset recharged 15 times.

and lamp capacity, the variation in inconvenience measured in this manner is not exogenously created. Yet, as argued below, we can consider this variation to be as good as random.

The recharge centers were established in villages right at the beginning of the experiments. If the location of the recharge center in a village was randomly selected, then the distances measured from that location are also random. However, the location of a recharge center may not be completely random because it is the location of the house of the VLE in that village, who is not randomly selected. Nevertheless, we attribute the emergence of VLEs more to their entrepreneurial inclination and less to their wealth, occupation, or any other characteristics that are correlated with their neighborhood. Hence, it is not unreasonable to assume that the location of VLE is not systematically correlated with the characteristics of consumers, such as income and family size, that may determine lamp usage.

To further test this claim, we collected information on income, occupation, family composition, and other characteristics of consumers in twelve randomly selected villages in our sample and found no significant correlation between consumer characteristics and distance to the recharge center. (In contrast to the survey that collected GPS coordinates, the consumer survey required asking multiple questions, and thus it was more expensive. Because of our budgetary constraints, we could not conduct the consumer survey in all sampled villages.) The detailed analysis is available in Appendix A. We extrapolate this observation to other villages in our sample. The results presented in Section 5 qualitatively remain unaffected if we use the data from only these 12 villages instead of the full sample in our structural analysis.

3.3. Data

Table 1 reports the total number of recharges recorded in the 29 sampled villages. We see from the last column that the villages with IDs 15, 17, 18, 19, 21, 23, and 29 recorded a much lower number of recharges when compared to the others in the sample. This is because the GSM component of the recharging device broke down in these villages during the experiments, thereby causing disruption to the experiment and partial loss of data. Therefore, we remove these 7 villages from our sample, and we report results using the data from the remaining 22 villages.

Figure 3(a) shows the distribution of interarrival times of recharges. The modal value of IAT observed is 7 days. The distribution is also long tailed, with some mass beyond 60 days. The average value of IAT is 14.5 days, with a standard deviation of 12.5 days. Overall, IATs in the sample display sizeable variation. As we discussed in Section 2.3, this variation is crucial in identifying and estimating our model. Moreover, the minimum IAT observed in the data is 3 days. Henceforth, we combine three days into one unit called a *period*. It will be evident in Section 4 that the computational complexity of our estimation procedure scales with the number of time periods; therefore, making the timeline coarser reduces computational time without losing any information on recharges.

Figure 3(b) plots the number of observed recharges across periods for three price conditions. We see that across all price conditions, the recharges are spread over the time horizon and seem to stabilize as time progresses. This suggests that consumers are not simply trying the lamps in the first few weeks (because of some enthusiasm to try a new technology) and then not using them at all. Figure 3(c) plots the average number of recharges recorded *per household* against the recharge price. Both Figure 3(b) and Figure 3(c)

Village ID	Total number of households	Number of households in the sample	Number of recharges recorded
1	138	88 (64%)	115
2	151	77 (51%)	372
3	170	78 (46%)	360
4	138	92 (67%)	158
5	158	92 (58%)	392
6	120	92 (77%)	206
7	120	88 (73%)	293
8	158	75 (47%)	146
9	104	75 (72%)	259
10	159	93 (58%)	345
11	200	83 (42%)	139
12	122	90 (74%)	281
13	103	86 (84%)	326
14	149	93 (62%)	283
15	160	91 (57%)	1
16	148	89 (60%)	192
17	252	84 (33%)	1
18	127	87 (68%)	61
19	125	77 (62%)	36
20	151	84 (56%)	392
21	138	79 (57%)	61
22	156	87 (56%)	248
23	150	88 (59%)	59
24	126	89 (71%)	247
25	156	87 (56%)	248
26	136	93 (68%)	276
27	137	90 (66%)	134
28	103	84 (82%)	165
29	146	89 (61%)	60

Table 1 Village-level statistics.

show that the number of recharges decreases as the price increases. It is noteworthy that in Figure 3(c), when price is zero, the average number of recharges per household is 6 in the three months of experimental duration, implying (on average) only 2 recharges per month. This shows that there are frictions beyond price at play in this context that hinder more frequent usage of lamps.

Figure 3(d) plots the average number of recharges per household for the two lamp capacity conditions in our experiments. One may expect the number of recharges to be lower for the lamps with higher capacity because they last longer. However, we record 2.35 recharges per household when $Q = 14$ and 3.11 recharges per household when $Q = 18$. We will make sense of this observation in Section 4, wherein we explore the theoretical relationship between the number of recharges and the lamp capacity in more detail.

Figure 3(e) plots the distribution of the distances of households from the (respective) recharge centers. (The distances are presented in buckets of length 50 meters.) We see that the modal distance between the VLE and consumers is 450 meters, and the average distance is 600 meters (with a standard deviation of 330 meters). The distribution is right-skewed, and some consumers live more than 1200 meters away from the VLE. Because of the hilly nature of Rwandan villages, we see from our surveys that it takes 25–30 minutes (on average) for a consumer to travel a kilometer. An average round trip to the recharge center

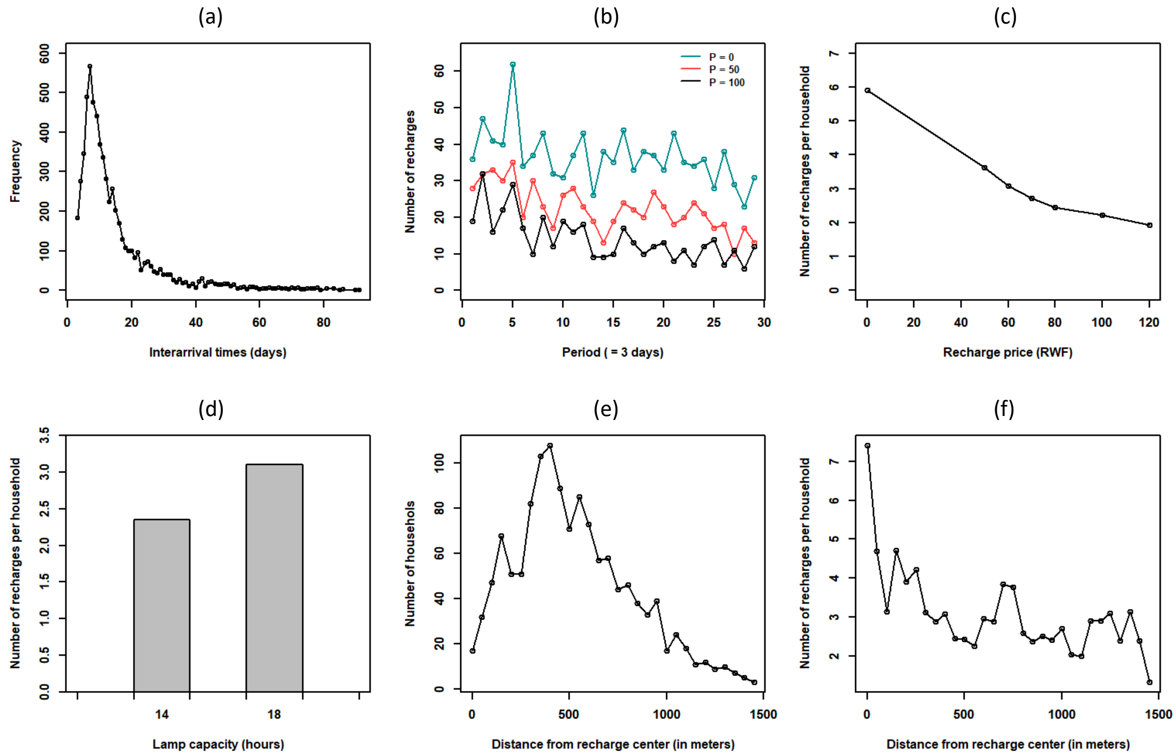

Figure 3 **Some patterns observed in the recharge data: (a) distribution of interarrival times, (b) recharge patterns over time for $P \in \{0, 50, 100\}$ RWF, (c) recharges per household as a function of recharge price, (d) recharges per household as a function of lamp capacity, (e) distribution of distances from the recharge center, (f) recharges per household as a function of distance from the recharge center.**

could last longer than a half hour, and for some consumers it could be longer than an hour. Figure 3(f) plots the average number of recharges per household as a function of distance from the VLE. The number of recharges decreases as the distance increases, suggesting that traveling to the recharge center is indeed an inconvenience that hinders the usage of lamps.

4. Model Formulation and Structural Estimation

4.1. Model of Recharge Decisions

The decision process in Figure 1 spans from the beginning of our experimental duration (denoted as period $t=1$) to the end of that duration (denoted as period $t=T$). We assume that our consumer is forward-looking and that her recharge decisions emerge from a stochastic dynamic program. As in Section 2.1, the sequence of the consumer's recharge decisions is denoted as $\mathfrak{r} = (r_1, \ldots, r_T)$. We denote its subsequence from period 1 to period t as $\mathfrak{r}\langle t \rangle$, such that $\mathfrak{r}\langle T \rangle = \mathfrak{r}$. Next, we construct the models for the monetary process $\mathbb{M}_t$ and the discharge process $\mathbb{D}_t$, and then formulate our consumer's DP problem. Since $\mathbb{S}_t$ captures the consumer's dynamics of cost considerations, the model for $\mathbb{S}_t$ stems from the Bellman equations of that DP.

Model of $\mathbb{M}_t$. In period t, $\mathbb{M}_t$ indicates whether or not a consumer has sufficient money for a recharge in that period. The money under consideration is the consumer's disposable income for lamp recharge. In other words, this is the money that consumer could use to recharge her lamp after accounting for all her other needs. If this disposable income is greater than or equal to recharge price P, then $\mathbb{M}_t = 1$; otherwise $\mathbb{M}_t = 0$.

We model $\mathbb{M}_t$ as a Markov process. The probability that the process jumps from $\mathbb{m}' \in \{0,1\}$ in period $t-1$ to $\mathbb{m} \in \{0,1\}$ in period t is denoted by

$$\check{v}(t, \mathbb{m}, \mathbb{m}'\,;\, \mathfrak{r}\langle t-1\rangle, P) \equiv \Pr(\mathbb{M}_t = \mathbb{m} \mid \mathbb{M}_{t-1} = \mathbb{m}'\,;\, \mathfrak{r}\langle t-1\rangle, P).$$

The serial dependence in the Markov chain reflects the possibility that the consumer's disposable income in the current period may depend on what she had in the previous period; the strength of this dependence may vary over time, which is reflected here by the non-stationarity of transition probabilities.

The transition probabilities may also depend on the history $\mathfrak{r}\langle t-1\rangle$ of the consumer's recharge decisions until time $t-1$ because every recharge is funded out of disposable income, and hence the subsequent probabilities of transition are affected by that recharge. We assume that the Markov chain renews after every recharge; this simplifies the relationship between $\mathfrak{r}\langle t-1\rangle$ and $\check{v}(t,)$ as shown below:

$$\check{v}(t, \mathbb{m}, \mathbb{m}'\,;\, \mathfrak{r}\langle t-1\rangle, P) = \check{v}(t, \mathbb{m}, \mathbb{m}'\,;\, l_t, P) \equiv v(t - l_t, \mathbb{m}, \mathbb{m}'\,;\, P), \tag{2}$$

where l_t is defined as the latest time period before t in which the lamp was recharged. Under the renewals assumption, l_t is the only piece of information from $\mathfrak{r}\langle t-1\rangle$ that affects the transition probabilities. Moreover, the transition probabilities at any time t depend only on the relative time period $\tau_t = t - l_t$, which is period t relative to the last recharge period. In (2), the transition probability function $\check{v}$ is defined on the absolute time period t, whereas the transition probability function v is defined on the relative time period τ.

The renewals assumption can be interpreted as the setting where the consumer spends all her disposable income when the recharge is done. We believe that this assumption is not unrealistic because the consumer lives in poverty, and so her disposable income for the lamp recharge would never significantly exceed the recharge price. Such renewals in disposable income are also consistent with the mental accounting model (Thaler 1985, 1999) of managing income, as discussed in Uppari et al. (2019). Accordingly, we assume that the Markov process starts from $\mathbb{M}_0 = 0$ at the outset and after every recharge.

Finally, with a slight abuse of notation, we denote by v_q the probability that the consumer has sufficient money for a recharge q periods after the recent lamp recharge, for $q \in \{1, 2, \ldots\}$. From the renewals assumption, it follows that

$$v_q(P) = \Pr(\mathbb{M}_q = 1 \mid \mathbb{M}_0 = 0\,;\, P).$$

The expression for v_q can be computed from the transition probabilities of $\mathbb{M}_t$.

Model of $\mathbb{D}_t$. We model $\mathbb{D}_t$ also as a Markov process. In period t, $\mathbb{D}_t = 0$ indicates that the lamp is not yet discharged, whereas $\mathbb{D}_t = 1$ indicates that it is discharged. Given the recharge history $\mathfrak{r}\langle t-1\rangle$, the probability of a jump from $\mathbb{d}' \in \{0,1\}$ in period $t-1$ to $\mathbb{d} \in \{0,1\}$ in period t is denoted by $\check{u}(t, \mathbb{d}, \mathbb{d}'\,;\, \mathfrak{r}\langle t-1\rangle, Q)$. As earlier, the serial dependence in the Markov chain and the non-stationarity of transition probabilities reflect the possibility that the lamp's discharge status in period t may depend on the status in period $t-1$.

Because the charge in the lamp is reset to Q hours after every recharge, the Markov chain of $\mathbb{D}_t$ renews after every recharge. Consequently, the Markov process starts from $\mathbb{D}_0 = 0$ at the outset and after every recharge. Moreover,

$$\check{u}(t, \mathbb{d}, \mathbb{d}'\,;\, \mathfrak{r}\langle t-1\rangle, Q) \equiv \Pr(\mathbb{D}_t = \mathbb{d} \mid \mathbb{D}_{t-1} = \mathbb{d}'\,;\, \mathfrak{r}\langle t-1\rangle, Q) = \check{u}(t, \mathbb{d}, \mathbb{d}'\,;\, l_t, Q) \equiv u(t - l_t, \mathbb{d}, \mathbb{d}'\,;\, Q).$$

Once the lamp is discharged, it remains in that state until it is recharged again. Therefore, we have $u(\tau,1,1\,;Q)=1$ for any given relative time period τ.

Let $\mathcal{Q}$ be the set containing the possible number of periods the lamp could last. Then, for $q\in\mathcal{Q}$, the probability that the lamp lasts exactly q periods after the recent lamp recharge (denoted as u_q) is given by

$$u_q(Q)=\Pr(\mathbb{D}_1=0,\ldots,\mathbb{D}_{q-1}=0,\mathbb{D}_q=1\,|\,\mathbb{D}_0=0\,;Q)=\prod_{\tau=1}^{q-1}u(\tau,0,0\,;Q)\times u(q,1,0\,;Q).$$

Model of $\mathbb{S}_t$. The consumer makes her recharge decisions by dynamically trading off her cost associated with recharging the lamp against the cost of not recharging it. On one hand, if the consumer chooses to recharge in period t, then she has the lamp's light available for the next few periods but she incurs an inconvenience cost of recharging the lamp in period t. We denote this cost by αI, where I is the distance between the consumer's household and the recharge center and the coefficient α encapsulates both the physical and psychological costs associated with traveling to the recharge center (α converts distance into RWF).

On the other hand, if the consumer chooses not to recharge her lamp in period t, then she incurs a blackout cost in that period. The blackout cost arises either from having no light at all or from switching to (inferior) alternative sources such as candles, flashlights or firewood. It is possible that this cost could fluctuate across periods because of any inherent variation in consumer's preferences and availability of alternative lighting sources. Therefore, we model the blackout cost in period t as a random variable $\tilde{\beta}_t$ (in RWF) and assume that $\tilde{\beta}_t$ is independently and identically distributed (i.i.d.) over time, with a finite mean β and a cumulative distribution function (CDF) F that is continuous and differentiable.[9]

We assume that the consumer minimizes her total cost from period 1 to T. The state space for the consumer's DP constitutes (i) the current time period t, (ii) the blackout cost $\tilde{\beta}_t=b$ realized in period t, (iii) the indicator $\mathbb{M}_t=\mathbb{m}$ indicating whether or not the consumer has sufficient money for the recharge in period t, and (iv) the last period l in which the lamp was recharged. The Bellman equation for cost $C(t,b,\mathbb{m},l)$ when $\mathbb{m}=1$ is as follows:

$$C(t,b,1,l)=\min\Big\{\overbrace{\alpha I}^{\text{inconvenience cost}}+\sum_{q\in\mathcal{Q}}u_q\big[\overbrace{v_q\bar{C}(t+q,1,t)}^{\text{enough money after } q \text{ periods}}+\overbrace{(1-v_q)\bar{C}(t+q,0,t)}^{\text{not enough money after } q \text{ periods}}\big],$$
$$\underbrace{b}_{\text{blackout cost}}+\underbrace{v(t-l+1,1,1)\bar{C}(t+1,1,l)}_{\text{enough money in the next period}}+\underbrace{v(t-l+1,0,1)\bar{C}(t+1,0,l)}_{\text{not enough money in the next period}}\Big\},\tag{3}$$

where $\bar{C}(t,\mathbb{m},l)=\mathbb{E}C(t,\tilde{\beta},\mathbb{m},l)$. The expectation is taken with respect to the distribution of $\tilde{\beta}_t$. (Here we suppressed the argument P in v and v_q and argument Q in u_q.)

The first term in the braces in (3) corresponds to the decision $r_t=1$, whereas the second one to $r_t=0$. If the consumer chooses to recharge, then she incurs the inconvenience cost αI and jumps (say) q periods ahead without experiencing any cost in those periods. The lamp is again discharged after those q periods,

[9] Similarly to blackout cost, consumer's inconvenience cost may also vary over time; call it $\tilde{\iota}_t$. It will be evident from the analysis that follows that the decision about whether to recharge or not is a function of $\tilde{\iota}_t-\tilde{\beta}_t$, not $\tilde{\iota}_t$ and $\tilde{\beta}_t$ separately. If we assume that $\tilde{\iota}_t=\alpha I+\tilde{\xi}_t^{(1)}$ and $\tilde{\beta}_t=\beta+\tilde{\xi}_t^{(2)}$, where $\tilde{\xi}_t^{(1)}$ and $\tilde{\xi}_t^{(2)}$ are zero-mean random variables, then $\tilde{\iota}_t-\tilde{\beta}_t=\alpha I-\beta-(\tilde{\xi}_t^{(2)}-\tilde{\xi}_t^{(1)})$. Therefore, using the data on recharge decisions, we cannot separately identify $\tilde{\xi}_t^{(1)}$ and $\tilde{\xi}_t^{(2)}$, and so we attribute time-varying nature of the inconvenience–blackout trade-off only to blackout cost.

and the consumer additionally experiences the expected cost of either $\bar{C}(t+q,1,t)$ (with probability v_q) or $\bar{C}(t+q,0,t)$ (with probability $1-v_q$). The last recharge period is set to t if the lamp is recharged. The exact number of periods that the lamp lasts (q) is uncertain at the point of recharge, so the consumer takes an expectation over its possible realizations (the realization q happens with probability u_q).

Instead, as shown by the second term in braces in (3), the consumer may opt not to recharge her lamp in period t and thereby incurs the blackout cost b and an additional expected cost of either $\bar{C}(t+1,1,l)$ or $\bar{C}(t+1,0,l)$. The former expected cost is incurred if the income process remains in state 1 in period $t+1$, which happens with probability $\check{v}(t+1,1,1\,;l) = v(t-l+1,1,1)$, whereas the latter expected cost is incurred if the income process jumps to state 0 in period $t+1$. The variable $\mathbb{S}_t$ in Figure 1 simply indicates whether the cost of recharging is lower than the cost of not recharging:

$$\mathbb{S}_t = \mathbb{1}\Big\{\alpha I + \sum_{q\in\mathcal{Q}} u_q\big[v_q\bar{C}(t+q,1,t) + (1-v_q)\bar{C}(t+q,0,t)\big] \\ - b - \sum_{\mathrm{m}\in\{0,1\}} v(t-l+1,\mathrm{m},1)\,\bar{C}(t+1,\mathrm{m},l) < 0\Big\}. \tag{4}$$

In the case when the consumer does not have sufficient money for the recharge (i.e., when $\mathrm{m}=0$), the Bellman equation is given by

$$C(t,b,0,l) = b + v(t-l+1,1,0)\bar{C}(t+1,1,l) + v(t-l+1,0,0)\bar{C}(t+1,0,l). \tag{5}$$

Here, the consumer experiences blackout cost b and the expected cost of $\bar{C}(t+1,\mathrm{m},l)$ depending on the realization of m in period $t+1$. Finally, we assume that the cost incurred after the terminal period T is zero, i.e., $C(T+n,b,\mathrm{m},l)=0$ for all $n\geq 1$ and all feasible (b,m,l).

Remark 2. Consistently with the structural assumptions A1–A3 discussed in Section 2.3, (i) of Q, P, and I, u is a function of only Q, and v is a function of only P; and (ii) I appears only in the expression for $\mathbb{S}_t$ and does not affect either u or v. Although it may seem that Q and P affect $\mathbb{S}_t$ in (4), they do so only through u and v respectively, and hence they do not *directly* influence $\mathbb{S}_t$.

Remark 3. One may question at this point why we do not incorporate the money-in-hand (call it m_t) directly into the decision model (i.e., use m_t, instead of $\mathbb{M}_t$, as a state variable in our DP). As we mentioned in Remark 1, it is difficult to observe the consumer's disposable income for the lamp's light at any point in time. Therefore, m_t will be an unobserved and serially correlated state variable in the DP. While estimating the model using recharge data, we need to account for all the possible paths the money process could have followed. Under a continuous state income process such as m_t, the estimation would involve T integrals per consumer, thereby imposing a huge computational burden (see Stinebrickner (2000) for the econometric issues associated with unobserved serially correlated state variables in dynamic programs).

However, if we discretize the income model, then it replaces the integrals with the summations over discrete states, which lowers the computational burden. Under a setting where no explicit information on disposable income is available, only the recharge timestamps are informative about the income process: the consumer has sufficient money (i.e., $\mathbb{M}_t = 1$) at the time of recharge. Therefore, if we discretize the process all the way down to a binary random variable ($\mathbb{M}_t$), we do not lose any information, but at the same time we also significantly reduce the computation time. For similar reasons, we do not model the charge remaining in the lamp; instead, we model only the corresponding indicator variable $\mathbb{D}_t$.

4.2. Structural Results

When the consumer's lamp is discharged and she has sufficient money for a recharge, she recharges her lamp if and only if $\mathbb{S}_t = 1$. It follows from (4) that this condition can be rewritten as $\tilde{\beta}_t > k_t$ for some threshold k_t. The following result shows how these *blackout-cost thresholds* can be computed. (All the proofs are in Appendix G.)

Proposition 1. *In a period $t \in \{1,\dots,T\}$ and for a last recharge period $l \in \{0,\dots,t-1\}$, the blackout-cost threshold $k(t,l)$ is found recursively as follows:*

$$\begin{aligned} k(t,l) &= \alpha I - \kappa(t+1,l,1) \\ &\quad + \mathbb{1}\{t+q \le T\} \sum_{q\in\mathcal{Q}} u_q \big[v_q \big(\mathbb{E}\min\{k(t+q,t),\tilde{\beta}\} + \kappa(t+q+1,t,1) \big) + (1-v_q)\big(\beta + \kappa(t+q+1,t,0)\big) \big], \end{aligned}$$

where, for feasible $(l,\mathbb{m})$, the function κ is given by $\kappa(t,l,\mathbb{m}) = 0$ for $t \ge T+1$, and for $1 \le t \le T$ it is

$$\kappa(t,l,\mathbb{m}) = v(t-l,1,\mathbb{m})\big[\mathbb{E}\min\{k(t,l),\tilde{\beta}\} + \kappa(t+1,l,1)\big] + v(t-l,0,\mathbb{m})\big[\beta + \kappa(t+1,l,0)\big].$$

In period t, since the consumer compares $\tilde{\beta}_t$ with $k(t,l)$ and recharges if the former is greater than the latter, we can interpret k as the effective cost (or a shadow cost) of recharging in period t, whereas $\tilde{\beta}_t$ is the cost of not recharging in that period. A lamp recharge in period t involves not only incurring an inconvenience cost of αI in that period, but also jumping few periods ahead without experiencing any cost in the interim periods; therefore, the effective cost k also accounts for the potential cost savings in those interim periods.

Using the threshold structure characterized in Propostion 1, we next write the probability of observing a recharge sequence under the decision model from the previous subsection. As in Section 2.1, we denote the recharge sequence that is a random variable as $\tilde{\mathfrak{r}}$ and the recharge sequence that is an instance of that random variable as $\mathfrak{r}$. Furthermore, we denote by (i) $\Theta = \{\alpha, u, v, F\}$ the set of all model parameters consisting of the inconvenience-sensitivity parameter and (with a slight abuse of notation) the probability functions' parameters; (ii) $\Gamma = (I,P,Q)$ the treatment condition of the consumer comprised of her inconvenience, recharge price, and lamp capacity; (iii) $\mathfrak{l} = (l_1,\dots,l_T)$ the sequence of observed last-recharge points, which can be computed using the recursion $l_t = (1-r_{t-1})l_{t-1} + r_{t-1}(t-1)$ for $t>1$ and $l_1 = 0$; and (iv) $\bar{F} = 1-F$. The following result forms the basis for writing the likelihood function for the recharge sequences observed in the data.

Proposition 2. *The probability of observing the recharge sequence $\mathfrak{r}$ is given by*

$$\Pr(\tilde{\mathfrak{r}} = \mathfrak{r}\,;\Theta,\Gamma,\mathfrak{l}) = \sum_{\mathbb{d}\in\{0,1\}} \sum_{\mathbb{m}\in\{0,1\}} \Omega(T,\mathbb{d},\mathbb{m}).$$

The function $\Omega(t,\mathbb{d},\mathbb{m})$, wherein $t\in\{1,\dots,T\}$, $\mathbb{d}\in\{0,1\}$ and $\mathbb{m}\in\{0,1\}$, can be computed recursively as

$$\begin{aligned} \Omega(t,\mathbb{d},\mathbb{m}) &= \Pr(\tilde{\mathfrak{r}}\langle t\rangle = \mathfrak{r}\langle t\rangle, \mathbb{D}_t = \mathbb{d}, \mathbb{M}_t = \mathbb{m}\,;\Theta,\Gamma,\mathfrak{l}) \\ &= \sum_{\mathbb{d}'\in\{0,1\}} \sum_{\mathbb{m}'\in\{0,1\}} \Omega(t-1,\mathbb{d}',\mathbb{m}') \times u(t-l_t,\mathbb{d},\mathbb{d}') \times v(t-l_t,\mathbb{m},\mathbb{m}') \\ &\qquad \times \big[\mathbb{d}\mathbb{m}\bar{F}(k(t,l_t))\big]^{r_t} \big[1-\mathbb{d}\mathbb{m}\bar{F}(k(t,l_t))\big]^{1-r_t} \quad \textit{for } 2\le t\le T, \textit{ and} \\ \Omega(1,\mathbb{d},\mathbb{m}) &= u(1,\mathbb{d},0)\times v(1,\mathbb{m},0)\times \big[\mathbb{d}\mathbb{m}\bar{F}(k(1,0))\big]^{r_1}\big[1-\mathbb{d}\mathbb{m}\bar{F}(k(1,0))\big]^{1-r_1}. \end{aligned}$$

It is noteworthy that both the thresholds in Proposition 1 and the probability of recharge in Proposition 2 can be expressed as recursive functions. This property is important in our estimation exercise because recursivity allows efficient computation in polynomial time using the *memoization* technique.[10]

To provide some empirical validity for our model, we examine the relationships, as predicted by our model, between the expected number of recharges and inconvenience, recharge price, and lamp capacity. For the sake of brevity, the details of that exercise are presented in Appendix B; here, we present only our main findings. Using a simpler version of the model that is amenable to formal analysis, we arrive at the following theoretical predictions:

(Π_1) The expected number of recharges decreases in inconvenience.

(Π_2) The expected number of recharges decreases in recharge price.

(Π_3) The expected number of recharges decreases in lamp capacity for relatively smaller values of inconvenience, and it is unimodal for relatively larger values of inconvenience.

Of the above, perhaps Π_3 is most surprising because one may intuit that as Q decreases, the expected number of recharges should increase because consumers get less light per recharge, and hence the time between successive recharges will decrease. However, the model predicts that this will be the case only for relatively smaller values of I. When the consumer's inconvenience of recharging is relatively high, a decrease in capacity results in a larger number of highly inconvenient trips to the recharge center, which in turn negatively affects the overall number of recharges, as stated in Π_3. (Indeed, this is the effect that we observe on average in the data as evident in Figure 3(d).) Through regression analysis in Appendix B, we find that the recharge data from the field supports Π_1–Π_3.

4.3. Empirical Models and Parameter Identification

For the purpose of both simulating and estimating the decision model, we use the empirical models described in this section. Without these models, we need to resort to nonparametric methods to estimate the probability functions $v(\tau, \mathbb{m}, \mathbb{m}'; P)$, $u(\tau, \mathbb{d}, \mathbb{d}'; Q)$, and $F(\cdot)$. That would result in too many parameters to be estimated and require too much variation in IATs (we would need to observe every possible value of IAT a significant number of times in each condition of (I, P, Q)). The absence of such massive variation in the IATs in our sample necessitates a parametric approach to modeling the aforementioned probability functions. This, of course, raises the issue of whether such restrictions are valid for the particular application at hand. We take a formal approach to validating our empirical models using out-of-sample testing in Section 4.4.

Empirical model of v. We impose a structure on the transition probabilities in (2) by explicitly modeling the underlying disposable income. We assume that the log of disposable income, denoted as m_t, follows an AR(1) process, i.e., $m_t = \rho m_{t-1} + \epsilon_t$; here, $\rho \in [0, 1)$ represents the strength of serial correlation in the AR(1) process, and the innovation $\epsilon_t \sim \mathrm{N}(\mu, \sigma^2)$ is an i.i.d. normal random variable.

[10] Memoization is a computational technique wherein computer programs are sped up by storing the results of expensive function calls and returning the cached results when the same inputs occur again. Without recursivity and memoization, computing the probability of a recharge sequence would require accounting for all the possible paths of (serially dependent) $\mathbb{D}_t$ and $\mathbb{M}_t$. Such path enumeration will result in an exponential time complexity.

Because the consumer starts to consider recharging the lamp only after she is given the lamp, we assume that this process starts afresh with initial state $m_0 = 0$. To reflect renewals, we assume that m_t is reset to zero after every recharge. Then, we have the following result, wherein Φ is the CDF of standard normal distribution and $\bar{\Phi} = 1 - \Phi$.

Lemma 1. *Let G_z represent the CDF of normal distribution* $\mathrm{N}\left(\frac{\mu(1-\rho^z)}{1-\rho}, \frac{\sigma^2(1-\rho^{2z})}{1-\rho^2}\right)$ *and $\bar{G}_z = 1 - G_z$. Under the above model of disposable income for the lamp's light, the following statements hold:*

(i) $v(1,1,\cdot\,;P) = \bar{G}_1(\log P\,;\mu,\sigma,\rho)$. *For relative time period $\tau > 1$,*

$$v(\tau,1,0\,;P) = \frac{1}{G_{\tau-1}(\log P\,;\mu,\sigma,\rho)} \int_{-\infty}^{\log P} \bar{\Phi}\left(\frac{\log P - \rho x - \mu}{\sigma}\right) dG_{\tau-1}(x\,;\mu,\sigma,\rho), \tag{6}$$

$$v(\tau,1,1\,;P) = \frac{1}{\bar{G}_{\tau-1}(\log P\,;\mu,\sigma,\rho)} \int_{\log P}^{\infty} \bar{\Phi}\left(\frac{\log P - \rho x - \mu}{\sigma}\right) dG_{\tau-1}(x\,;\mu,\sigma,\rho). \tag{7}$$

(ii) $v_q(P) = \bar{G}_q(\log P\,;\mu,\sigma,\rho)$.

Empirical model of *u*. We build the model by imposing a structure on the number of periods $\tilde{N}$ that the lamp lasts after a recharge. We assume that $\tilde{N} - 1$ is distributed as Poisson($Q\lambda$), such that $\tilde{N} \geq 1$ always. λ is the fraction of a period that is served, on average, by one hour of the lamp's light (or alternatively, assuming a sufficiently small λ, $Q\lambda$ is the probability that the lamp lasts longer than a period). As expected, the lamp lasts longer with higher Q and higher λ. Moreover, since $u(\tau,1,0\,;Q)$ is the probability that the lamp discharges in the τ^{th} period, given that it has not discharged in the $(\tau-1)^{\text{th}}$ period, it is simply the hazard rate of the random variable $\tilde{N}$. The following lemma formalizes these statements.

Lemma 2. *Let H represent the probability mass function of* Poisson($Q\lambda$). *Under the above model of consumption of the lamp's light, the following statements hold:*

(i) $u_q(Q) = H(q-1\,;Q\lambda)$.

(ii) $u(1,1,\cdot\,;Q) = H(0\,;Q\lambda)$. *For relative time period $\tau > 1$,*

$$u(\tau,1,0\,;Q) = \frac{H(\tau-1\,;Q\lambda)}{1 - \sum_{n=1}^{\tau-1} H(s-1\,;Q\lambda)} \quad \textit{and} \quad u(\tau,1,1\,;Q) = 1.$$

Empirical model of *F*. In our setting, $\tilde{\beta}_t$ is the disutility that the consumer experiences when she does not have lamp's light. In other words, it captures the valuation that the consumer places on the lamp's light *relative* to the valuation of her alternative options such as relying on kerosene and candles or simply not using light at all (and thereby experiencing a blackout). Because we do not model the consumption of and preferences for alternative lighting solutions, we model $\tilde{\beta}_t$ over the real line such that it can take both positive values (e.g., when the consumer strongly values the lamp's light) and negative values (e.g., when the consumer has a stock of alternative sources). We assume that $\tilde{\beta}_t = \beta + \tilde{\xi}_t$ where $\tilde{\xi}_t \sim \mathrm{N}(0, \sigma_\xi^2)$ and that F is the CDF of $\tilde{\beta}_t$.

Identification of parameters. With the empirical models just discussed, $(\alpha, \beta, \sigma_\xi)$ are the parameters of $\mathbb{S}_t$, (μ, σ, ρ) are the parameters of $\mathbb{M}_t$, and λ is the parameter of $\mathbb{D}_t$. The set of all the parameters, which we denoted as Θ in Section 4.2, reduces to $\Theta = \{\alpha, \beta, \sigma_\xi, \mu, \sigma, \rho, \lambda\}$. We now write the likelihood function, which we later maximize to estimate Θ. We use the index set $\mathfrak{V}$ for villages and the index set function $\mathfrak{J}(v)$ for the consumers in village $v \in \mathfrak{V}$. For a consumer $j \in \mathfrak{J}(v)$ in village $v \in \mathfrak{V}$, we denote by (i) $\mathfrak{r}_{jv}$ the sequence of observed recharge decisions, (ii) Γ_{jv} the treatment condition, and (iii) $\mathfrak{l}_{jv}$ the sequence of observed last recharge points. The collections of (i)–(iii) for the entire consumer pool are written simply as $\{\mathfrak{r}_{jv}\}$, $\{\Gamma_{jv}\}$, and $\{\mathfrak{l}_{jv}\}$ respectively. The likelihood function for the consumer pool in village v is then given by

$$L(\Theta\,;\, v, \mathfrak{J}(v), \{\mathfrak{r}_{jv}\}, \{\Gamma_{jv}\}, \{\mathfrak{l}_{jv}\}) = \prod_{j \in \mathfrak{J}(v)} \Pr\left(\tilde{\mathfrak{r}} = \mathfrak{r}_{jv}\,;\, \Theta, \Gamma_{jv}, \mathfrak{l}_{jv}\right) \quad \text{for } v \in \mathfrak{V}, \tag{8}$$

where $\Pr(\tilde{\mathfrak{r}})$ is given by Proposition 2. The complex nature of the likelihood function in (8) makes intractable the mathematical analysis of the identifiability of model parameters (e.g., showing that L is unimodal in Θ). Therefore, we resort to intuitive arguments on the identification of parameters, which parallel the arguments made in Section 2.3. A detailed discussion of which sources of variation in the data identify which parameters is in Appendix C; here, we present a summary of that discussion.

Broadly, the variation in IATs due to variation in I (resp., P and Q) helps identify $\mathbb{S}_t$ (resp., $\mathbb{M}_t$ and $\mathbb{D}_t$). If we assume that λ is known, then we can control for consumption time, and the residual variation in IATs is due to the variation in blackout times. For consumers facing the zero-price condition, the BTs consist of only the strategic component. The variation in IATs across those consumers (with varying inconvenience levels) identifies α, whereas the variation in IATs within consumers identifies β. As is common in the discrete choice literature, we cannot identify the variance of the error term, and so we normalize σ_ξ to one. With these parameters identified, we can control for the strategic component of BT. The residual variation in IATs across individuals with varying price levels can be attributed to the liquidity component of BT, and that variation identifies μ and σ. We cannot identify ρ because of the assumed renewal structure for the disposable-income process. We treat ρ as a *hyperparameter*, i.e., we set it exogenously and then examine the sensitivity of other parameter estimates by varying ρ. All the aforesaid parameters are identified as a function of λ, and they control for blackout times. The remaining variation in IATs across individuals with varying capacity levels is attributed to the variation in CTs, which thereby identifies λ.

4.4. Model Estimation and Predictive Ability

We now estimate the empirical models described in the previous subsection using our data. We also estimate some alternative specifications that extend those models and choose the specification that best fits the data. First, we discuss our model estimation procedure and goodness-of-fit criteria and then give the details of the candidate specifications.

All the model specifications that we investigate are estimated at the village level, thereby incorporating heterogeneity across villages. For each village $v \in \mathfrak{V}$, we split $\mathfrak{J}(v)$ (exclusively and exhaustively) into three sets: (i) *training set* $\mathfrak{J}^{tr}(v)$, which constitutes the data from the experimental conditions $(P, Q) \in \{\{0, 50, 60, 70, 80, 120\} \times \{18\}\} \cup \{\{80, 100\} \times \{14\}\}$; (ii) *cross-validation set* $\mathfrak{J}^{cv}(v)$, consisting of the data from consumers facing $(P,Q)=(100,18)$, which is the current business model; and (iii) *test set* $\mathfrak{J}^{ts}(v)$, consisting of the data from our experimental condition wherein every fourth recharge was offered free.

Our parameter estimation procedure is as follows. Because ρ is a hyperparameter, we fix it exogenously and estimate the remaining parameters (denoted by set ϑ) using training set: $\hat{\vartheta}(\rho,v)=\arg\max_{\vartheta} L(\vartheta\cup\{\rho\}\,;v,\mathfrak{J}^{tr}(v),\cdot)$. We vary ρ from 0 to 0.9 in steps of 0.1 and select the value of ρ that maximizes the likelihood on cross-validation set: $\hat{\rho}(v)=\arg\max_{\rho} L(\hat{\vartheta}(\rho,v)\cup\{\rho\}\,;v,\mathfrak{J}^{cv}(v),\cdot)$. Overall, $\hat{\Theta}(v)=\hat{\vartheta}(\hat{\rho}(v),v)\cup\{\hat{\rho}(v)\}$.

As we are interested in the models' ability to predict counterfactual policies, we use the test set as the experimental condition wherein the consumer's decision process is *structurally different* from the decision process in the experimental conditions of training and cross-validation sets. While the decision process in the latter case is the same as in Figure 1, in the former case, the price faced by the consumer dynamically varies depending on her recharge decisions, and she needs to keep track of an additional state variable, which is the number of recharges done so far after the previous free recharge was availed. (The corresponding decision process and the Bellman equations are given in Appendix F). Accordingly, we use as our goodness-of-fit criterion the mean absolute percentage error (MAPE) of the *predicted* number of recharges with respect to the *actual* number of recharges on the test set.

We resort to simulations to obtain the predicted number of recharges. In village v, we generate the recharge sequence $\hat{\mathfrak{r}}_{jv,n}$ for consumer j in simulation round n using (i) the decision process in Figure 1 for $j\in\mathfrak{J}^{tr}(v)\cup\mathfrak{J}^{cv}(v)$ and the decision process in Figure 11(a) for $j\in\mathfrak{J}^{ts}(v)$; (ii) the treatment condition of that consumer Γ_{jv}; and (iii) the probability models of $\mathbb{D}_t$, $\mathbb{M}_t$, and $\mathbb{S}_t$ along with their estimated parameters from $\hat{\Theta}(v)$. The sum of the recharge decisions in $\hat{\mathfrak{r}}_{jv,n}$ is denoted by $\hat{R}_{jv,n}$. Furthermore, the sums of recharges aggregated at the village level and the entire-sample level are, respectively, given by $\hat{R}^{\mathfrak{s}}_{v,n}=\sum_{j\in\mathfrak{J}^{\mathfrak{s}}(v)}\hat{R}^{\mathfrak{s}}_{jv,n}$ and $\hat{R}^{\mathfrak{s}}_{n}=\sum_{v\in\mathfrak{V}}\hat{R}^{\mathfrak{s}}_{v,n}$ for $\mathfrak{s}\in\{tr,cv,ts\}$. Similarly, the actual sums of recharges observed in the sample are represented by R_{jv}, $R^{\mathfrak{s}}_{v}$, and $R^{\mathfrak{s}}$. We define our MAPE measure at the village level as follows:

$$\text{MAPE}^{\mathfrak{s}}_{n}=\frac{1}{|\mathfrak{V}|}\sum_{v\in\mathfrak{V}}\left|\frac{R^{\mathfrak{s}}_{v}-\hat{R}^{\mathfrak{s}}_{v,n}}{R^{\mathfrak{s}}_{v}}\right|\quad\text{for }\mathfrak{s}\in\{tr,cv,ts\}.$$

Table 2 reports the averages of $\text{MAPE}^{\mathfrak{s}}_{n}$ and $\hat{R}^{\mathfrak{s}}_{n}$ across N_s simulation rounds for all the model specifications that we estimate. We use $N_s=100$ in all the simulations in our paper.

In Table 2, specification S_1 is exactly the one described in Sections 4.1–4.3. S_1 fits the data in the training and cross-validation sets reasonably well. The out-of-sample MAPE on the test set is (on average) 10.8%. S_1, however, assumes that the consumers are forward-looking with a discount factor of $\delta=1$, which may be unreasonable. To understand the relevance of this assumption, we next investigate the performance under a specification that lies on the other extreme. S_2 assumes that the consumers are myopic with $\delta=0$. The fit of S_2 to training and cross-validation datasets is almost indistinguishable from that of S_1. However, the limitation of assuming myopia is evident from the poor performance of S_2 on the test set. Because every fourth recharge is free in the test set, the current recharge decision of consumers has an impact on their future recharge price, and hence in reality, consumers plausibly account for the future costs while making the recharge decisions in such setting. Since S_2 rules out forward-looking behavior, it poorly predicts recharge decisions in the test set.

		Training set		Cross-validation set		Test set	
		Actual = 4640		Actual = 429		Actual = 560	
		Pred.	MAPE	Pred.	MAPE	Pred.	MAPE
Structural	S_1	4627.66	0.9%	452.54	5.2%	500.02	10.8%
		(50.58)	(0.7%)	(22.31)	(3.1%)	(21.38)	(3.8%)
	S_2	4556.92	1.8%	452.89	6.0%	472.03	15.7%
		(54.21)	(1.1%)	(19.65)	(4.0%)	(21.27)	(3.8%)
	S_3	4619.21	0.9%	444.99	4.1%	511.35	8.8%
		(48.72)	(0.7%)	(11.01)	(1.9%)	(15.59)	(2.1%)
	S_4	4669.92	0.7%	446.82	4.1%	515.73	7.5%
		(25.21)	(0.4%)	(7.55)	(1.8%)	(7.94)	(1.4%)
Atheoretical	A_1	4600.58	1.2%	423.89	3.5%	451.29	19.4%
		(59.12)	(0.9%)	(18.84)	(2.8%)	(22.91)	(4.1%)
	A_2	4601.32	1.2%	428.58	3.4%	459.41	18.0%
		(55.03)	(0.8%)	(18.42)	(2.6%)	(23.37)	(4.2%)

Table 2 Goodness of fit of various model specifications.

It is possible that the consumers are neither completely forward looking (as in S_1) nor completely myopic (as in S_2); their discount factor δ could be between 0 and 1. Specification S_3 incorporates discounting in the Bellman equations of our DP model in Section 4.1. We cannot estimate δ using the variations in our data (this is a common problem in several DP-based structural models; see e.g., Rust 1994), and so we take δ as a hyperparameter and estimate it along with ρ using cross validation. The predictive ability of S_3 is better than that of both S_1 and S_2, thereby suggesting that our consumers are partially forward looking.

In addition to village-level heterogeneity, there could be heterogeneity within a village across consumers. We incorporate this by assuming that the blackout cost is heterogenous (and model it as a random variable – i.e., a random effect); we call the corresponding specification S_4. Instead of estimating parameter β, we assume that β for a consumer is drawn from the distribution Normal($\mu_\beta, \sigma_\beta^2$), and we estimate the parameters μ_β and σ_β. S_4 improves further upon S_3 and has an out-of-sample MAPE of 7.5%. We estimate a few other specifications with heterogeneity incorporated in income and consumption processes as well; however, they do not improve upon the performance displayed by S_4 and are also computationally intensive (because of more random effect terms), and hence we do not consider them further.

All the above specifications are different versions of a basic theoretical structure that we impose on the data. We now compare the performance of our structural approach with some atheoretical approaches to see if there are any benefits to assuming the decision-making structure. (The terminology of structural vs. atheoretical is borrowed from Keane 2010.) Model A_1 is a Poisson regression model, similar to the one that we use in our reduced-form analysis in Appendix B, except that we estimate it at the village level (instead of using village-level fixed effects). This model assumes that the recharges observed in the experimental duration are the realizations of a Poisson process, in contrast to being the realizations of the controlled decision processes in Figure 1 or Figure 11(a). The arrival rate of the Poisson process is estimated as a function of I, P, and Q using the training data. This model fits the training data well, and its performance on the cross-validation set is slightly better than that of our structural models.

However, atheoretical models such as A_1 are usually limited in terms of generating counterfactual settings. For example, it is not trivial to compute the performance of A_1 on the test set where every fourth recharge is free. One plausible way to model this counterfactual is by assuming that the recharges observed in the test set are the realizations of a non-homogenous Poisson process whose instantaneous arrival rate depends on the number of recharges done so far after the previous free recharge. As we see in Table 2, this model performs much worse on the test set than our structural models.[11]

Yet another atheoretical approach is to assume that consumers behave in a completely random manner (and make decisions in each period perhaps by simply tossing a coin). This model is called A_2. We determine the probability of recharge in a period as a function of I, P, and Q using the logistic regression on the training data. Similar to A_1, this model also fits the training and cross-validation sets well but performs badly on the test set. We believe that this is because the atheoretical approaches completely ignore the decision-making process of the consumers and merely model it as a stochastic process, which thereby limits their performance in counterfactual settings wherein the decision-making aspects play an important role.

To summarize, in Table 2, the model with the best predictive ability is the one which (i) is estimated at the village-level, (ii) incorporates discounting, and (iii) accounts for blackout cost heterogeneity. This model has an out-of-sample MAPE of 7.5%, indicating good predictive ability. It also performs better than the atheoretical approaches, thereby suggesting that the assumed structure is both useful and powerful. In the interest of brevity, we present the parameter estimates $\hat{\Theta}(v)$ along with their interpretation under specification S_4 in Appendix D and the elasticity of the expected number of recharges with respect to variables I, P, and Q in Appendix E.

5. Counterfactual Analysis

We broadly classify the counterfactual policies that we study here based on the factor(s) that they address: (i) inconvenience-based, (ii) liquidity-based, and (iii) price-and-capacity-based. We distinguish between liquidity-based policies and price-based policies based on whether they affect the recharge price: the latter directly modify the distribution of $\mathbb{M}_t$ by varying the price, whereas the former do not vary the price but create an *option* to recharge even when $\mathbb{M}_t = 0$.

For any given counterfactual policy $\mathcal{P}$, we are interested in the expected sum of recharges across all villages $R(\mathcal{P}) = \sum_{v\in\mathfrak{V}}\sum_{j\in\mathfrak{J}(v)}\mathbb{E}\tilde{R}_{jv}(\mathcal{P})$ resulting under that policy and in the corresponding expected revenue $V(\mathcal{P}) = P \times R(\mathcal{P})$. As in Section 4.4, we approximate the expectations with sample averages of recharge decisions in the simulations: $\mathbb{E}\tilde{R}_{jv} \approx \hat{R}_{jv,n}/N_s$. We generate $\hat{R}_{jv,n}$ using (i) the decision process corresponding to the counterfactual policy $\mathcal{P}$, (ii) the condition $(I,P,Q)_{jv}$ as determined by the policy $\mathcal{P}$, and (iii) the probability models under specification S_4 with the estimated parameters $\hat{\Theta}(v)$.

Throughout this section, we assume that the current business model – the decision process in Figure 1 and $(I,P,Q)_{jv} = (I_{jv}, 100\text{ RWF}, 18\text{ hours})$ – is the base case to which the performance of other counterfactuals is

[11] Under model A_1, the Poisson regression estimates the aggregate arrival rate of recharges $\Lambda(I,P,Q)$ over a duration of T periods as a function of I, P, and Q using the data from the training set. The per-period arrival rate, or approximately the probability of recharge in a period, is given by Λ/T. To simulate recharges for a consumer j in test set, we model the probability of recharge as $\Lambda(I_{jv},P_{jv},Q_{jv})/T$ in the periods where the upcoming recharge is not free and as $\Lambda(I_{jv},0,Q_{jv})/T$ in the periods where the upcoming recharge is free. In contrast, model A_2 directly estimates the probability of recharge as a function of I, P, and Q using logistic regression on the period-level data from the training set. The logic for simulating test-set recharges is same as that of A_1.

compared. It should be noted that this base case is also a counterfactual policy because not every consumer in our sample was subjected to the aforementioned price-and-capacity condition. We find that the expected number of recharges in this base case is 4540 (32). (This number is unsurprisingly lower than the actual number of recharges (5577) observed in the sample because our experiments included many consumers who faced price values lower than 100 RWF and therefore recharged more often.) Hereafter, we measure the impact of alternative policies in terms of the resultant (i) percentage increase in the number of recharges and (ii) percentage increase in the firm's revenue, when compared to the base case; both (i) and (ii) are equivalent if the recharge price remains unchanged.

5.1. Inconvenience-based Counterfactual Policies

Start more recharge centers. Figure 4 plots the impact of reducing inconvenience in various counterfactual settings. As a benchmark, we first examine the case wherein we set $I = 0$ for all the consumers. We see in Figure 4 that the expected number of recharges increases by 60% under this benchmark case over the base case. This reaffirms our observation from previous sections that recharge inconvenience is a significant contributor to the inefficiency in this business model, leading to low recharge rates.

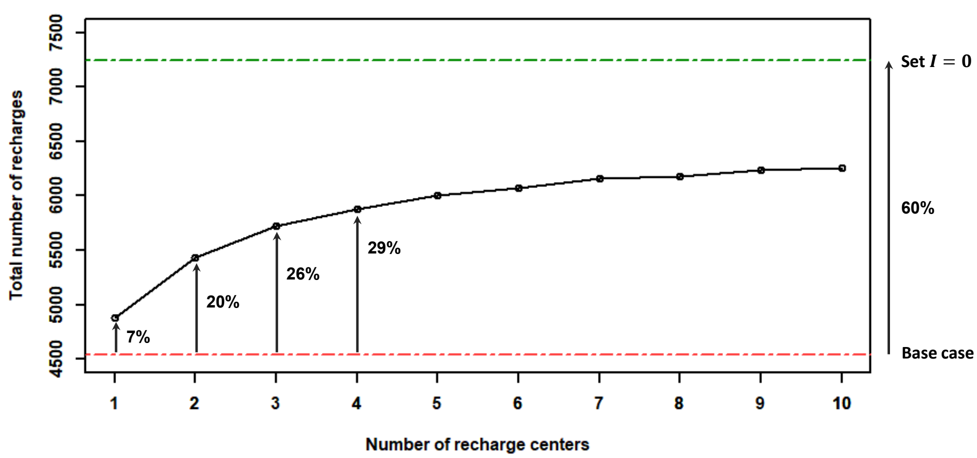

Figure 4 Impact of increasing the number of recharge centers on total expected number of recharges.

One way to reduce consumer inconvenience is by opening more recharge centers per village. Figure 4 shows the expected number of recharges under settings with different number of (optimally-located) recharge centers. (In simulations, we cluster the households in a village through a k-means clustering algorithm and then measure the impact of locating the recharge centers at the centroids of those clusters). We find that moving the recharge center to a more central location in respective villages improves the expected recharges by 7%. Moreover, by establishing 1–3 more (optimally-located) recharge centers, the firm can improve the number of recharges by 20–29%. Although there are gains to be made beyond adding three additional recharge centers, the marginal value that they bring decreases.[12]

[12] The standard errors around the estimated mean number of recharges under *all* the counterfactual policies discussed in this and subsequent sections are between 25 and 40 recharges. Therefore, the reported improvements in recharges are highly statistically significant.

There are two main practical insights from this analysis: First, there are significant gains to be made by moving the recharge center to an *inconvenience-minimizing location*. In reality, this location may or may not be the center of the village. It could also be a school or a retail store or any other place that is frequented by consumers. Because we do not have information on visit patterns to different locations in villages, we cannot measure the potential impact of moving the recharge center to one of those alternative locations. However, one would choose to move to these locations instead of the village center if they are more inconvenience-minimizing than the latter; hence, the above estimate can be thought of as a lower bound on the resulting improvement in recharges.

Second, it may be trivial that adding more recharge centers reduces consumer inconvenience and increases the number of recharges, but it is often not clear how many additional recharge centers are required to bring a significant improvement in recharges. The above analysis shows that just two or three additional recharge centers are sufficient to capture half the benefits of reducing inconvenience to zero. It is possible that as we increase the number of recharge centers, VLEs at those centers may be less incentivized (or may even drop out) because of their reduced market share. Then, instead of adding the recharge centers, the firm could add some "drop-off points" in a village, where consumers could safely drop their lamps and collect them later after they are recharged.

Make periodic visits to consumers. An alternative way of reducing consumer inconvenience is by delivering a door-to-door recharge service: instead of the consumer traveling to the recharge center, the VLE or a representative of the firm can travel to the consumer. This service can also be implemented in collaboration with firms that employ agents who visit consumers frequently. For example, the agents hired by Living Goods go door-to-door to sell health products, the agents of Vision Spring conduct regular eye tests and sell eyeglasses, and Shakti agents sell fast-moving consumer goods (FMCG) products by Unilever. Such firms sometimes tend to act as distribution platforms in the BoP markets, and their services can be leveraged if feasible.

Here, we consider a simpler version wherein the firm's VLE visits the households in her village once every n periods. If $n = 1$, then the VLE visits every period, whereas if $n = 2$ (resp., $n = 5$), then the VLE visits (approximately) once a week (resp., once every two weeks). On the days of the VLE's visit, consumers experience zero inconvenience as they can hand over the lamp to the VLE for the recharge. On such days, since the consumer must decide whether or not to give the (plausibly not-yet-discharged) lamp to the VLE, the consumer's decision process under this business model will be different from the one in Figure 1. Figure 5(a) shows the consumer's decision process under the *periodic-visit model* just described.

The indicator variable $\mathbb{V}_t$ in Figure 5(a) is equal to one in period t if the VLE visits the consumer in that period. $\mathbb{V}_t$ evolves deterministically: it is equal to one once every n periods, and it is zero in the remaining periods. As expected, the decision process coincides with that shown in Figure 1 when $\mathbb{V}_t$ is zero. When $\mathbb{V}_t$ is one, the decision process branches at $\mathbb{D}_t$. Even when the lamp is not completely discharged (i.e., $\mathbb{D}_t = 0$), the consumer may decide to recharge the lamp because she can give the lamp to the VLE and thereby experience zero inconvenience. Otherwise, the consumer herself needs to visit the recharge center in a later period after the lamp discharges. In Figure 5(a), $r_t^{(v)}$ indicates the recharges where the lamps were handed over to VLE.

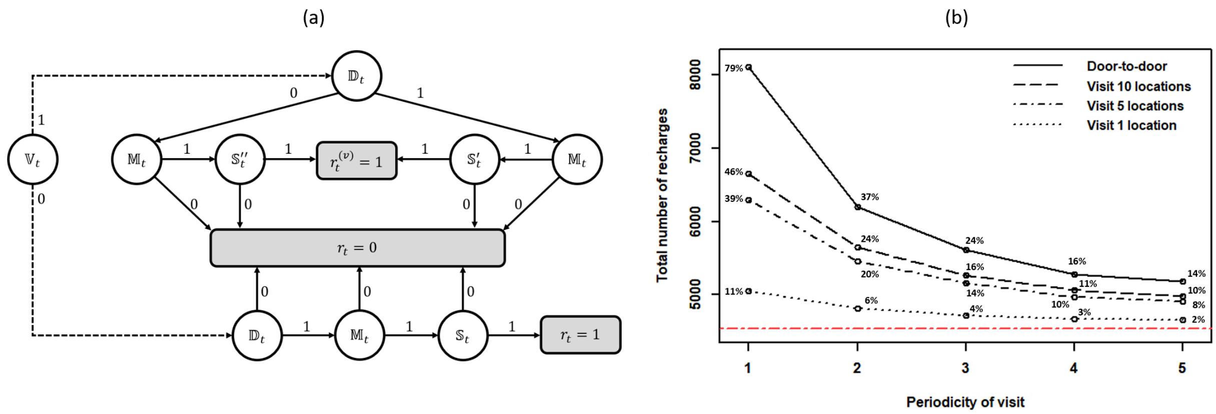

Figure 5 Periodic-visit model: (a) decision process, and (b) performance as a function of the periodicity of VLE's visits when the visits are door-to-door, or to 1, 5, and 10 locations per village.

Since the trade-offs and the future costs (and hence the Bellman equations) differ across the alternative branches of the decision process, we represent $\mathbb{S}_t$ separately for each branch. For brevity, we present the Bellman equations for this decision process in Appendix F.

Using the decision process in Figure 5(a) as the data-generating process, we simulate the recharge decisions for visit frequencies $n \in \{1, \ldots, 5\}$. The corresponding expected numbers of recharges are plotted in Figure 5(b). It is noteworthy that for $n = 1$, wherein the consumer experiences zero inconvenience in all periods, we observe a 79% increase in recharges, which is higher than the improvement achieved by setting $I = 0$ in Figure 4. This is because the consumer decision process differs across these two cases: under the latter case, only the discharged lamps arrive for recharge, whereas under the periodic-visit model, some partially-discharged lamps are recharged too, thereby resulting in a higher number of recharges. As n increases, the improvements in recharges decline steeply. If the VLE visits once per week, the recharges increase by 37%, whereas if she visits once every two weeks, we observe a 14% increase in recharges.

The periodic-visit model considered so far is a door-to-door service requiring the VLE to visit each household in the village. Such door-to-door visits may be costly for a VLE, and hence in practice, she may choose to periodically visit only some select locations in the village (at some pre-decided and publicly known time), and the consumers must visit those locations to hand over the lamps to VLE and then to collect them back later. Figure 5(b) also shows the improvement in recharges when the VLE visits one, five, and ten (optimal) locations in the village with varying frequencies. We do not see much improvement when the visits are to only one location, and the benefits marginally decrease as the number of visit locations increases, as there is little difference between visiting five versus ten locations. However, it is remarkable that just by visiting five locations per village, half the benefits from visiting door-to-door are captured (for all n) even though the latter option requires visiting ∼90 households per village.

5.2. Liquidity-based Counterfactual Policies

The current business model imposes two constraints that critically interact with consumers' liquidity constraints. First, the consumer is required to always *fully* recharge her lamp and accordingly pay full-recharge price. Sometimes, the consumer may have a strong need for light but may not have sufficient money to fully recharge the lamp. She could use her money to partially recharge the lamp and use that light to satisfy her need; instead, she is compelled to experience a blackout in the current business model. Therefore, providing the option to partially recharge the lamp may bring benefits to both the firm and the consumer. Second, payments are *coupled* with recharges in the exisiting model: payment for a recharge happens at the same time as the recharge. Because of the stochastic nature of the consumer's needs and disposable income, even if a consumer has sufficient money for the recharge today, she may not have it later when she desires to recharge the lamp. Hence, it may be beneficial to offer some flexibility in payments by decoupling payments from recharges, e.g., by providing the option to prepay for the upcoming recharge or to recharge on credit.

Before we discuss the efficacy of the aforementioned policies, we consider a benchmark decision process shown in Figure 6(a). Here, the money process $\mathbb{M}_t$ does not act as a constraint in the consumer's decision process: in a period, if the lamp is discharged and if it is convenient to recharge, then the consumer simply proceeds with the recharge. It is assumed under this benchmark case that the consumer always has sufficient money for the recharge and pays when she recharges the lamp – thus, this case removes liquidity constraints from the model. Figure 6(g) presents the performance of all the liquidity-based counterfactual policies discussed in this section. We see that under this benchmark model, the recharges increase by 123% over the base case. Such a high increase may not be that surprising, given that our context is poverty, whose defining feature is a liquidity constraint, and expectedly that constraint is a major contributor to the inefficiency in the business model. We also now know that the best that any policy addressing consumers' liquidity constraints can achieve is a 123% increase in recharges.

Allow partial recharges. Uppari et al. (2019) theoretically show that the firm can benefit by allowing partial recharges. We now quantify those benefits using our experimental data. We consider only two options: half recharge (denoted as $r_t^{(0.5)}$) and full recharge (denoted as $r_t^{(1)}$). Although our model can be extended to an arbitrary number of options, we note that offering a variety of partial recharge levels could negatively impact the lamp's battery life while also requiring recharge centers to upgrade their technology to track the charge level in the lamp. Therefore, the assumed setup offers both analytical and practical simplicity.

Figure 6(b) shows how the provision of partial recharges changes the consumer decision process. (The Bellman equations for all decision processes in this section are given in Appendix F.) The variable $\mathbb{M}_t^{(0.5)}$ (resp., $\mathbb{M}_t^{(1)}$) indicates whether or not there is enough money for a half (resp., full) recharge. When the lamp is discharged, if the consumer does not have money for a full recharge but has money for a half recharge, then she considers a half recharge. By comparing Figure 1 with Figure 6(b), we see that in the former case, the consumer could proceed only when $\mathbb{M}_t = 1$, whereas in the latter case, she can proceed even when there is no money for a full recharge. Thus, the decision process branches at the $\mathbb{M}_t$ node, thereby alleviating the liquidity constraint to some extent. We simulate the recharge decisions with Figure 6(b) as the data-generating process. Since the consumer only pays $P/2$ for a half recharge, to retain the equivalence between the increase in recharges and the increase in revenue, we count a half recharge as $r = 0.5$ in our simulations. Figure 6(g) shows that providing the partial recharging option increases the number of recharges (and revenue) by 19% over the base case.

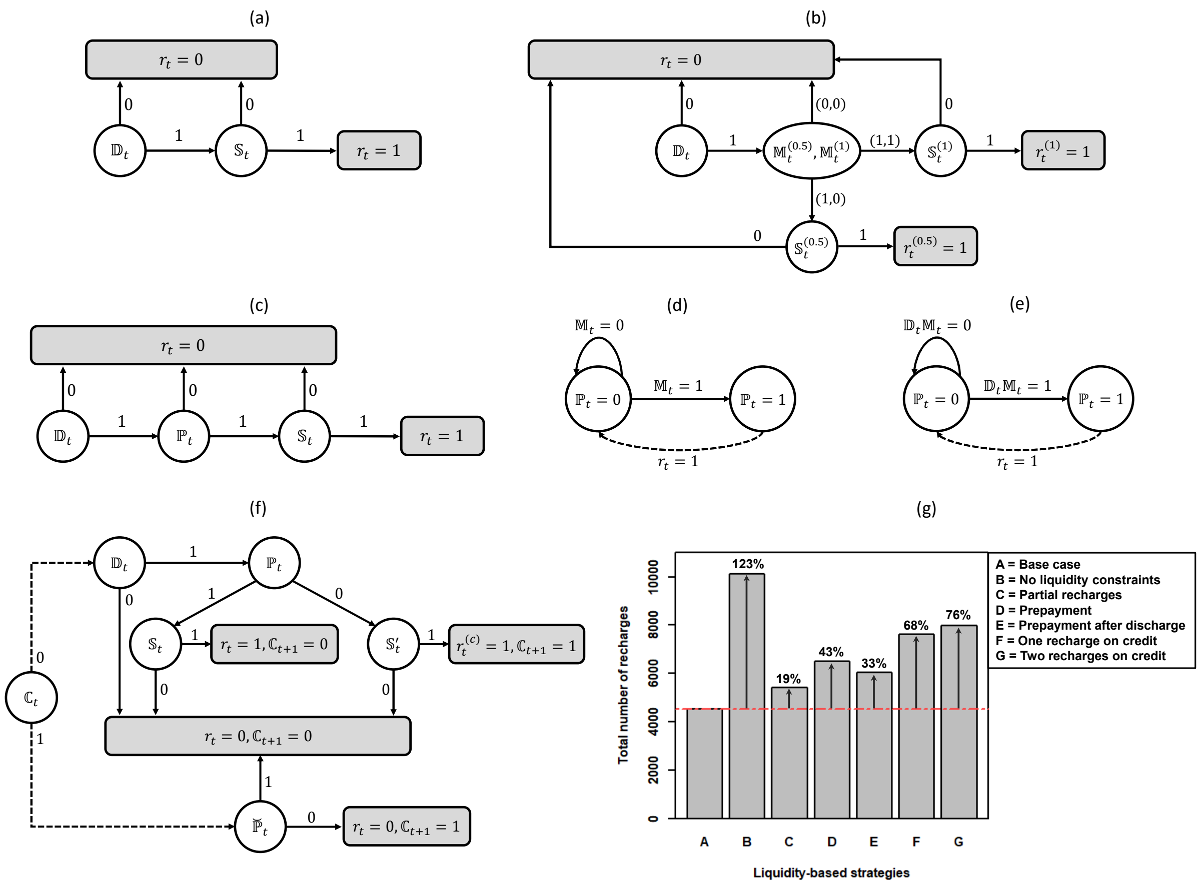

Figure 6 Liquidity-based counterfactuals. Decision processes for (a) no liquidity constraints benchmark, (b) option to partially recharge, (c) option to prepay, and (f) one recharge on credit. (d) and (e) show the evolution of $\mathbb{P}_t$ for the case of prepayment and prepayment after discharge, respectively. (g) The performance of the liquidity-based counterfactuals.

Decouple payments from recharges. We now turn our attention to decoupling payments from recharges and note two important points. First, when payments and recharges can occur at separate points in time, there must be a mechanism in place for the consumer to make the payment without traveling to the recharge center; otherwise, decoupling may not bring any benefits to the consumer. The flexible payment schemes can be implemented in practice by a mobile payment mechanism, wherein the consumer transfers money through her mobile phone without any need to travel. Such mobile transfers have become fairly prominent in sub-Saharan Africa and in developing Asia with the increased market penetration of mobile technology in those regions (GSMA 2015). Second, as we mentioned earlier, we analyze decoupling using two mechanisms: the option to prepay and the option to recharge on credit. One might ask, given that the consumers are cash constrained, whether they will be inclined to prepay for the recharges. Using the payment data for solar lamps in sub-Saharan Africa, Guajardo (2019) demonstrates that consumers sometimes bundle their payments instead of paying a fixed amount weekly – such bundles constitute both the payments that were skipped in the past weeks as well as the advance payments for the upcoming weeks. We infer from this finding that, because of stochastic income and liquidity constraints, there is a demand for both prepayments and on-credit recharges on the consumer side. Morever, evidence in the microfinance literature demonstrates that introducing such flexibility in payment schemes reduces financial stress and enables consumers to manage their income better (Laureti and Hamp 2011, Field et al. 2012, Barboni 2017).

Figure 6(c) shows the decision process when the consumer is provided the option to prepay for the recharge. We replace the variable $\mathbb{M}_t$ with the variable $\mathbb{P}_t$ in the decision process. $\mathbb{M}_t$ indicates whether or not there is enough money in hand to pay for the recharge, whereas $\mathbb{P}_t$ indicates whether or not the payment for the upcoming recharge is already done. $\mathbb{P}_t$, naturally, is a function of $\mathbb{M}_t$, and Figures 6(d)–(e) show two plausible evolutions of $\mathbb{P}_t$. Figure 6(d) shows the setting where as soon as the consumer has enough money for the recharge, she pays for it through mobile money – she neither waits for the lamp to discharge nor does she consider inconvenience–blackout trade-offs. Thus, when $\mathbb{M}_t = 1$, $\mathbb{P}_t$ immediately transits from zero to one. Assuming that the consumer pays at the first instance when she has money may be optimistic, and so we consider an alternative case in Figure 6(e). In this case, the consumer pays at the first instance when she has enough money *after* the lamp is discharged (i.e., to transit to $\mathbb{P}_t = 1$, we need both $\mathbb{M}_t = 1$ and $\mathbb{D}_t = 1$). It is straightforward to show that $\mathbb{P}_t$ in Figures 6(d)–(e) stochastically dominates $\mathbb{M}_t$; therefore, the liquidity constraints are milder under the prepayment option. The two prepayment models result, respectively, in 33% and 43% increases in recharges over the base case.

The decision process when the firm allows only one recharge on credit is shown in Figure 6(f). The consumer can recharge her lamp once without paying, and then she must pay off that debt before the next recharge. The variable $\mathbb{C}_t$ in Figure 6(f) indicates whether the consumer previously recharged on credit. $\mathbb{C}_t$ evolves together with the recharge decisions of the consumer. When $\mathbb{C}_t$ is zero, the consumer can choose to recharge even when the payment for that recharge is not done yet (i.e., the decision process branches at $\mathbb{P}_t$). If the recharge is done without payment (denoted by $r_t^{(c)} = 1$), then $\mathbb{C}_{t+1}$ is set to one, indicating for future reference that the consumer has recharged earlier on credit. When $\mathbb{C}_t$ is one, the consumer has no choice other than paying for the previous recharge. The variable $\breve{\mathbb{P}}_t$ indicates whether the payment for the previous recharge is done. If $\breve{\mathbb{P}}_t = 1$, then $\mathbb{C}_{t+1}$ is reset to zero, indicating that the consumer is no longer in debt. Otherwise, $\mathbb{C}_{t+1}$ remains to be one. We can similarly look into the case when the firm allows two recharges on credit. The corresponding decision process is shown in Figure 11(b) in Appendix F. The simulation results show a 68% increase in recharges when the firm allows one recharge on credit and a 76% increase with the option of two recharges on credit.

To summarize, because the consumers are poor and have erratic cash flows, there are significant gains to be made by providing flexibility in payments. The maximum benefit is seen when the firm offers the option to recharge on credit, which is followed by the option to prepay and the option to partially recharge the lamp. It is worth noting that just by providing 1–2 on-credit recharges, the firm can attain more than half the benefit arising from eliminating liquidity constraints altogether.

5.3. Price/Capacity-based Counterfactual Policies

In all the counterfactuals that we discussed thus far, the recharge price P was fixed at 100 RWF and the lamp capacity Q at 18 hours. We now consider the counterfactual settings wherein the firm changes P and Q. These changes are assumed to be in the status quo business model, and so the decision process in these price/capacity-based counterfactuals is the same as in Figure 1. We rewrite expected recharges $R(\mathcal{P})$ and expected revenue $V(\mathcal{P})$, respectively, as $R(P,Q)$ and $V(P,Q)$ to make it explicit that we now view them as functions of price and capacity.

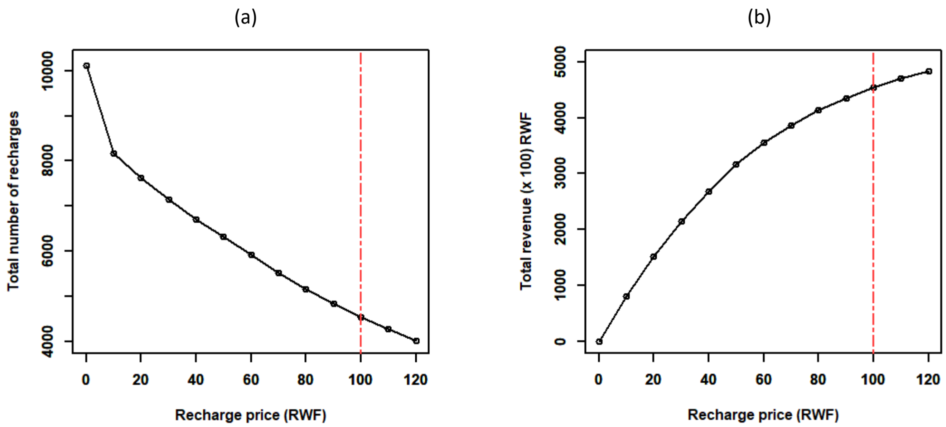

Figure 7 **At $Q = 18$ hours, (a) expected number of recharges and (b) expected revenue at different price levels.**

Set price optimally. We first fix the lamp capacity at $Q = 18$ hours and examine the impact of price changes. Unlike the strategies in Sections 5.1 and 5.2, which increase both R and V simultaneously, varying price has different impacts on R and V. Figure 7(a) shows the total number of recharges expected at different price levels, and Figure 7(b) shows the total revenue expected at those price levels. The number of recharges decreases steeply as we increase the price. The revenue increases in price because of the relatively inelastic nature of recharges with respect to price.[13] (See Appendix E for a discussion on the price elasticity of the expected number of recharges.) Consequently, if the firm focuses solely on maximizing revenue, then the optimal price (i.e., $\arg\max_P V(P,Q)$) will not be an interior solution. By increasing price, the firm gains revenue, but it also loses the market penetration of its product.

However, several firms operating in the BoP markets are social enterprises with a dual objective that values both the usage of their products by poor consumers *and* revenue for the firms' sustenance, even when improving the former may hurt the latter. Assuming that the firm has a Cobb-Douglas-type preference for improvements in usage and revenue relative to the status quo, we model this dual objective as

$$W(P,Q;\gamma) = \left[\frac{R(P,Q)}{R(P_0,Q_0)}\right]^{\gamma} \left[\frac{V(P,Q)}{V(P_0,Q_0)}\right]^{1-\gamma},$$

where $(P_0,Q_0) = (100 \text{ RWF}, 18 \text{ hours})$ is the status quo, and $\gamma \in [0,1]$ is the weight given to usage relative to revenue. Depending on the financial status of the firm and the usage requirements imposed either by the investors or the regulators, the firm may place different weights on usage at different points in time. Here, we exogenously vary the values of γ and examine the sensitivity of optimal gains in revenue and usage.

[13] At a given Q, the derivative of $V(P,Q)$ with respect to P is $R(P,Q)(1+e(P;Q))$, which is positive if the price elasticity $e(P;Q) > -1$ (the condition for relative inelasticity). Although in Figure 7 we show revenue only for prices between 0 RWF and 120 RWF (which was our experimental range), we numerically verified that for prices up to 200 RWF and for capacity levels between 6 hours and 30 hours, the revenue continues to increase in price.

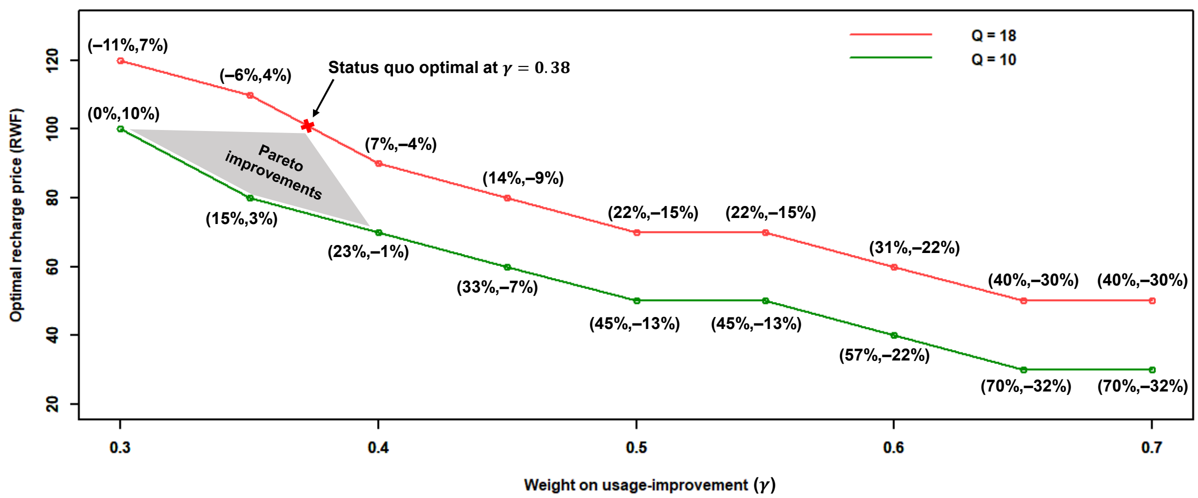

Figure 8 **Optimal price $P^*(Q\,;\gamma)$ along with percentage improvements in usage and revenue for $Q = 18$ hours (status quo capacity) and $Q = 10$ hours (optimal capacity) at different values of γ.**

The optimal price $P^*(Q\,;\gamma) = \arg\max_P W(P, Q\,;\gamma)$ balances the two conflicting objectives of increasing both R and V at a given value of Q. Figure 8 presents $P^*(18\,;\gamma)$ in red for different values of γ, along with the corresponding percentage improvements in usage and revenue. (For these values of γ, we see numerically that $W(P, Q\,;\gamma)$ is unimodal in price; thus, $P^*(Q\,;\gamma)$ exists and is unique.) When the firm places an equal weight on both usage- and revenue-improvements (i.e., $\gamma = 0.5$), it is optimal to reduce the price to 70 RWF. This leads to an increase in recharges of 22% and a drop in revenue of 15%. As we move leftward (resp., rightward) from $\gamma = 0.5$, then the firm becomes more revenue focused (resp., usage focused); therefore, it is optimal to increase (resp., decrease) the price. In fact, the status quo price and capacity are optimal if the firm places a 38% weight on usage.

Set both price and capacity optimally. It is clear from both Figure 7 and the red line in Figure 8 that any movement from the status quo either decreases usage or revenue: the higher the increase in usage, the higher the corresponding drop in revenue, and vice versa. However, Pareto improvements along both the dimensions are plausible if firm also varies capacity along with price. We define $Q^*(\gamma) = \arg\max_Q W(P^*(Q\,;\gamma), Q\,;\gamma)$. As we discussed under Π_3 in Section 4.2, as Q decreases, the number of recharges increase but only for relatively large values of capacity. Reducing Q to relatively smaller values results in too many trips and too much inconvenience for consumers, which in turn negatively affects usage. The optimal value $Q^*(\gamma)$ balances this trade-off. We find that $Q^*(\gamma) = 10$ hours for all $\gamma \in [0.3, 0.7]$.

Figure 8 shows $P^*(10\,;\gamma)$ in green. As expected, the optimal price values are lower when capacity is reduced. We see in general that setting both price and capacity optimally leads to an increase in recharges that is higher than, and a drop in revenue that is lower than, the case wherein the firm sets only price optimally. Pareto improvements are indeed possible when the firm's γ is between 0.3 and 0.4. For the status quo weight on usage (i.e., $\gamma = 0.38$), we find that the optimal price is 80 RWF, and the resultant improvements in usage and revenue are 15% and 3% respectively. Overall, the firm benefits by *reducing both price and capacity*. We next note two points regarding this finding.

First, Prahalad and Hart (2002) exemplify FMCG companies for having adapted some of their products (e.g., shampoo, tea, and cold medicines) for the BoP market by repackaging goods in smaller volumes to make them more affordable. Delivering light through a rechargeable lamp is equivalent to selling light in a small package. Our analysis suggests that – given consumer behavior in the market – it is better for the firm to sell light in a package of even smaller size by reducing price and capacity. Prahalad and Hart (2002) also argue that the consumers at the top of the pyramid (ToP) have enough disposable income, buy in bulk, and shop less frequently, i.e., they use their spending money to "inventory convenience," whereas the consumers at the BoP have limited cash, shop every day, and look for smaller packages. Although this may seem true on the surface, our analysis conclusively shows that (in)convenience is an important factor not only for the ToP consumers, but also for the BoP consumers, and that it must be considered while deciding package size in BoP markets.

Second, the ratio of capacity and price at the status quo is $18/100 = 0.18$ hours/RWF, whereas it is $10/80 = 0.125$ hours/RWF at the optimum at $\gamma = 0.38$ – the *bang for the buck* is lower at the reduced price and capacity levels. Because of the consumer's cash constraints, lowering the price increases her ability to pay for the recharges; but to generate enough revenue at the lowered price, the firm must also then substantially reduce capacity to induce a higher recharge frequency. In other words, the consumer pays a *poverty premium* when the light is provided in a smaller, more affordable package. Mendoza (2011) calls this the *size effect* – the penalty that the poor pay for being served in portions of smaller size. For a comprehensive discussion on poverty premia in other contexts, we refer the reader to Mendoza (2011) and Davies et al. (2016).

5.4. Discussion

Sections 5.1–5.3 evaluated the efficacy of several counterfactual policies that target inconvenience, liquidity constraints, recharge price, and lamp capacity; our analysis is summarized in Table 3. From now on, we refer to the inconvenience- and liquidity-based strategies together as *operations-based* strategies because they improve performance, not by varying the economic variables (namely, price and lamp capacity – the amount paid and the amount obtained in return) but by addressing the sources of inefficiencies (namely, inconvenience and liquidity constraints) in the current business model and by changing the (recharge and payment) processes within the firm.

We discuss three important implications of our analysis and findings. First, we see from Table 3 that *simple strategies can achieve good performance*. For example, just by allowing the consumers to recharge on credit 1–2 times or by starting 2–3 more recharge centers/dropoff points per village, the firm can reap half the benefits from *completely* removing inconvenience and liquidity constraints from the business model. It is important to recognize that while evaluating a counterfactual strategy, we have incorporated only those structural changes in the model that are required by that strategy. That way, we isolate the impact of implementing a particular strategy. Therefore, when combined together, the strategies in Table 3 may result in an even stronger impact on lamp usage and the firm's revenue. Analyzing the performance of any combinatorial strategy is a straightforward extension of the analysis conducted in our paper.

Moreover, Banerjee et al. (2017) note that in "all external decision making problems, inference is unavoidably subjective. In structural modeling, the source of subjectivity is the model itself." Admittedly, the

Counterfactual policy	Usage-improvement	Revenue-improvement
Price/Capacity-based		
Set P optimally, $\gamma: 0.3 \to 0.7$	$-11\% \to 40\%$	$7\% \to -30\%$
Set both P and Q optimally, $\gamma: 0.3 \to 0.7$	$0\% \to 70\%$	$10\% \to -32\%$
Set both P and Q optimally, $\gamma = 0.38$	15%	3%
Inconvenience-based		
Benchmark: {set $I = 0$, visit door-to-door every period}	60%, 79%	
Start 2–3 more recharge centers	26%–29%	
Visit door-to-door once in {1 week, 2 weeks}	14%, 37%	
Visit 5 locations once in {every period, 1 week, 2 weeks}	39%, 20%, 8%	
Liquidity-based		
Benchmark: no liquidity constraints	123%	
Allow partial (half) recharges	19%	
Allow prepayments	33%–43%	
Allow 1–2 recharges on credit	68%–76%	

Table 3 Summary of the performance of counterfactual policies.

decision models in counterfactual settings need not coincide with the ones that we assumed in our paper. Nevertheless, because our process of external extrapolation is transparent, one can further enrich the analysis by introducing additional, plausibly subjective, components to those decision models. For example, in the periodic-visit model, the consumer may not always be at home when the VLE visits to collect her lamp, which may in turn affect the performance of that strategy; one can augment the decision process in Figure 5(a) with a probabilistic node representing the presence of consumer at home; the corresponding probability cannot be estimated from the data that we have, and hence it must be introduced either through subjective beliefs or through market research. Such extensions are also straightforward.

Second, *operations-based strategies tend to perform better than price/capacity-based strategies.* To increase the penetration of its product, a firm operating in the BoP market may have a natural tendency to reduce the price because (economic) poverty, by definition, relates to lack of money, and hence the inability to pay a higher price. Furthermore, several blog posts by entrepreneurs, technical reports by policy organizatons (e.g., Bates et al. 2012, Girardeau and Pattanayak 2018), and much of the development economics literature cited in Section 1 place emphasis on pricing strategies, thereby making them more salient. Of course, price is an important lever in determining product adoptions under poverty, but as we saw in Section 5.3, reducing price in our context simply decreases revenue. Therefore, operating at a lower price without the support from either donors or investors to fulfill any financial deficit (due to subsidies) may not be a sustainable strategy in the long run. Changing capacity along with price improves matters; however, the resultant smaller packaging makes consumers pay a poverty premium, and as seen in Table 3, the improvements are also quite limited in magnitude.

In contrast, operations-based strategies increase usage and revenue simultaneously without consumers paying any poverty premium. This observation underscores the importance of removing, to the extent possible, the inefficiencies embedded in the business model by design – e.g., constraints such as making the consumer travel to a single village-level recharge center and allowing her to pay only when she recharges her lamp – that critically interact with the consumer behavior and limit the product adoption. We acknowledge that the firm may need to incur some costs to implement the operations-based strategies, yet we do not closely examine them in our paper. The costs of counterfactuals – such as the cost of starting a new recharge center in a particular village, the cost of sending the VLE regularly to the households in a village, and the cost of setting up a mobile payment scheme – do not arise from a generalized framework like the revenues arise from the framework of our structural model. Assessing implementation costs is a context-specific exercise, which must be carried out by the firm before it implements a strategy. Besides, given the significant benefits arising from even simple operations-based strategies, it may be worthwhile for the firm to incur those costs. Moreover, the funds that the BoP firms receive initially may be allocated to cover the costs involved in implementing an appropriate operational model and then scale up *efficiently* with that model, as opposed to increasing the outreach *inefficiently* by giving subsidies using those funds.

Third, *our research template can facilitate better experimentation with strategies.* Several firms operating in the BoP markets are budding startups. Although there is some emphasis on setting up the right business model to deliver life-improving goods/services to BoP consumers (e.g., IFC 2012), there is no formal method developed to arrive at one. The lean startup philosophy (Ries 2011) advocates constant experimentation for rapid improvements in the business model. However, as Felin et al. (2019) put it, "the favored hypothesis-generating tool of lean startup—the business model canvas—lacks specificity in helping startups craft unique, firm-specific hypotheses and critical experiments for testing theories." The practitioners can adopt the methodology in this paper for the purpose of hypothesis generation. After a firm has (i) a *minimum viable product* (MVP), (ii) a plausible theory on consumer behavior arising under MVP, and (iii) a model that formalizes that behavior, it can conduct experiments – with the MVP – consisting of the minimal set of treatment conditions that are required to estimate the model. Unlike the ToP markets, there is a dearth of reliable datasets in the BoP markets; therefore, the data arising from such experiments can be used for a variety of analyses, including structural estimation. The estimated model can then guide the firm on what to do next and what is expected in return, thereby resulting in hypotheses that are grounded in both theory and data, which can later be tested by further experimentation.

6. Concluding Remarks

In this paper, we rigorously analyzed consumer behavior and the operational inefficiencies that result under the rechargeable lamp-based off-grid lighting model. Our work has implications for firms, policymakers, academics, and consumers. We build a model of consumer recharge behavior and estimate it using field data from Rwanda. The model's requirements for estimation are minimal: the recharge timestamps, the lamps' price and capacity, and a proxy for the inconvenience of the consumers. We validate the predictive ability of the estimated model and use it to evaluate the efficacy of several inconvenience-based, liquidity-based, and price/capacity-based strategies. The estimated model can serve as a decision support system to assess the potential revenue opportunities of any alternative strategies ex-ante. Such analysis can also guide the treatment conditions for future field experiments. Firms operating rechargeable lamp businesses in other countries can also collect data and fit it to our model and thereafter use it for their decision making.

Although we do not analyze all off-grid solutions currently in the market, we delve deep in terms of analyzing the rechargeable lamp model. Such analysis helps policymakers in Rwanda and other similar countries that want to scale up their off-grid connectivity to weigh rechargeable lamp technologies against the benefits and limitations of other technologies. For instance, although solar home systems remove inconvenience to a great extent (which is a main source of inefficiency in rechargeable lamp technologies), they are unaffordable to poor consumers and hence need to be heavily subsidized either by the government or donor organizations. In contrast, it is much cheaper to get rechargeable lamps into the hands of consumers, but their usage is limited by recharge inconvenience. The current study estimates consumer usage under the latter model. By similarly estimating usage under the solar home systems model, and by accounting for the corresponding costs, policymakers can assess which model works better at offering light to consumers.

In terms of academic relevance, we contribute to the nascent literature in OM that studies operational issues in poor countries. Our dynamic model of off-grid light consumption incorporates consumer inconvenience, income and consumption uncertainties, and liquidity constraints. We characterize the optimal solution of consumer's dynamic program, and examine some properties of our model. The theoretical analysis in the paper is used only to the extent of generating predictions that can be tested using the field data. Future research can further explore our base model and its counterfactual versions; e.g., one could investigate the theoretically optimal way of providing off-grid light to consumers and what payment plans offer the optimal amount of flexibility to alleviate liquidity constraints. One could also conduct experiments with more sophisticated treatment conditions, collect more detailed data, and employ a more elaborate model to estimate the additional effects – such as the impacts of consuming alternative lighting sources and consumers' behavioral biases on recharge behavior – that we could not because of our budgetary limitations and the difficulties in collecting data from our remotely located villages.

Finally, better business models and energy policies that account for consumer inconvenience, liquidity constraints, and actual usage data should result in higher usage of cleaner and cheaper lighting sources, which in turn contributes to increasing consumers' productivity, improving their health, alleviating poverty, and promoting economic growth.

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Appendix A: A Note on Variation in Inconvenience

Because no strategic choices were made either by the firm or by the VLEs while placing a recharge center in our experimental villages, we hypothesize that the inconvenience faced by a consumer may not be systematically related to any relevant observable characteristics of that consumer. To test this claim, we collected data on households from twelve villages with IDs $3, 4, 5, 6, 10, 12, 13, 14, 20, 24, 26$, and 27. This set of villages were randomly selected from the set of all sampled villages.

Variable	Coefficient	Standard Error	p-value
Household (HH) composition			
Number of members in the HH	−1.284	10.304	0.901
Number of females in the HH	5.524	12.630	0.662
Number of children in the HH	21.539	37.549	0.566
Age of the HH head	−0.318	0.796	0.689
Maximum education level of members in the HH	−8.667	14.096	0.539
Features of the dwelling			
$\mathbb{1}${Dwelling has roof with guttering}	−22.185	64.117	0.729
$\mathbb{1}${Dwelling has wooden walls}	−15.461	71.070	0.828
$\mathbb{1}${Dwelling has brick walls without cement covering}	22.157	25.229	0.380
$\mathbb{1}${HH owns livestock}	30.914	24.401	0.205
Number of rooms in the dwelling	−6.337	7.536	0.401
Number of mobile phones owned by the HH	−15.289	14.472	0.291
Economic activities			
$\mathbb{1}${A HH member owns a small business}	−64.749	41.016	0.115
$\mathbb{1}${A HH member is a farmer}	56.527	39.638	0.154
$\mathbb{1}${A HH member has a regularly-paid job}	73.874	55.812	0.186
$\mathbb{1}${A HH member engages in part-time jobs}	43.118	27.464	0.117
$\mathbb{1}${A HH member is retired and gets pension regularly}	260.895	172.103	0.130
Total income of HH per day	−0.009	0.017	0.597
Amount in hand on day of survey	0.051	0.031	0.104
Alternative lighting sources			
$\mathbb{1}${HH uses kerosene}	41.658	71.014	0.558
$\mathbb{1}${HH uses candles}	2.715	48.734	0.956
$\mathbb{1}${HH uses flashlight}	34.364	27.373	0.210
$\mathbb{1}${HH uses a solar lantern}	−7.586	39.603	0.848
$\mathbb{1}${HH uses some other lighting sources}	27.243	62.117	0.661
Expenditure on alternative sources in the last month	0.001	0.006	0.803

Table 4 Regression of distance on consumer covariates.

Our dependent variable is the distance (measured in meters) between the GPS coordinates (recorded in three dimensions) of a household in a village and the recharge center in that village. We regress this variable against several consumer characteristics (listed in Table 4) and village-level fixed effects (thereby accounting for the time-invariant unobservables at the village level). The results of our regression analysis are presented in Table 4. All p-values are greater than 0.05, and thus we find no significant relationship between the dependent and independent variables.[14]

A few things are to be noted with regard to the regressors in Table 4: (i) the base category for the dummy indicating a roof with guttering is a roof without guttering; (ii) the base category for the wall-related dummies is a dwelling with brick walls and cement covering; (iii) the base category for the economic activity dummies is one that captures all the "other" types of occupations; (iv) consumers use multiple lighting sources, and so there is no collinearity problem when we use a dummy for "other" lighting sources (e.g., fire-sticks, biogas, second-hand automobile batteries); (v) all money-related variables are measured in RWF; and (vi) the education level is coded as 0 if the consumer has no schooling, 1 if she is educated up to some primary level, 2 if she is educated up to some secondary level, 3 if she completed secondary education, and 4 if the consumer received some vocational training.

Appendix B: Testable Predictions and Reduced-Form Analysis

B.1. A Simpler Model and Testable Predictions

We are interested in understanding how inconvenience, recharge price and lamp capacity affect the expected number of recharges $\mathbb{E}\tilde{R}$ (defined in (1)) under our model to test the validity of those relationships vis-á-vis data. Although using Proposition 2, we can obtain the expression for $\mathbb{E}\tilde{R}$, analytically characterizing it is difficult because there are 2^T possible combinations of recharge sequences, and the time-varying thresholds along with the uncertainties in liquidity and consumption make analysis of recharge probabilities cumbersome.

Instead, we analyze a version of the model that incorporates all factors of interest, but in the simplest possible manner, and is amenable to formal analysis. To build this model, we assume that (i) the consumption time is deterministic, and upon recharge, the lamp lasts for exactly $q\,(\geq 1)$ periods; and (ii) the disposable income process is stochastic but i.i.d. over time periods, and the probability that the consumer has sufficient money for recharge in a period is given by $v_\perp(P)$. With these assumptions, the blackout-cost threshold given in Proposition 1 (now denoted simply as k_t) simplifies to

$$k_t = \alpha I - \sum_{i=1}^{\check{q}-1} \Big[v_\perp \mathbb{E} \min\{k_{t+i}, \tilde{\beta}\} + (1 - v_\perp)\beta \Big], \tag{9}$$

where $\check{q} = \min\{q, T - t + 1\}$. To understand the threshold structure, recall that k_t is the effective cost of recharging in period t, whereas $\tilde{\beta}_t$ is the cost of not recharging in that period. As we discussed in Section 4.2, the effective cost of recharging in a period must account for both the inconvenience cost and the potential cost savings from jumping q periods ahead. The cost that the consumer would have incurred in period $t+i$ (for $1 \leq i \leq q-1$, assuming no end of horizon) if she does not recharge in period t is equal to (i) the

[14] We note that by the virtue of random assignment to the experimental conditions, the variables P and Q are neither (significantly) correlated with the variable I, nor with the covariates in Table 4.

(expected) minimum of k_{t+i} and $\tilde{\beta}_{t+i}$ if she has sufficient money for recharge in period $t+i$ (and has the option to recharge); and (ii) $\mathbb{E}\tilde{\beta} = \beta$ if she does not have enough money in that period (and has no option to recharge). Since the probability of the former event is $v_\perp$ and of the latter is $1 - v_\perp$, the expected cost saving in an interim period is $v_\perp \mathbb{E}\min\{k_{t+i}, \tilde{\beta}\} + (1 - v_\perp)\beta$. These expected cost terms for the interim periods are deducted from αI in the expression for k_t in (9).

We simplify the model further by assuming that the thresholds are stationary (which is equivalent to having an infinite horizon) to obtain a simple expression for interarrival times. We denote the time-invariant threshold by k^*, which is the (unique) solution to the following equation:

$$k = \alpha I - (q-1)\big\{v_\perp \mathbb{E}\min\{k, \tilde{\beta}\} + (1 - v_\perp)\beta\big\}. \tag{10}$$

To see why, we compare (10) with (9). The former is obtained by using the time-invariance property of the thresholds on the latter. We formalize the above arguments in the following result:

Lemma 3. *The sequence $\{k_T, k_{T-1}, \ldots\}$ is convergent. The limit of this sequence is k^*.*

Now we characterize the expected IAT under this model. After $q-1$ periods of consumption, the consumer waits until she has sufficient money for recharge *and* her blackout cost is above the threshold. The probability of this event in any period is $v_\perp \bar{F}(k^*)$, and the wait time is geometrically distributed. Hence, the expected blackout time is the mean of this geometric distribution, and the expected interarrival time Ψ is given by

$$\underbrace{\Psi}_{\text{IAT}} = \underbrace{q-1}_{\text{CT}} + \underbrace{\underbrace{1/v_\perp}_{\text{BT}_\text{L}} \times \underbrace{1/\bar{F}(k^*)}_{\text{BT}_\text{S}}}_{\text{BT}}. \tag{11}$$

The above expression for IAT parallels the structural pattern presented in Figure 2. The time between successive recharges (IAT) is the sum of consumption time and blackout time. The blackout time has two components: one because of liquidity constraints (BT_L) and the other because of strategic behavior (BT_S). As evident from Figure 2, BT ends when both constraints $\mathbb{M}_t = 1$ and $\mathbb{S}_t = 1$ are satisfied together (i.e., $\mathbb{M}_t = 1 \wedge \mathbb{S}_t = 1$), and parallely, BT is given by $\text{BT}_\text{L} \times \text{BT}_\text{S}$.

Because the recharges follow a renewal process in the above formulation with mean inter-renewal interval Ψ, by the elementary renewal theorem, the expected number of recharges $\mathcal{R}$ in a duration T that is large enough is given by $\mathcal{R} \approx T/\Psi$. The following result establishes the relationship between $\mathcal{R}$ and the variables I, P, and Q (which is proxied here by q). Although q is a discrete variable, in the result below, we treat it as a continuous variable (satisfying $q \geq 1$) to simpify the analysis.

Proposition 3. *If we assume that $v_\perp(P)$ is monotonically decreasing in P and that F has increasing hazard rate, then the following statements hold:*

(i) *$\mathcal{R}$ is decreasing in I.*

(ii) *$\mathcal{R}$ is decreasing in P.*

(iii) *There exists a threshold $\hat{I} \geq 0$ such that $\mathcal{R}$ is unimodal in q for $I \geq \hat{I}$, and it is decreasing in q for $I < \hat{I}$.*

The three subparts of Proposition 3 parallel the three predictions stated in Section 4.2. As we have only two lamp capacity conditions in our experiments, we restate Π_3 as follows:

(Π_3) The difference between the number of recharges at $Q = 18$ hours and at $Q = 14$ hours is (a) negative for low values of inconvenience and (b) positive for high values of inconvenience.

B.2. Testing the Model's Predictions

To test Π_1–Π_3, we use the data from nine experimental conditions (i.e., $(P,Q) \in \{\{0,50,60,70,80,100,120\} \times \{18\}\} \cup \{\{80,100\} \times \{14\}\}$) and run the following Poisson regression:

$$\tilde{R}_{jv} \sim \text{Pois}(\lambda_{jv}), \quad \text{where} \quad \log \lambda_{jv} = a_0 + a_1 I_{jv} + a_2 P_{jv} + a_3(I_{jv})\mathbb{Q}_{jv} + e_v. \tag{12}$$

Here, subscript j corresponds to an individual consumer and subscript v to a village. $\tilde{R}_{jv}$ is the total number of recharges of consumer j in village v in the three months of the experimental duration, P_{jv} is the recharge price (in RWF) assigned to this consumer, I_{jv} is her inconvenience (in kilometers), $\mathbb{Q}_{jv}$ is a dummy variable indicating whether or not the consumer's lamp capacity is 18 hours (as opposed to 14 hours), and e_v indicates a village-level fixed effect. As per Π_1 and Π_2, we expect a_1 and a_2 to be negative. Because the impact of capacity on recharges depends on consumer inconvenience, the coefficient of $\mathbb{Q}_{jv}$ in (12) is a function of I_{jv}. According to Π_3, $a_3(I)$ should be negative for smaller values of I and positive for larger values of I.

The results of our regression analysis are presented in Table 5, wherein we analyze three specifications of (12). In specification I, function a_3 is assumed to be constant. The resultant regression model presents the average effects of variables I, P, and $\mathbb{Q}$ in the sample. The coefficients of I and P in specification I are negative and statistically significant. The coefficient of $\mathbb{Q}$ is positive yet lacks significance, perhaps because the capacity conditions used in the experiments were 14 hours and 18 hours, which are not far apart from each other. The economic interpretation of these coefficients is as follows: a 10 RWF increase in recharge price, all else equal, decreases the expected number of recharges by 9% ($= \exp(-0.0097 \times 10) - 1$). An equivalent decrease in recharges is obtained by increasing inconvenience by 165 meters (i.e., $-0.5866 \times 0.165 = -0.0097 \times 10$). This 9% decrease in expected number of recharges can be compensated exactly by increasing the lamp capacity by 11.5 hours (because $\exp(0.0308 \times 11.5/4) - 1 = 9\%$).

Variable	(I)	(II)	(III)
I	-0.5866^{***}	-0.6159^{***}	-0.8374^{***}
	(0.0484)	(0.1019)	(0.0848)
P	-0.0097^{***}	-0.0097^{***}	-0.0097^{***}
	(0.0004)	(0.0004)	(0.0004)
$\mathbb{Q}$	0.0308	0.0133	−0.0482
	(0.0393)	(0.0661)	(0.0444)
$\mathbb{Q} \times I$		0.0366	
		(0.1117)	
$\mathbb{Q} \times I^2$			0.2507^{***}
			(0.0679)
N	1709	1709	1709
Pseudo-R^2	0.2596	0.2597	0.2622

Table 5 **Impact of inconvenience, recharge price, and lamp capacity. Superscript '***' is used when $p \leq 0.001$ and no superscript is used when $p > 0.1$.**

In specification II, we set $a_3(I) = a_{30} + a_{31} I$. However, this specification is not informative to either support or reject the hypothesized structure of $a_3(I)$ as both the coefficients related to $\mathbb{Q}$ lack significance. This may be because of some nonlinearity in $a_3(I)$; therefore, we consider specification III, wherein we set

$a_3(I) = a_{30} + a_{31}I^2$. Here, the coefficients of I and P have the same signs as in specification I. The sign of the coefficient of $\mathbb{Q}$ flips (but the coefficient lacks significance), and the interaction term is positive (and significant). We see that, under specification III, a 4-hour increase in lamp capacity results in a 5% *drop* in the expected number of recharges for consumers living close to the recharge center, whereas it results in a 1% and 22% *rise* for consumers living, respectively, 500 meters and a kilometer away from the recharge center.

Together, these findings provide support for Π_1, Π_2, and Π_{3b}. To comment on Π_{3a} with some confidence, we resort to a subsample analysis, wherein we run the Poisson regression with $\log \lambda_{jv} = a_0 + a_1 I_{jv} + a_2 P_{jv} + a_3 \mathbb{Q}_{jv}$ over a subsample with $I < \mathcal{I}$, where $\mathcal{I} \in \{10, 20, 30, 50, 100\}$ meters. We find that the coefficient of $\mathbb{Q}$ is negative for all values of $\mathcal{I}$ and that it is statistically significant for smaller values of $\mathcal{I}$ (thereby providing support for Π_{3a}) and loses significance as $\mathcal{I}$ increases.[15] Our findings remain unaffected under negative binomial and zero-inflated Poisson regression specifications. (The details are available upon request.) We conclude that recharge data from the field supports Π_1–Π_3.

Appendix C: Identifiability of Model Parameters

In this section, we present some intuitive arguments on the identification of parameters. We discuss which sources of variation in the data and which theoretical assumptions in the model drive the quantitative values of the parameter estimates. We first examine identification in simpler models wherein only a subset of parameters appear. If a parameter cannot be identified in such simpler models, then it cannot also be identified in more general models. Thereafter, we move to the general versions and examine whether the parameters identifiable in simpler models continue to be identifiable in the general ones.

Identifiability of $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\sigma_\xi})$. The simplest model to understand the identifiability of these parameters is the one that assumes away consumption uncertainty and liquidity constraints: the consumer always has sufficient money for the recharge and the lamp lasts for exactly $q\,(\geq 1)$ periods upon recharge. Using the normal distribution assumption of $\tilde{\xi}_t$, we can rewrite (3) as

$$C(t, \tilde{z}_t) = \min\left\{\alpha I + \bar{C}(t+q), \beta + \sigma_\xi \tilde{z}_t + \bar{C}(t+1)\right\}, \tag{13}$$

where $\bar{C}(t) = \int_{-\infty}^{\infty} C(t,z)\, d\Phi(z)$. We cannot identify σ_ξ because equation (13) is only identified up to scale; multiplying both sides by a positive constant does not change the recharge decision. This continues to be the case in more complex models too. Therefore, we follow a common standard in the discrete choice literature and normalize the variance of the error term to one.

The recharge decision does not change in (13) also when we increase αI and β by the same amount. Thus, at the individual level, we cannot identify α and β separately; we can only identify the difference $\alpha I - \beta$. Nevertheless, because we have recharge data for multiple consumers and the value of inconvenience I varies across them, we can estimate α using the variation in inconvenience across individuals. Now assuming that the value of α is given, we investigate whether or not we can identify β at the individual level.

We assumed under this model that we know consumption time perfectly and that it is equal to q periods. To be consistent with this assumption, we also assume that all interarrival times of the focal consumer are

[15] The coefficients of $\mathbb{Q}$, along with the p-values in parentheses, for the aforementioned values of $\mathcal{I}$ are as follows: $-0.669(0.020), -0.702(0.014), -0.375(0.081), -0.177(0.328), -0.166(0.189)$.

greater than or equal to q periods; otherwise, this model assigns zero likelihood. Then, the BT is given by the difference between IAT and q. Because there are no liquidity constraints under this model, BT here is purely due to the strategic behavior of the consumer. Specifically, the blackout time here corresponds to the hitting time of the blackout cost, i.e., the time it takes (after the lamp's light is consumed) for the realized blackout cost to go beyond the (time-varying) threshold. Since these times are purely a function of β, the variation in these blackout times can identify β.

To summarize, we cannot identify σ_ξ, and so we normalize it to one. Hence, α and β become unitless quantities. At the individual level, we cannot identify α and β separately. Using the variation in inconveniences across individuals, we can estimate α. Given a value of α, using the variation in blackout times of an individual, we can estimate β at the individual level. Therefore, β is overidentified when we jointly estimate α and β using the recharge data of multiple individuals. This means that we can actually identify more parameters that represent heterogeneity in β (e.g., through random effects).

Identifiability of (μ, σ, ρ). The blackout times in the model just discussed were purely due to the strategic behavior of the consumer. In this section, we discuss identification under a model that is the other extreme. We assume that after the lamp is discharged, the consumer recharges her lamp whenever she has sufficient money for a recharge. She is neither sensitive to inconvenience nor to blackouts and hence does not desire to balance out inconvenience and blackout costs. So in this case, BTs are purely due to liquidity constraints.

We first assume that $\rho \geq 0$ is given. Recall that the consumer's disposable income starts growing immediately after a recharge. To keep the discussion simple, we assume that the consumer never has sufficient money for a recharge in the first period when her lamp is discharged; then, all hitting times are greater than q. (The arguments can be easily extended to the cases where this assumption is relaxed.) Consequently, BT is the time that it takes beyond the consumption time for the income process to hit the threshold P. As we argue next, the variation in these BTs forms the source of identification for μ and σ.

The variation in BTs helps us identify transition probabilities $v(t,1,0)$ (for some t) of the Markov chain. Moreover, the estimate of $v(t,1,0)$ is equal to the sample hazard rate, i.e., it is the proportion of instances in the sample where the income did not hit the threshold P in $t-1$ periods but hits it at t. We illustrate this point using the following example: assume that $q = 1$, and that we observe the following hitting times: $\{3,2,3,4,2\}$. Then the corresponding likelihood function is $[v(2,0,0)v(3,1,0)][v(2,1,0)][v(2,0,0)v(3,1,0)][v(2,0,0)v(3,0,0)v(4,1,0)][v(2,1,0)]$. If we treat each $v(t,\mathbb{m},\mathbb{m}')$ as a separate variable, then by noting that $v(t,1,0) = 1 - v(t,0,0)$, we obtain the estimate $v(2,1,0) = 2/5$ by maximizing the above likelihood with respect to $v(2,1,0)$. This estimate is exactly equal to the proportion of instances in which the income did not hit the threshold in one period but hits it in two. Similarly, $v(3,1,0) = 2/3$. We cannot identify $v(4,1,0)$ from the above hitting time data. However, since $v(2,1,0)$ and $v(3,1,0)$ are two distinct functions of μ and σ, we can identify them from the estimates of those transition probabilities. In a general setting, to identify μ and σ, we should be able to estimate at least two transition probabilities, and hence we need to observe at least three distinct hitting time values in the data with two of them occurring more than once.

The identification of μ and σ is also possible when we use the data from multiple consumers with at least two distinct price levels. What we need is multiple distinct equations, corresponding to multiple distinct estimates of transition probabilities, to estimate μ and σ. Because the transition probabilities are functions of recharge price too, we obtain distinct equations (corresponding to distinct price levels) in this case as well. This feature is important (i) when the Markov chain is either serially independent or stationary, wherein $v(t, \mathbb{m}, \mathbb{m}')$ becomes independent of t, and (ii) when we estimate parameters in more complex models wherein BTs may not be purely due to liquidity constraints.

Now we argue that ρ is non-identifiable. ρ represents serial correlation in the income process. Therefore, its source of identification is the variation in the number of times the process transitions from 0 to 0, 0 to 1, 1 to 0, and 1 to 1 in the successive periods. To be able to estimate three parameters jointly, we need to observe at least three types of transitions. However, because we assume a renewal structure for the income process, we never observe the latter two types of transitions. The former two types of transitions are useful in estimating only μ and σ (for a given ρ).

In summary, for a given ρ, the variation in blackout times – either within an individual or across individuals facing different price levels – identifies μ and σ. We cannot identify ρ either at the individual level or at the aggregate level because of the assumed renewal structure; therefore, we treat it as a hyper-parameter.

Joint identifiability of (α, β) and (μ, σ). To understand the joint identifiability of these parameters, we combine the features of the two simplified models discussed above. Now the consumer is liquidity constrained and accounts for inconvenience–blackout trade-offs, yet her consumption time is deterministic (and equal to q periods). In addition, we assume that (i) σ_ξ is normalized to one, (ii) ρ is exogenously specified, and (iii) all the observed IATs are greater than q such that BTs are interarrival times minus q.

We could earlier identify β and (μ, σ) separately at the individual level by attributing the variation in BTs purely either to strategic behavior or to liquidity constraints. Now, both the strategic behavior of the consumer and her liquidity constraints contribute to her blackout times; they cannot be disentangled at the individual level. The variation in recharge price across individuals plays an important role in disentangling the two components. We have recharge data of consumers who faced zero recharge price. The liquidity constraints play no role in the decision-making process of these consumers; their decision model reduces to the one that we discussed at the beginning of this section. Therefore, using the data of zero-price consumers, we can estimate α and β. Given these estimates of α and β, we can control for the strategic part of BTs for the consumers facing nonzero price. The residual variation in BTs (across consumers facing different price levels) can then be purely attributed to liquidity constraints, which then identifies μ and σ.

The above procedure of estimating parameters sequentially is mentioned only to demonstrate that the variation in recharge price allows us to identify $(\alpha, \beta, \mu, \sigma)$. This procedure, however, is inefficient because it uses only partial data to estimate each set of parameters. We can jointly estimate all four parameters by maximizing the likelihood for zero-price and nonzero-price consumers together.

Identifiability of $\boldsymbol{\lambda}$ along with other parameters. Until now, we have assumed that q is known, which allowed us to compute blackout times directly from the observed interarrival times. If q is uncertain, but the capacity Q and the parameter λ are known, then we can still identify all of $(\alpha, \beta, \mu, \sigma)$ because given a realization of q, we can disentangle CT from BT, and the variation in BTs identifies the parameters. However, if λ is also a parameter that needs to be estimated, then there is no source of variation that disentangles consumption and blackout times.

To identify λ, we need something that affects CT but not BT. The variation in lamp capacity serves this purpose. If we assume that λ is known, then all the aforesaid parameters are identified as a function of λ, and they can control for blackout times. The remaining variation in IATs across individuals with varying capacity levels is attributed to the variation in CTs, which thereby identifies λ.

Appendix D: Parameter Estimates and Interpretation

The parameter vector Θ under specification S_4 is given by $\Theta = (\alpha, \mu_\beta, \sigma_\beta, \mu, \sigma, \rho, \lambda, \delta)$, where (i) α denotes the sensitivity to inconvenience, (ii) μ_β is the mean of the blackout cost β (when modeled as a random effect) and σ_β is its standard deviation, (iii) μ and σ are the mean and standard deviation of the innovations in the AR(1) process of disposable income, (iv) ρ is the serial correlation in that AR(1) process, (v) λ is the fraction of a period served by one hour of lamp's light, and (vi) δ is the discount factor in the Bellman equations. The maximum likelihood estimates $\hat{\Theta}(v)$ for all villages are presented in Table 6.

Village ID	$\hat{\delta}$	$\hat{\rho}$	$\hat{\alpha}$	$\hat{\mu}_\beta$	$\hat{\sigma}_\beta$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\lambda}$
1	0.3	0.00	1.56	−1.02	0.77	24.67	24.15	0.044
2	0.3	0.05	0.83	0.04	0.95	4.01	1.55	0.052
3	0.4	0.00	2.66	0.87	0.94	4.17	0.95	0.047
4	0.3	0.35	0.10	−0.81	0.49	−25.84	55.03	0.036
5	0.2	0.00	1.30	0.26	0.76	4.54	0.69	0.066
6	0.1	0.00	0.97	−0.49	0.45	3.93	1.29	0.031
7	0.8	0.05	2.94	0.22	0.94	4.10	0.32	0.062
8	0.5	0.00	2.47	−0.21	0.57	2.11	3.94	0.025
9	0.2	0.25	1.22	−0.22	0.79	3.89	6.63	0.043
10	0.8	0.10	1.10	0.36	0.42	3.72	1.16	0.062
11	0.7	0.00	0.23	−1.54	0.20	4.90	0.16	0.001
12	0.1	0.05	1.12	−0.16	0.75	3.79	1.15	0.053
13	0.8	0.25	2.32	0.64	0.66	1.80	3.61	0.045
14	0.1	0.20	0.61	−1.06	0.41	4.13	0.23	0.003
16	0.7	0.25	3.22	0.61	0.74	−0.64	6.09	0.082
20	0.5	0.10	0.03	−0.12	0.78	3.66	1.05	0.059
22	0.1	0.15	0.46	−0.79	0.38	3.82	0.64	0.034
24	0.6	0.00	0.61	−0.08	0.83	3.03	2.51	0.092
25	0.4	0.15	1.11	−0.22	0.48	4.20	1.40	0.056
26	0.7	0.15	1.58	−0.42	0.49	−4.09	122.85	0.025
27	0.1	0.15	0.03	−1.88	0.69	4.13	0.26	0.024
28	0.1	0.00	0.69	−0.85	0.44	3.08	2.52	0.004

Table 6 **Maximum likelihood estimates under specification S_4 for all villages.**

For the sake of brevity, here we interpret in detail the parameter estimates for a single village with ID 5; the interpretable measures discussed in the remainder of this section are presented for all villages later in

Table 7. The estimate $\hat{\delta} = 0.2$ suggests that a representative consumer from village 5 is somewhat myopic about weighing the future costs and benefits associated with using lamps. Specifically, the consumer places a weight of less than 1% beyond 3 periods (= 9 days) since $\hat{\delta}^3 \leq 0.01$, or $\lceil \log(0.01)/\log(\hat{\delta}) \rceil = 3$.[16]

We modeled the number of periods that a lamp lasts as $\tilde{N} - 1 \sim \text{Poisson}(Q\lambda)$ in Section 4.3. Using the estimate $\hat{\lambda} = 0.066$ period/hour, Figure 9(a) presents the probability that the lamp is discharged within n periods for 14-hour and 18-hour lamps and for $n \in \{1, \ldots, 5\}$. As expected, an 18-hour lamp lasts longer than a 14-hour lamp. The charge in an 18-hour (resp., a 14-hour) lamp is totally consumed within one period with probability 30% (resp., 40%), and within two periods ($\approx$ a week) with probability 67% (resp., 76%). Both types of lamps are almost certainly discharged within 4 periods.

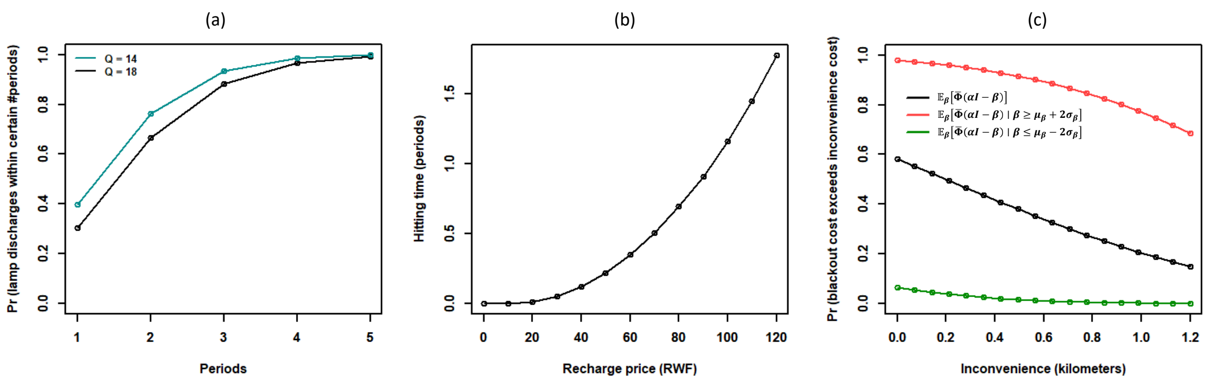

Figure 9 Interpretation of parameters estimated for village 5: (a) probability that the lamp is discharged within certain number of periods for $Q \in \{14, 18\}$ hours, (b) average time it takes for the disposable income to hit different price levels, and (c) probability that the blackout cost exceeds inconvenience cost at different levels of I.

Under the disposable income model discussed in Section 4.3, the estimates $\hat{\mu}$, $\hat{\sigma}$, and $\hat{\rho}$ imply that m_t – the log of the consumer's disposable income for the lamp's light in period t – grows as $m_t = 0 \times m_{t-1} + 4.54 + 0.69\tilde{z}_t$, where $\tilde{z}_t$ is an i.i.d. standard normal random variable. (Here, m_t is i.i.d. because $\hat{\rho} = 0$. In fact, the average value of ρ across villages is 0.1, suggesting that the disposable income of consumers is almost independent across periods.) A better way to interpret the income process is in terms of its hitting times. For recharge price P, if we denote by $\tilde{h}(P)$ the corresponding (first) hitting time, then

$$\Pr\left\{\tilde{h}(P) = k\right\} = \Pr\left\{\mathbb{M}_1 = \ldots = \mathbb{M}_{k-1} = 0, \mathbb{M}_k = 1;\, P\right\} \quad \text{for } k \in \{1, 2, \ldots\}. \tag{14}$$

[16] The estimates of δ presented in Table 6 are considerably lower than the discount factors that are usually seen in the behavioral economics literature (see Cohen et al. 2020, for a review of the methods to estimate discount factors; when considering consumption decisions, δ in the short run is almost close to one). Since the consumer has several important tasks in her daily life, the extent to which she looks forward at the consequences of those tasks may differ across tasks; the parameter δ in our context corresponds *solely* to the extent to which the consumer looks forward while accounting for the costs associated with using lamps. Moreover, in our paper, δ is not estimated directly using some variation in the choice data of consumers; it is estimated using the method of cross-validation. The values of $\hat{\delta}$ in Table 6 suggest that the observed data is best explained if we assume that the consumers are relatively more myopic than forward looking when it comes to recharging their lamps.

Using the above probability mass function of $\tilde{h}(P)$, we can compute the mean $\mathbb{E}\tilde{h}(P)$ for any given P. We plot the average hitting times for different recharge price values in Figure 9(b). We see that $\mathbb{E}\tilde{h}(P)$ is convex in P, implying that accruing money for a recharge becomes increasingly tougher as price increases. It takes (i) almost no time to accrue 20 RWF, (ii) on average a day to accrue 60 RWF, and (iii) 5.34 days on average for the disposable income to hit 120 RWF.

Because σ_ξ is normalized to one, the coefficients α and β (thereby μ_β and σ_β) become unitless quantities. Therefore, we interpret the estimates $\hat{\alpha}$, $\hat{\mu}_\beta$, and $\hat{\sigma}_\beta$ through their impact on inconvenience and blackout costs. In any given period, the probability that the blackout cost in that period exceeds inconvenience cost is given by

$$\Pr\left\{\alpha I - \tilde{\beta}_t \le 0\right\} = \Pr\left\{\alpha I/\sigma_\xi - \beta/\sigma_\xi \le \tilde{z}_t\right\} = \bar{\Phi}(\alpha I - \beta), \tag{15}$$

where $\tilde{z}_t$ is the unit-normal random variable. The mean, β, of $\tilde{\beta}_t$ is a random variable in S_4 with distribution Normal$(\mu_\beta, \sigma_\beta^2)$. We compute three versions of (15) and plot them as a function of I in Figure 9(c). First, we compute (15) averaged across all the possible values of β: $\mathbb{E}_\beta[\bar{\Phi}(\alpha I - \beta)]$. Recall that β in our model captures the disutility that the consumer experiences when she does not have the lamp's light. The aforementioned measure captures (on average) the fraction of times a representative consumer (or equivalently, the fraction of consumers in the village) needs the lamp's light enough that its value exceeds the inconvenience of recharging. As expected, this measure is decreasing in I, and it is equal to 34% and 15% when I is equal to 0.6 kms and 1.2 kms, respectively. It must be noted that this measure is not 100%, but 58%, as $I \to 0$. Although the households that are very close to the recharge center face almost no inconvenience, they may not always need the lamp's light (and hence their blackout cost may not exceed inconvenience cost) because some of those households may not have much activity in the night time (e.g., the ones with older people), or on some days they may not mind the lack of lamp's light because they have a stock of alternative lighting sources.

Second, we compute (15) conditional on β being relatively high: $\mathbb{E}_\beta[\bar{\Phi}(\alpha I - \beta) \mid \beta \ge \mu_\beta + 2\sigma_\beta]$. This measures the proportion of instances when the blackout cost exceeds inconvenience cost for consumers who usually value the lamp's light highly (e.g., the ones with school-going children at home), or equivalently for a representative consumer on the days when she is in great need of lamp's light (e.g., on days when children have exams or guests visit the home). It is equal to 89% and 67% when I is equal to 0.6 kms and 1.2 kms respectively, and it is ∼100% as $I \to 0$. Third, we compute $\mathbb{E}_\beta[\bar{\Phi}(\alpha I - \beta) \mid \beta \le \mu_\beta - 2\sigma_\beta]$, which measures the probability that blackout cost is greater than inconvenience cost conditional on β being relatively low (for the reasons discussed in the previous paragraph). It is equal to 7%, 1%, and ∼0% when I is 0, 0.6, and 1.2 kilometers respectively.

Table 7 restates the estimates from Table 6 in a different and interpretable manner. It presents for each village (i) the number of weeks beyond which a representative consumer in that village places a weight of less than 1% on future costs and benefits of using lamps, (ii) the expected number of days an 18-hour lamp lasts, (iii) the expected number of days it takes for the disposable income process to hit 100 RWF, (iv) the fraction of instances when the consumer – with $I = \bar{I}_v$, which is the average inconvenience in village v – needs the lamp's light enough that its value exceeds the inconvenience of recharging, and (v) the recharges

Village ID	$\lceil \log(0.01)/\log(\hat{\delta}) \rceil$ (weeks)	$\mathbb{E}[\tilde{N}\,;\,Q=18]$ (days)	$\mathbb{E}\tilde{h}(100)$ (days)	$\mathbb{E}_{\beta}[\bar{\Phi}(\alpha \bar{I}_v - \beta)]$	Recharges recorded per household
1	1.71	5.38	0.76	0.07	1.31
27	0.86	4.30	4.01	0.06	1.49
11	5.57	3.05	0.10	0.05	1.67
4	1.71	4.94	11.60	0.21	1.72
8	3.00	4.35	8.40	0.19	1.95
28	0.86	3.22	8.01	0.17	1.96
16	5.57	7.43	14.41	0.15	2.16
6	0.86	4.67	6.99	0.16	2.24
24	4.29	7.97	8.31	0.38	2.78
22	0.86	4.84	7.06	0.18	2.85
25	2.57	6.02	3.47	0.24	2.85
26	5.57	4.35	3.73	0.21	2.97
14	0.86	3.16	3.15	0.11	3.04
12	0.86	5.86	7.94	0.32	3.12
7	9.00	6.35	15.92	0.20	3.33
9	1.29	5.32	3.65	0.22	3.45
10	9.00	6.35	7.29	0.25	3.71
13	9.00	5.43	9.98	0.33	3.79
5	1.29	6.56	3.49	0.33	4.26
3	2.57	5.54	6.28	0.41	4.62
20	3.00	6.19	8.59	0.46	4.67
2	1.71	5.81	5.01	0.41	4.83

Table 7 Interpretation of parameter estimates for all villages.

recorded per household in our experiments (which is equal to fourth column of Table 1 divided by its third column). The table is sorted such that the values in the last column appear in an acending order.

We see in Table 7 that in almost all villages, the charge in the lamp is consumed within a week. In villages with relatively low recharge rates, consumers either place a low valuation on lamp's light (e.g., villages 1, 27, and 11) or take too long to accrue money for the recharge (e.g., villages 4, 8, 28, and 16). When consumers look far ahead while accounting for costs, they tend to recharge relatively more often even when it takes longer for them to accumulate money for the recharge (e.g., villages 16, 7, and 13).[17]

Appendix E: Elasticity of Expected Number of Recharges

We examine the impact of variables I, P, and Q on the expected number of recharges by computing the latter's elasticity with respect to (wrt) those variables. For a *representative consumer*, we write the expected number of recharges under the treatment (I,P,Q) as

$$\mathcal{R}(I,P,Q) = \sum_{v\in\mathfrak{V}} \mathbb{E}\tilde{R}_{jv}(I,P,Q) \times |\mathfrak{J}(v)| / \textstyle\sum_{i\in\mathfrak{V}} |\mathfrak{J}(i)|,$$

where the first term of the summand is the expected number of recharges for a representative consumer j in village v and the second term is the (sample) probability that the consumer is from village v. We approximate

[17] The values of $\hat{\mu}$ and $\hat{\sigma}$ for villages 1, 4, and 26 in Table 6 may seem to be outliers, yet we note that those estimates result in reasonable hitting time values (as can be seen in Table 7) and reasonable probability values (e.g., the value of $v(1,1,0\,;\,P=100)$ for villages 1, 4, and 26 are 0.80, 0.29, and 0.47 respectively). A higher σ simply indicates that there is substantial uncertainty in the consumer's life with regard to (in other words, there can be huge swings in) the money that she can spend on lamp recharges. A negative μ indicates that the consumer's disposable income is usually low (but not negative, because we modeled the log of the income process).

expectations with sample averages in our simulations. We generate $\hat{R}_{jv,n}$ in simulation round n using (i) the decision process in Figure 1, (ii) the treatment condition (I, P, Q), and (iii) the probability models under specification S_4 with the estimated parameters $\hat{\Theta}(v)$. Then, $\mathbb{E}\tilde{R}_{jv} \approx \hat{R}_{jv,n}/N_s$.

To compute elasticities, we take the values of I, P, and Q, respectively, from the sets $\mathfrak{I} = \{0, 0.1, \ldots, 1.2\}$ kilometers, $\mathfrak{P} = \{0, 10, \ldots, 120\}$ RWF and $\mathfrak{Q} = \{2, 4, \ldots, 30\}$ hours. We denote the $\mathfrak{k}^{\text{th}}$ element of set $\mathfrak{I}$ as $I^{(\mathfrak{k})}$ and define for $1 \le \mathfrak{k} \le |\mathfrak{I}| - 1$:

$$e_I(I^{(\mathfrak{k})}; P, Q) = \frac{\mathcal{R}(I^{(\mathfrak{k}+1)}, P, Q) - \mathcal{R}(I^{(\mathfrak{k})}, P, Q)}{I^{(\mathfrak{k}+1)} - I^{(\mathfrak{k})}} \times \frac{I^{(\mathfrak{k}+1)} + I^{(\mathfrak{k})}}{\mathcal{R}(I^{(\mathfrak{k}+1)}, P, Q) + \mathcal{R}(I^{(\mathfrak{k})}, P, Q)}, \quad \text{and} \tag{16}$$

$$\bar{e}_I(I^{(\mathfrak{k})}) = \frac{1}{|\mathfrak{P}| \times |\mathfrak{Q}|} \sum_{P \in \mathfrak{P}} \sum_{Q \in \mathfrak{Q}} e_I(I^{(\mathfrak{k})}; P, Q). \tag{17}$$

By fixing price and capacity values at P and Q respectively, (16) computes the arc elasticity of recharges for the pair $I^{(\mathfrak{k})}$ and $I^{(\mathfrak{k}+1)}$. (Although this elasticity is a function of the pair $(I^{(\mathfrak{k})}, I^{(\mathfrak{k}+1)})$, we denote it as $e_I(I^{(\mathfrak{k})}; P, Q)$ for notational simplicity.) In (17), $\bar{e}_I(I^{(\mathfrak{k})})$ is obtained by averaging the elasticities in (16) across different values of P and Q. In a similar manner, we compute the elasticities $\bar{e}_P$ wrt P and $\bar{e}_Q$ wrt Q.

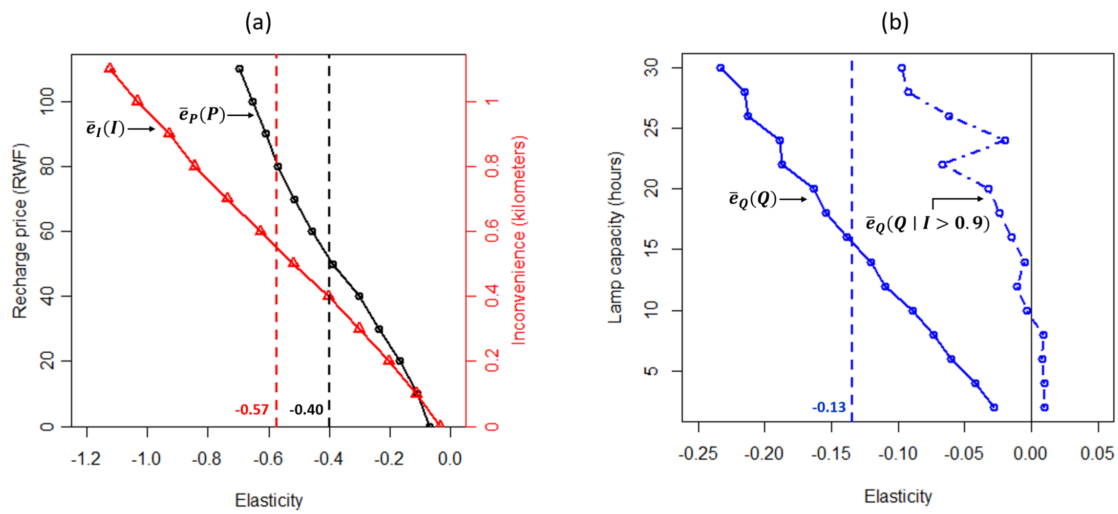

Figure 10 The average elasticities of expected number of recharges with respect to (a) recharge price and inconvenience, and (b) lamp capacity.

Figure 10(a) displays the values of $\bar{e}_P$ and $\bar{e}_I$. Because recharges decrease in P and I, $\bar{e}_P$ and $\bar{e}_I$ are always negative. Interestingly, in the price range shown, the expected number of recharges is on average *relatively inelastic* wrt price, i.e., $\bar{e}_P(P) > -1 \, \forall P \in \mathfrak{P}$. (This is not an artefact of any modeling assumptions. All price elasticities calculated from the raw data in Figure 3(c) are also greater than -1.) The essential nature of need for light and the lack of any attractive substitutes for cleaner light may have contributed to the relative inelasticity of recharges wrt price.

To assess the relative impact of price and inconvenience, we now compare the values of $\bar{e}_P$ and $\bar{e}_I$. Given that the consumers are cash constrained, we might expect the consumers to react more strongly to changes in price than to changes in inconvenience. However, we see that the average elasticity wrt inconvenience is $\sum_{I \in \mathfrak{I}} \bar{e}_I(I)/|\mathfrak{I}| = -0.57$, which is slightly higher (in absolute terms) than the average elasticity wrt price $\sum_{P\in\mathfrak{P}} \bar{e}_P(P)/|\mathfrak{P}| = -0.40$, thereby reflecting the significant opportunity cost in recharging the lamp. Although an average measure is sensitive to the range over which the average is taken, the ranges of price and inconvenience considered here are reasonable as they are in accord with our experimental data.

An alternative way to compare the elasticities is by considering the marginal measures. A consumer living 200 meters away from the recharge center responds to a 1% increase in I by dropping her recharges by 0.2%, which, as seen in Figure 10(a), is same as the response to a 1% increase in P of a consumer facing a price of ∼25 RWF. Similarly, $-0.4 = \bar{e}_I$(400 meters) $= \bar{e}_P$(50 RWF), and $-0.6 = \bar{e}_I$(600 meters) $= \bar{e}_P$(90 RWF). A 0.8% drop in recharges with a 1% increase in I is seen for a consumer living 750 meters away, but there is no price in $\mathfrak{P}$ at which an elasticity of -0.8 is observed. Given such strong response of the (representative) consumer to changes in I, we infer that inconvenience is a strong lever (and plausibly stronger than price in some cases) in terms of moving the expected number of recharges.

Figure 10(b) shows the values of $\bar{e}_Q$. On average, the elasticity of recharges wrt capacity is negative. A 1% increase in lamp capacity decreases the number of recharges, on average, by 0.13%. Compared to the aforementioned average elasticities wrt I and P, this estimate is lower because $\bar{e}_Q$ need not always be negative. As we discussed under Π_3 in Section 4.2, at higher values of I, reducing capacity decreases recharges because the lower capacity results in a higher number of overly inconvenient trips. Consequently, as can be seen in Figure 10(b), the elasticity wrt capacity can be positive for high values of inconvenience.

It is important to note that the values of elasticities shed light on how the expected number of recharges reacts to changes in I, P, and Q, but they do not give any *actionable insights* to the firm. Consider the following consequential questions: (i) We find that recharges react strongly to changes in I; however, unlike P and Q, I cannot be changed arbitrarily by the firm – it needs to be varied by making changes in the business model; how can the firm make those changes, and what is their impact on recharges? (ii) We see that decreasing P increases recharges by alleviating the consumer's liquidity constraints; however, given the relative inelasticity of recharges wrt price, dropping the price also plausibly drops revenue; are there ways to alleviate liquidity constraints without negatively affecting revenue? (iii) We see that the elasticity wrt Q can be both positive and negative, and hence it must also be equal to zero at some capacity values; given the distribution of consumer preferences in the market, is there an optimal capacity? Such questions are answered through an extensive business model analysis in Section 5.

Appendix F: Bellman Equations

Here, we present the Bellman equations for all the counterfactuals discussed in the main text. We write the equations only for $t \le T$, and in all the cases the cost $C(T+n;\cdot) = 0\,\forall\, n \ge 1$. Moreover, we assume that the discount factor $\delta = 1$ in the equations; they can be easily extended to any arbitrary $\delta \in [0,1)$. For notational simplicity, we denote $\tau = t - l$ in the equations.

Every fourth recharge free: The decision process when the firm offers every fourth recharge for free is shown in Figure 11(a). The variable $\mathbb{F}_t$ keeps track of the number of recharges done by the consumer *after* the previous free recharge was availed. Therefore, $\mathbb{F}_t \in \{0,1,2,3\}$. If $\mathbb{F}_t < 3$, then the decision process coincides with that in Figure 1. When $\mathbb{F}_t$ hits 3, the consumer becomes eligible for a free recharge, and hence her

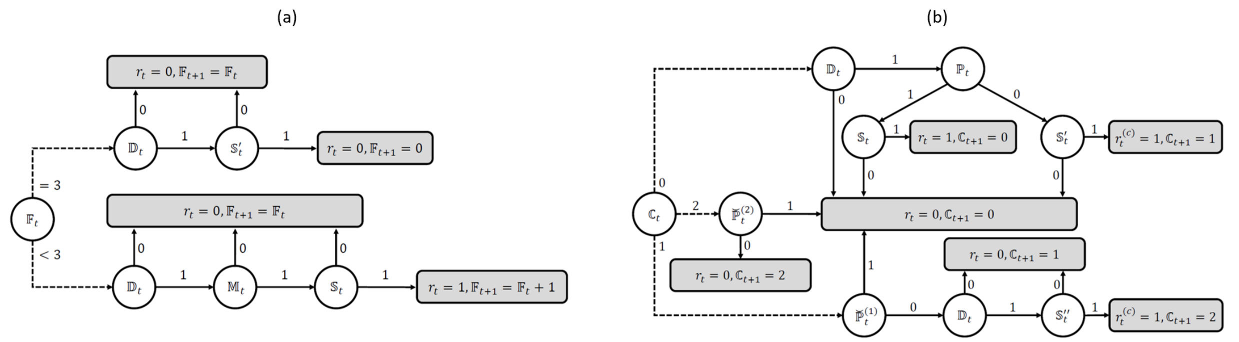

Figure 11 **Decision process when (a) every fourth recharge is free, and (b) two recharges are allowed on credit.**

liquidity constraint disappears (i.e., $\mathbb{M}_t = 1$ until the next recharge). Before the free recharge is availed, $\mathbb{F}_t$ increments by one with every recharge, and $\mathbb{F}_t$ resets to zero after the free recharge. The Bellman equations for this decision process are given below. The state space now includes an extra variable i to keep track of $\mathbb{F}_t$. Below, $i \in \{0,1\}$.

$$
\begin{aligned}
C(t,b,0,l,i) &= b + v(\tau+1,0,0)\bar{C}(t+1,0,l,i) + v(\tau+1,1,0)\bar{C}(t+1,1,l,i),\\
C(t,b,1,l,i) &= \min\Big\{\alpha I + \sum_{q\in\mathcal{Q}} u_q\Big[v_q\bar{C}(t+q,1,t,i+1) + (1-v_q)\bar{C}(t+q,0,t,i+1)\Big],\\
&\qquad b + v(\tau+1,0,1)\bar{C}(t+1,0,l,i) + v(\tau+1,1,1)\bar{C}(t+1,1,l,i)\Big\},\\
C(t,b,0,l,2) &= b + v(\tau+1,0,0)\bar{C}(t+1,0,l,2) + v(\tau+1,1,0)\bar{C}(t+1,1,l,2),\\
C(t,b,1,l,2) &= \min\Big\{\alpha I + \sum_{q\in\mathcal{Q}} u_q\,\bar{C}(t+q,1,t,3),\\
&\qquad b + v(\tau+1,0,1)\bar{C}(t+1,0,l,2) + v(\tau+1,1,1)\bar{C}(t+1,1,l,2)\Big\},\\
C(t,b,0,l,3) &= b + \bar{C}(t+1,1,l,3),\\
C(t,b,1,l,3) &= \min\Big\{\alpha I + \sum_{q\in\mathcal{Q}} u_q\Big[v_q\bar{C}(t+q,1,t,0) + (1-v_q)\bar{C}(t+q,0,t,0)\Big], b + \bar{C}(t+1,1,l,3)\Big\}.
\end{aligned}
$$

Periodic-visit model: The VLE visits once every n days. Instead of $\mathbb{V}_t$, we keep track of the variable k – the number of days left to the VLE's visit – in the state space. The cost function is given by $C(t,b,\mathbb{d},\mathbb{m},l,k)$, where $\mathbb{d}\in\{0,1\}$ is the lamp's discharge status in period t, and the other variables in the state space are as in the rest of the paper. In the equations below, $k\in\{1,\dots,n-1\}$.

$$
\begin{aligned}
C(t,b,0,\mathbb{m},l,k) &= \sum_{(\mathbb{d}',\mathbb{m}')\in\{0,1\}^2} u(\tau+1,\mathbb{d}',0)v(\tau+1,\mathbb{m}',\mathbb{m})\bar{C}(t+1,\mathbb{d}',\mathbb{m}',l,k-1),\\
C(t,b,1,0,l,k) &= b + \sum_{\mathbb{m}'\in\{0,1\}} v(\tau+1,\mathbb{m}',0)\bar{C}(t+1,1,\mathbb{m}',l,k-1),\\
C(t,b,1,1,l,k) &= \min\Big\{\alpha I + \sum_{(\mathbb{d}',\mathbb{m}')\in\{0,1\}^2} u(1,\mathbb{d}',0)v(1,\mathbb{m}',0)\bar{C}(t+1,\mathbb{d}',\mathbb{m}',t,k-1),\\
&\qquad b + \sum_{\mathbb{m}'\in\{0,1\}} v(\tau+1,\mathbb{m}',1)\bar{C}(t+1,1,\mathbb{m}',l,k-1)\Big\},\\
C(t,b,0,0,l,0) &= \sum_{(\mathbb{d}',\mathbb{m}')\in\{0,1\}^2} u(\tau+1,\mathbb{d}',0)v(\tau+1,\mathbb{m}',0)\bar{C}(t+1,\mathbb{d}',\mathbb{m}',l,n-1),
\end{aligned}
$$

$$C(t,b,1,0,l,0) = b + \sum_{\mathbb{m}'\in\{0,1\}} v(\tau+1,\mathbb{m}',0)\bar{C}(t+1,1,\mathbb{m}',l,n-1),$$

$$C(t,b,0,1,l,0) = \min\Big\{ \sum_{(\mathbb{d}',\mathbb{m}')\in\{0,1\}^2} u(1,\mathbb{d}',0)v(1,\mathbb{m}',0)\bar{C}(t+1,\mathbb{d}',\mathbb{m}',t,n-1), \sum_{(\mathbb{d}',\mathbb{m}')\in\{0,1\}^2} u(\tau+1,\mathbb{d}',0)v(\tau+1,\mathbb{m}',1)\bar{C}(t+1,\mathbb{d}',\mathbb{m}',l,n-1)\Big\},$$

$$C(t,b,1,1,l,0) = \min\Big\{ \sum_{(\mathbb{d}',\mathbb{m}')\in\{0,1\}^2} u(1,\mathbb{d}',0)v(1,\mathbb{m}',0)\bar{C}(t+1,\mathbb{d}',\mathbb{m}',t,n-1), b + \sum_{\mathbb{m}'\in\{0,1\}} v(\tau+1,\mathbb{m}',1)\bar{C}(t+1,1,\mathbb{m}',l,n-1)\Big\}.$$

No liquidity constraints benchmark: Because the consumer does not experience any liquidity constraints, she need not keep track of her monetary status $\mathbb{m}$ and the last recharge point l in the state space. The Bellman equations simplify to

$$C(t,b) = \min\Big\{\alpha I + \sum_{q\in\mathcal{Q}} u_q\,\bar{C}(t+q),\ b+\bar{C}(t+1)\Big\}.$$

Allowing partial recharges: Instead of $\mathbb{M}_t$, the consumer's cost function now keeps track of the state variable $(\mathbb{M}_t^{(0.5)},\mathbb{M}_t^{(1)})$, whose possible values are $(0,0)$, $(1,0)$, and $(1,1)$. Analogous to $v(\tau,\mathbb{m},\mathbb{m}')$ and v_q, we define $w(\tau,(\mathbb{m}^{(0.5)},\mathbb{m}^{(1)}),(\mathbb{m}'^{(0.5)},\mathbb{m}'^{(1)}))$ and $w_q(\mathbb{m}^{(0.5)},\mathbb{m}^{(1)})$ as the probability transition functions for $(\mathbb{M}_t^{(0.5)},\mathbb{M}_t^{(1)})$. The expressions for w and w_q can be derived from the empirical model for $\mathbb{M}_t$ discussed in Section 4.3, and are given as follows: $w_q(0,0)=G_q(\log P/2)$ and $w_q(1,0)=G_q(\log P)-G_q(\log P/2)$; $w(1,(0,0),\cdot)=G_1(\log P/2)$ and $w(1,(1,0),\cdot)=G_1(\log P)-G_1(\log P/2)$; for relative time period $\tau>1$,

$$w(\tau,(0,0),(0,0)) = \frac{1}{G_{\tau-1}(\log P/2)}\int_{-\infty}^{\log P/2}\Phi\Big(\frac{\log P/2-\rho x-\mu}{\sigma}\Big)\,dG_{\tau-1}(x),$$

$$w(\tau,(1,0),(0,0)) = \frac{1}{G_{\tau-1}(\log P/2)}\int_{-\infty}^{\log P/2}\Big\{\Phi\Big(\frac{\log P-\rho x-\mu}{\sigma}\Big)-\Phi\Big(\frac{\log P/2-\rho x-\mu}{\sigma}\Big)\Big\}\,dG_{\tau-1}(x),$$

$$w(\tau,(0,0),(1,0)) = \frac{1}{G_{\tau-1}(\log P)-G_{\tau-1}(\log P/2)}\int_{\log P/2}^{\log P}\Phi\Big(\frac{\log P/2-\rho x-\mu}{\sigma}\Big)\,dG_{\tau-1}(x),$$

$$w(\tau,(1,0),(1,0)) = \frac{1}{G_{\tau-1}(\log P)-G_{\tau-1}(\log P/2)}\int_{\log P/2}^{\log P}\Big\{\Phi\Big(\frac{\log P-\rho x-\mu}{\sigma}\Big) - \Phi\Big(\frac{\log P/2-\rho x-\mu}{\sigma}\Big)\Big\}\,dG_{\tau-1}(x),$$

$$w(\tau,(0,0),(1,1)) = \frac{1}{\bar{G}_{\tau-1}(\log P)}\int_{\log P}^{\infty}\Phi\Big(\frac{\log P/2-\rho x-\mu}{\sigma}\Big)\,dG_{\tau-1}(x),$$

$$w(\tau,(1,0),(1,1)) = \frac{1}{\bar{G}_{\tau-1}(\log P)}\int_{\log P}^{\infty}\Big\{\Phi\Big(\frac{\log P-\rho x-\mu}{\sigma}\Big)-\Phi\Big(\frac{\log P/2-\rho x-\mu}{\sigma}\Big)\Big\}\,dG_{\tau-1}(x).$$

In addition, we denote by $u_q^{(0.5)}$ the probability that a half-charged lamp lasts for q periods and $u_q^{(1)}=u_q$ as expressed in Section 4.3. Then, the Bellman equations are given as follows:

$$C(t,b,(0,0),l) = b + w(\tau+1,(0,0),(0,0))\bar{C}(t+1,(0,0),l) + w(\tau+1,(1,0),(0,0))\bar{C}(t+1,(1,0),l) + w(\tau+1,(1,1),(0,0))\bar{C}(t+1,(1,1),l),$$

$$C(t,b,(1,0),l) = \min\Big\{\alpha I + \sum_{q\in\mathcal{Q}} u_q^{(0.5)}\Big[w_q(0,0)\bar{C}(t+q,(0,0),t) + w_q(1,0)\bar{C}(t+q,(1,0),t)$$

$$
\begin{aligned}
&\qquad\qquad + w_q(1,1)\bar{C}(t+q,(1,1),t)\Big],\\
&\quad b + w(\tau+1,(0,0),(1,0))\bar{C}(t+1,(0,0),l) + w(\tau+1,(1,0),(1,0))\bar{C}(t+1,(1,0),l)\\
&\qquad\qquad + w(\tau+1,(1,1),(1,0))\bar{C}(t+1,(1,1),l)\Big\},\\
C(t,b,(1,1),l) = \min\Big\{&\alpha I + \sum_{q\in\mathcal{Q}} u_q^{(1)}\Big[w_q(0,0)\bar{C}(t+q,(0,0),t) + w_q(1,0)\bar{C}(t+q,(1,0),t)\\
&\qquad\qquad + w_q(1,1)\bar{C}(t+q,(1,1),t)\Big],\\
&\alpha I + \sum_{q\in\mathcal{Q}} u_q^{(0.5)}\Big[w_q(0,0)\bar{C}(t+q,(0,0),t) + w_q(1,0)\bar{C}(t+q,(1,0),t)\\
&\qquad\qquad + w_q(1,1)\bar{C}(t+q,(1,1),t)\Big],\\
&b + w(\tau+1,(0,0),(1,1))\bar{C}(t+1,(0,0),l) + w(\tau+1,(1,0),(1,1))\bar{C}(t+1,(1,0),l)\\
&\qquad\qquad + w(\tau+1,(1,1),(1,1))\bar{C}(t+1,(1,1),l)\Big\}.
\end{aligned}
$$

Prepayment: The differences between the Bellman equations under Figure 1 and the prepayment model are: (i) v_q is replaced by $\breve{w}_q$, where $\breve{w}_q = \Pr(\mathbb{P}_t = 1 \mid \mathbb{P}_0 = 0) = \Pr(\tilde{h}(P) \le q)$, i.e., the probability that the hitting time, as defined in (14), is less than or equal to q periods; and (ii) the second term within braces in the first equation below does not incorporate the possibility of losing the accrued money for the recharge, because once the income hits the threshold P, the payment is done through mobile money. Correspondingly the Bellman equations are given by

$$
\begin{aligned}
C(t,b,1,l) &= \min\Big\{\alpha I + \sum_{q\in\mathcal{Q}} u_q\big[\breve{w}_q\bar{C}(t+q,1,t) + (1-w_q)\bar{C}(t+q,0,t)\big],\ b + \bar{C}(t+1,1,l)\Big\},\\
C(t,b,0,l) &= b + v(t-l+1,1,0)\bar{C}(t+1,1,l) + v(t-l+1,0,0)\bar{C}(t+1,0,l).
\end{aligned}
$$

Prepayment after discharge: Unlike the prepayment model discussed above, the consumer here pays for the recharge only after the lamp is discharged; therefore, v_q is not replaced by $\breve{w}_q$ under this model. Consequently, the cost functions are written as

$$
\begin{aligned}
C(t,b,1,l) &= \min\Big\{\alpha I + \sum_{q\in\mathcal{Q}} u_q\big[v_q\bar{C}(t+q,1,t) + (1-v_q)\bar{C}(t+q,0,t)\big],\ b + \bar{C}(t+1,1,l)\Big\},\\
C(t,b,0,l) &= b + v(t-l+1,1,0)\bar{C}(t+1,1,l) + v(t-l+1,0,0)\bar{C}(t+1,0,l).
\end{aligned}
$$

One recharge on credit: Here, the cost function is denoted as $C(t,b,\mathbb{d},l_r,l_p,s)$, wherein l_r is the last *recharge* point and l_p is the last *payment* point; because the recharges and payments are decoupled under this model, we need to keep track of l_r and l_p separately, which earlier coincided with a single variable l in other settings. The additional state variable s indicates the debt status of the consumer: (i) $s=1$ indicates that there is a debt of one recharge, (ii) $s=0$ indicates that there is no debt and the money for the next recharge is not paid, and (iii) $s=0'$ indicates that there is no debt and the money for the next recharge is paid. The cost functions are given by

$$
\begin{aligned}
C(t,b,1,l_r,l_p,1) &= b + v(t-l_p+1,0,0)\bar{C}(t+1,1,l_r,l_p,1) + v(t-l_p+1,1,0)\bar{C}(t+1,1,l_r,t+1,0),\\
C(t,b,0,l_r,l_p,1) &= \sum_{\mathbb{d}'\in\{0,1\}} u(t-l_r+1,\mathbb{d}',0)v(t-l_p+1,0,0)\bar{C}(t+1,\mathbb{d}',l_r,l_p,1)
\end{aligned}
$$

$$+\sum_{\mathbb{d}'\in\{0,1\}} u(t-l_r+1,\mathbb{d}',0)v(t-l_p+1,1,0)\bar{C}(t+1,\mathbb{d}',l_r,t+1,0),$$

$$C(t,b,1,l_r,l_p,0)=\min\Big\{b+v(t-l_p+1,0,0)\bar{C}(t+1,1,l_r,l_p,0)+v(t-l_p+1,1,0)\bar{C}(t+1,1,l_r,t+1,0'),$$

$$\alpha I+\sum_{\mathbb{d}'\in\{0,1\}} u(1,\mathbb{d}',0)v(t-l_p+1,0,0)\bar{C}(t+1,\mathbb{d}',t,l_p,1)$$

$$+\sum_{\mathbb{d}'\in\{0,1\}} u(1,\mathbb{d}',0)v(t-l_p+1,1,0)\bar{C}(t+1,\mathbb{d}',t,t+1,0)\Big\},$$

$$C(t,b,0,l_r,l_p,0)=\sum_{\mathbb{d}'\in\{0,1\}} u(t-l_r+1,\mathbb{d}',0)v(t-l_p+1,0,0)\bar{C}(t+1,\mathbb{d}',l_r,l_p,0)$$

$$+\sum_{\mathbb{d}'\in\{0,1\}} u(t-l_r+1,\mathbb{d}',0)v(t-l_p+1,1,0)\bar{C}(t+1,\mathbb{d}',l_r,t+1,0'),$$

$$C(t,b,1,l_r,l_p,0')=\min\Big\{\alpha I+\sum_{\mathbb{d}'\in\{0,1\}} u(1,\mathbb{d}',0)v(1,0,0)\bar{C}(t+1,\mathbb{d}',t,t,0)$$

$$+\sum_{\mathbb{d}'\in\{0,1\}} u(1,\mathbb{d}',0)v(1,1,0)\bar{C}(t+1,\mathbb{d}',t,t+1,0'),b+\bar{C}(t+1,1,l_r,l_p,0')\Big\},$$

$$C(t,b,0,l_r,l_p,0')=\sum_{\mathbb{d}'\in\{0,1\}} u(t-l_r+1,\mathbb{d}',0)\bar{C}(t+1,\mathbb{d}',l_r,l_p,0').$$

Two recharges on credit: The decision process when the consumer is allowed to recharge two times on credit is shown in Figure 11(b). As in the previous model, the cost function is denoted as $C(t,b,\mathbb{d},l_r,l_p,s)$, where l_r and l_s are the last recharge and payment points respectively, and s indicates the debt status of the consumer: (i) $s=2$ indicates that there is a debt of two recharges, (ii) $s=1'$ indicates that there is a debt of two recharges, of which one is paid for, (iii) $s=1$ indicates that there is a debt of one recharge, (iv) $s=0$ indicates that there is no debt and the money for the next recharge is not paid, and (v) $s=0'$ indicates that there is no debt and the money for the next recharge is paid. The corresponding Bellman equations are:

$$C(t,b,1,l_r,l_p,2)=b+v(t-l_p+1,0,0)\bar{C}(t+1,1,l_r,l_p,2)+v(t-l_p+1,1,0)\bar{C}(t+1,1,l_r,t+1,1'),$$

$$C(t,b,0,l_r,l_p,2)=\sum_{\mathbb{d}'\in\{0,1\}} u(t-l_r+1,\mathbb{d}',0)v(t-l_p+1,0,0)\bar{C}(t+1,\mathbb{d}',l_r,l_p,2)$$

$$+\sum_{\mathbb{d}'\in\{0,1\}} u(t-l_r+1,\mathbb{d}',0)v(t-l_p+1,1,0)\bar{C}(t+1,\mathbb{d}',l_r,t+1,1'),$$

$$C(t,b,1,l_r,l_p,1')=b+v(t-l_p+1,0,0)\bar{C}(t+1,1,l_r,l_p,1')+v(t-l_p+1,1,0)\bar{C}(t+1,1,l_r,t+1,0),$$

$$C(t,b,0,l_r,l_p,1')=\sum_{\mathbb{d}'\in\{0,1\}} u(t-l_r+1,\mathbb{d}',0)v(t-l_p+1,0,0)\bar{C}(t+1,\mathbb{d}',l_r,l_p,1')$$

$$+\sum_{\mathbb{d}'\in\{0,1\}} u(t-l_r+1,\mathbb{d}',0)v(t-l_p+1,1,0)\bar{C}(t+1,\mathbb{d}',l_r,t+1,0),$$

$$C(t,b,1,l_r,l_p,1)=\min\Big\{b+v(t-l_p+1,0,0)\bar{C}(t+1,1,l_r,l_p,1)+v(t-l_p+1,1,0)\bar{C}(t+1,1,l_r,t+1,0),$$

$$\alpha I+\sum_{\mathbb{d}'\in\{0,1\}} u(1,\mathbb{d}',0)v(t-l_p+1,0,0)\bar{C}(t+1,\mathbb{d}',t,l_p,2)$$

$$+\sum_{\mathbb{d}'\in\{0,1\}} u(1,\mathbb{d}',0)v(t-l_p+1,1,0)\bar{C}(t+1,\mathbb{d}',t,t+1,1')\Big\},$$

$$C(t,b,0,l_r,l_p,1)=\sum_{\mathbb{d}'\in\{0,1\}} u(t-l_r+1,\mathbb{d}',0)v(t-l_p+1,0,0)\bar{C}(t+1,\mathbb{d}',l_r,l_p,1)$$

$$+\sum_{\mathbb{d}'\in\{0,1\}} u(t-l_r+1,\mathbb{d}',0)v(t-l_p+1,1,0)\bar{C}(t+1,\mathbb{d}',l_r,t+1,0),$$

$$
\begin{aligned}
C(t,b,1,l_r,l_p,0) = \min\Big\{&b + v(t-l_p+1,0,0)\bar{C}(t+1,1,l_r,l_p,0) + v(t-l_p+1,1,0)\bar{C}(t+1,1,l_r,t+1,0'),\\
&\alpha I + \sum_{\mathbb{d}'\in\{0,1\}} u(1,\mathbb{d}',0)v(t-l_p+1,0,0)\bar{C}(t+1,\mathbb{d}',t,l_p,1)\\
&+ \sum_{\mathbb{d}'\in\{0,1\}} u(1,\mathbb{d}',0)v(t-l_p+1,1,0)\bar{C}(t+1,\mathbb{d}',t,t+1,0)\Big\},\\
C(t,b,0,l_r,l_p,0) = &\sum_{\mathbb{d}'\in\{0,1\}} u(t-l_r+1,\mathbb{d}',0)v(t-l_p+1,0,0)\bar{C}(t+1,\mathbb{d}',l_r,l_p,0)\\
&+ \sum_{\mathbb{d}'\in\{0,1\}} u(t-l_r+1,\mathbb{d}',0)v(t-l_p+1,1,0)\bar{C}(t+1,\mathbb{d}',l_r,t+1,0'),\\
C(t,b,1,l_r,l_p,0') = \min\Big\{&\alpha I + \sum_{\mathbb{d}'\in\{0,1\}} u(1,\mathbb{d}',0)v(1,0,0)\bar{C}(t+1,\mathbb{d}',t,t,0)\\
&+ \sum_{\mathbb{d}'\in\{0,1\}} u(1,\mathbb{d}',0)v(1,1,0)\bar{C}(t+1,\mathbb{d}',t,t+1,0'), b + \bar{C}(t+1,1,l_r,l_p,0')\Big\},\\
C(t,b,0,l_r,l_p,0') = &\sum_{\mathbb{d}'\in\{0,1\}} u(t-l_r+1,\mathbb{d}',0)\bar{C}(t+1,\mathbb{d}',l_r,l_p,0').
\end{aligned}
$$

Appendix G: Proofs

Proof of Proposition 1. From (4), we know that the threshold $k(t,l)$ is given by

$$
k(t,l) = \alpha I \; - \sum_{\mathbb{m}\in\{0,1\}} v(t-l+1,\mathbb{m},1)\,\bar{C}(t+1,\mathbb{m},l) \; + \sum_{q\in\mathcal{Q}} u_q\big[v_q\bar{C}(t+q,1,t) + (1-v_q)\bar{C}(t+q,0,t)\big]. \quad (18)
$$

We then obtain the following expected cost functions using (3), (5), and (18):

$$
\bar{C}(t,1,l) = \mathbb{E}\min\{k(t,l),\tilde{\beta}\} + v(t-l+1,1,1)\bar{C}(t+1,1,l) + v(t-l+1,0,1)\bar{C}(t+1,0,l), \quad (19)
$$

$$
\bar{C}(t,0,l) = \beta + v(t-l+1,1,0)\bar{C}(t+1,1,l) + v(t-l+1,0,0)\bar{C}(t+1,0,l). \quad (20)
$$

If we define $\kappa(t,l,\mathbb{m}) = v(t-l,1,\mathbb{m})\bar{C}(t,1,l) + v(t-l,0,0)\bar{C}(t,0,l)$ and substitute it back in (18), (19), and (20), then we obtain the expression for the theshold as given in the statement of the proposition. $\square$

Proof of Proposition 2. The following equalities are obtained from the definition of $\Omega(t,\mathbb{d},\mathbb{m})$. (We suppressed the arguments Θ and Γ in the probability expressions.)

$$
\begin{aligned}
\Omega(t,\mathbb{d},\mathbb{m}) &= \Pr(\tilde{\mathfrak{r}}\langle t\rangle = \mathfrak{r}\langle t\rangle, \mathbb{D}_t = \mathbb{d}, \mathbb{M}_t = \mathbb{m}\,;\,\mathfrak{l})\\
&= \sum_{\mathbb{d}'\in\{0,1\}}\sum_{\mathbb{m}'\in\{0,1\}} \Pr(\tilde{\mathfrak{r}}\langle t\rangle = \mathfrak{r}\langle t\rangle, \mathbb{D}_t = \mathbb{d}, \mathbb{M}_t = \mathbb{m}, \mathbb{D}_{t-1} = \mathbb{d}', \mathbb{M}_{t-1} = \mathbb{m}'\,;\,\mathfrak{l})\\
&= \sum_{\mathbb{d}'\in\{0,1\}}\sum_{\mathbb{m}'\in\{0,1\}} \Pr(\tilde{\mathfrak{r}}\langle t-1\rangle = \mathfrak{r}\langle t-1\rangle, \mathbb{D}_{t-1} = \mathbb{d}', \mathbb{M}_{t-1} = \mathbb{m}'\,;\,\mathfrak{l})\\
&\qquad \times \Pr(\tilde{r}_t = r_t, \mathbb{D}_t = \mathbb{d}, \mathbb{M}_t = \mathbb{m} \mid \tilde{\mathfrak{r}}\langle t-1\rangle = \mathfrak{r}\langle t-1\rangle, \mathbb{D}_{t-1} = \mathbb{d}', \mathbb{M}_{t-1} = \mathbb{m}'\,;\,\mathfrak{l})\\
&= \sum_{\mathbb{d}'\in\{0,1\}}\sum_{\mathbb{m}'\in\{0,1\}} \Omega(t-1,\mathbb{d}',\mathbb{m}') \times \Pr(\mathbb{D}_t = \mathbb{d} \mid \mathbb{D}_{t-1} = \mathbb{d}'\,;\,l_t)\\
&\qquad \times \Pr(\mathbb{M}_t = \mathbb{m} \mid \mathbb{M}_{t-1} = \mathbb{m}'\,;\,l_t) \times \Pr(\tilde{r}_t = r_t \mid \mathbb{D}_t = \mathbb{d}, \mathbb{M}_t = \mathbb{m}\,;\,l_t)\\
&= \sum_{\mathbb{d}'\in\{0,1\}}\sum_{\mathbb{m}'\in\{0,1\}} \Omega(t-1,\mathbb{d}',\mathbb{m}') \times u(t-l_t,\mathbb{d},\mathbb{d}')\\
&\qquad \times v(t-l_t,\mathbb{m},\mathbb{m}') \times \big[\mathbb{d}\mathbb{m}\bar{F}(k(t,l_t))\big]^{r_t}\big[1-\mathbb{d}\mathbb{m}\bar{F}(k(t,l_t))\big]^{1-r_t}.
\end{aligned}
$$

Here, the second equality follows from the law of total probability, the third equality from Bayes' law, and the fourth equality from recognizing that – conditional on $\mathfrak{l}$, other parameters and the previous period's states $\mathbb{d}'$ and $\mathbb{m}'$ – the evolution of processes $\mathbb{D}_t$ and $\mathbb{M}_t$ and the decision $\tilde{r}_t$ are independent of previous recharge decisions $\tilde{\mathfrak{r}}\langle t-1\rangle$. Finally, the realized recharge decision r_t is equal to one if and only if $\mathbb{d}=1$, $\mathbb{m}=1$, and the blackout cost is above the threshold $k(t,l_t)$; thus, $\Pr(\tilde{r}_t=1 \mid \mathbb{D}_t=\mathbb{d}, \mathbb{M}_t=\mathbb{m}\,; l_t)=\mathbb{d}\mathbb{m}\bar{F}(k(t,l_t))$. The expression of Ω for period $t=1$ can be obtained in a similar manner. □

Proof of Lemma 1. From the definition of m_τ, we obtain $m_\tau=\rho m_{\tau-1}+\epsilon_\tau=\epsilon_\tau+\rho\epsilon_{\tau-1}+\cdots+\rho^{\tau-1}\epsilon_1+m_0$. Since $\epsilon_t\sim \mathrm{N}(\mu,\sigma^2)$, the CDF of m_τ is given by G_τ as defined in the statement of Lemma 1. Then

$$\begin{aligned}
v(\tau,1,0)&=\Pr(\mathbb{M}_\tau=1 \mid \mathbb{M}_{\tau-1}=0)=\Pr(m_\tau\geq \log P \mid m_{\tau-1}<\log P)\\
&=\Pr(m_\tau\geq \log P, m_{\tau-1}<\log P)/\Pr(m_{\tau-1}<\log P)\\
&=\Pr(\epsilon_\tau\geq \log P-\rho m_{\tau-1}, m_{\tau-1}<\log P)/\Pr(m_{\tau-1}<\log P)\\
&=\frac{1}{G_{\tau-1}(\log P)}\int_{-\infty}^{\log P}\Pr(\epsilon_\tau\geq \log P-\rho x)\,dG_{\tau-1}(x),
\end{aligned}$$

which is same as the expression in (6). The expression in (7) can be derived in a similar manner. Moreover, $v_q=\Pr(\mathbb{M}_q=1 \mid \mathbb{M}_0=0)=\Pr(m_q\geq \log P)=\bar{G}_q(\log P)$. □

Proof of Lemma 2. The result trivially follows from the definitions of u and u_q, and the assumption $\tilde{N}-1\sim \text{Poisson}(Q\lambda)$. □

Proof of Lemma 3. We split the sequence $\{k_T,k_{T-1},\ldots\}$ as $\{k_T,\ldots,k_{T-q+1}\}\cup\{k_{T-q},k_{T-q-1},\ldots\}$. Since the first subsequence is of finite length, it suffices to show that the latter subsequence is convergent. We do that by showing that it is a bounded sequence. It then follows from the Bolzano-Weierstrass theorem that this sequence is convergent.

We first note that when $t\leq T-q$, we see from (9) that threshold k_t is a function of the next $q-1$ thresholds. We denote this function as ζ; hence,

$$k_t=\zeta(k_{t+1},\ldots,k_{t+q-1})=\alpha I-\sum_{i=1}^{q-1}\left\{v_\perp\mathbb{E}\min\{k_{t+i},\tilde{\beta}\}+(1-v_\perp)\beta\right\}=\alpha I-(q-1)\beta+v_\perp\sum_{i=1}^{q-1}\mathbb{E}[\tilde{\beta}-k_{t+i}]^+.$$

Because $\mathbb{E}\min\{k_{t+i},\tilde{\beta}\}\leq\beta$, $k_t\geq \alpha I-(q-1)\beta\equiv \underline{k}$, and because $\mathbb{E}[\tilde{\beta}-k_{t+i}]^+\leq\mathbb{E}[\tilde{\beta}-\underline{k}]^+$, $k_t\leq \underline{k}+v_\perp(q-1)\mathbb{E}[\tilde{\beta}-\underline{k}]^+\equiv\overline{k}$. Therefore, the sequence is bounded below by $\underline{k}$ and above by $\overline{k}$.

Given that the sequence is convergent, say its limit is $k_{-\infty}$. Then, because ζ is continuous in all its arguments, it follows that

$$\begin{aligned}
k_{-\infty}=\lim_{n\to\infty}k_{T-q-n}&=\lim_{n\to\infty}\zeta(k_{T-q-n+1},\ldots,k_{T-q-n+q-1})\\
&=\zeta(\lim_{n\to\infty}k_{T-q-n+1},\ldots,\lim_{n\to\infty}k_{T-q-n+q-1})=\zeta(k_{-\infty},\ldots,k_{-\infty}).
\end{aligned}$$

The fixed-point equation $k_{-\infty}=\zeta(k_{-\infty},\ldots,k_{-\infty})$ is same as (10). Therefore, the limit of the sequence $k_{-\infty}$ is same as the k^* that solves (10). It only remains to show that k^* exists and is unique. For that purpose, we define the following function:

$$\mathfrak{K}(k,I,v_\perp,q)=k-\alpha I+(q-1)\left\{v_\perp\mathbb{E}\min\{k,\tilde{\beta}\}+(1-v_\perp)\beta\right\}, \tag{21}$$

such that k^* is the solution to the implicit equation $\mathfrak{K}(k,\cdot)=0$. The function $\mathfrak{K}$ is increasing in k because $\partial\mathfrak{K}/\partial k = 1+(q-1)v_\perp \bar{F}(k)\geq 0$. Moreover, $\mathfrak{K}(\underline{k},\cdot) = -v_\perp(q-1)\mathbb{E}\big[\tilde{\beta}-\underline{k}\big]^+\leq 0$ and $\mathfrak{K}(\overline{k},\cdot)=v_\perp(q-1)\big\{\mathbb{E}\big[\tilde{\beta}-\underline{k}\big]^+ - \mathbb{E}\big[\tilde{\beta}-\overline{k}\big]^+\big\}\geq 0$. Thus, there exists a unique k^* that satisfies $\mathfrak{K}(k^*,\cdot)=0$. □

Proof of Proposition 3. By applying the implicit function theorem to (21), we obtain

$$\frac{\partial k^*}{\partial I} = -\frac{\partial\mathfrak{K}/\partial I}{\partial\mathfrak{K}/\partial k} = \frac{\alpha}{1+(q-1)v_\perp\bar{F}(k^*)}\geq 0, \tag{22}$$

$$\frac{\partial k^*}{\partial v_\perp} = -\frac{\partial\mathfrak{K}/\partial v_\perp}{\partial\mathfrak{K}/\partial k} = \frac{(q-1)\mathbb{E}[\tilde{\beta}-k^*]^+}{1+(q-1)v_\perp\bar{F}(k^*)}\geq 0, \tag{23}$$

$$\frac{\partial k^*}{\partial q} = -\frac{\partial\mathfrak{K}/\partial q}{\partial\mathfrak{K}/\partial k} = \frac{v_\perp\mathbb{E}[\tilde{\beta}-k^*]^+-\beta}{1+(q-1)v_\perp\bar{F}(k^*)} = \frac{k^*-\alpha I}{(q-1)(1+(q-1)v_\perp\bar{F}(k^*))}. \tag{24}$$

We see from (22) that $\bar{F}(k^*)$ is decreasing in I, and from (11) that Ψ is increasing in I; hence, $\mathcal{R}$ is decreasing in I. Next, using (23), we see that $v_\perp\bar{F}(k^*)$ is increasing in $v_\perp$:

$$\begin{aligned}\frac{\partial(v_\perp\bar{F}(k^*))}{\partial v_\perp} &= \bar{F}(k^*) - f(k^*)v_\perp\frac{(q-1)\mathbb{E}[\tilde{\beta}-k^*]^+}{1+(q-1)v_\perp\bar{F}(k^*)} \geq \bar{F}(k^*) - f(k^*)\frac{\mathbb{E}[\tilde{\beta}-k^*]^+}{\bar{F}(k^*)}\\ &= \int_{k^*}^{\infty}\bar{F}(s)\left[\frac{f(s)}{\bar{F}(s)}-\frac{f(k^*)}{\bar{F}(k^*)}\right]ds\geq 0.\end{aligned}$$

The last inequality is because the function $f/\bar{F}$ is increasing, wherein f is the density function of $\tilde{\beta}$. Thus, from (11), Ψ is decreasing in $v_\perp$. As $v_\perp$ is decreasing in P, $\mathcal{R}$ is also decreasing in P. To examine the behavior of Ψ wrt q, we note that

$$\frac{\partial\Psi}{\partial q} = 1+\frac{f(k^*)}{v_\perp\bar{F}(k^*)^2}\frac{\partial k^*}{\partial q} \tag{25}$$

$$= \left\{\frac{v_\perp\bar{F}(k^*)^2}{f(k^*)}(1+(q-1)v_\perp\bar{F}(k^*))-\beta+v_\perp\mathbb{E}[\tilde{\beta}-k^*]^+\right\}\times\frac{f(k^*)}{v_\perp\bar{F}(k^*)^2(1+(q-1)v_\perp\bar{F}(k^*)^2)}. \tag{26}$$

We see from (24) that whether k^* is increasing or decreasing in q depends on the sign of $k^*-\alpha I$. It is easy to verify from (22) that $k^*-\alpha I$ is decreasing in I and $\lim_{I\to\infty}k^*-\alpha I=-\infty$. Therefore, there exists a threshold $\hat{I}_1\geq 0$ such that for $I\geq\hat{I}_1$, k^* is decreasing in q and for $I<\hat{I}_1$, k^* is increasing in q. In the latter case, it follows from (25) that Ψ is increasing in q. Now it remains to examine what happens when k^* decreases in q.

The term outside the braces in (26) is positive; therefore, the sign of $\partial\Psi/\partial q$ depends only on the term inside the braces, which we denote as $\mathfrak{b}(q)$. We note that, when $I\geq\hat{I}_1$, $\mathfrak{b}(q)$ is increasing in q because (i) $\bar{F}$ is decreasing, (ii) $\bar{F}/f$ is decreasing, (iii) $\mathbb{E}[\tilde{\beta}-k]^+$ is decreasing in k, and (iv) k^* is decreasing in q. Moreover, $\lim_{q\to\infty}\mathfrak{b}(q)=\infty$ and $\lim_{q\to 1}\mathfrak{b}(q)=v_\perp\bar{F}(\alpha I)^2/f(\alpha I)-\beta+v_\perp\mathbb{E}[\tilde{\beta}-\alpha I]^+\equiv\mathfrak{z}(I)$. Furthermore, $\mathfrak{z}(I)$ is decreasing in I and $\lim_{I\to\infty}\mathfrak{z}(I)=-\beta<0$ (because when k^* is decreasing in q, from (24), $\beta> v_\perp\mathbb{E}[\tilde{\beta}-k^*]^+>0$). It follows that there exists a threshold $\hat{I}_2\geq 0$ such that (a) for $I\geq\hat{I}_2$, $\mathfrak{z}(I)=\lim_{q\to 1}\mathfrak{b}(q)$ is negative, and hence $\partial\Psi/\partial q$ single crosses the horizontal axis from below; in other words, Ψ is U-shaped in q, and $\mathcal{R}$ is unimodal in q, and (b) for $I<\hat{I}_2$, $\mathfrak{z}(I)=\lim_{q\to 1}\mathfrak{b}(q)$ is positive, Ψ is increasing in q and $\mathcal{R}$ is decreasing in q.

Overall, since $\hat{I}_2\geq\hat{I}_1\geq 0$, we define the threshold $\hat{I}=\hat{I}_2$, and conclude that $\mathcal{R}$ is unimodal in q when $I\geq\hat{I}$ and $\mathcal{R}$ is decreasing in q when $I<\hat{I}$. □

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