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How to deal with exchange rate risk in infrastructure and other long-lived projects

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Luciano de Castro[1], Claudio Frischtak[2], and Arthur Rodrigues[3]

Executive summary

Currency risk remains a significant deterrent to foreign investment flows to developing and emerging economies, particularly for long-duration infrastructure investments. Given the nature of such long-term investments, that take place in non-tradable sectors for which natural hedge is unlikely, hedging instruments are frequently costly, if at all available. Thus, either government assumes the implicit foreign exposure liability by providing guarantees, or contents itself with a smaller inflow of foreign capital. The fact that countries which are most in need of to build or modernize their infrastructure and attract foreign investment flows typically face balance of payment problems and a high opportunity cost of foreign exchange, makes the search for hedging substitutes a policy priority.

In this paper, we propose an alternative approach to address currency risk in infrastructure and long-term investments grounded on the notion that it would be more efficient for the government to provide a hedging or guarantee mechanism which adjusts the length of the concession through some publicized function defined ex-ante and dependent on the exchange rate (a "target function"). Not only the risk is symmetrical to both parties (government and the external provider of finance), insofar as an appreciation of the local currency leads to a shortened length of the concession and conversely, but more fundamentally, a risk-neutral government would be indifferent to the purely financial implications of providing the mechanism, even if it were not to charge for access. Moreover, considering the likely increase in foreign investors' willingness to bid for the countries' projects, the mechanism would in fact lead to gains on a social welfare function independent on how the government values consumer (user) surplus and efficiency of firms providing the service. This finding is intuitive to the extent that by providing hedge to suppliers of foreign capital, governments would attract a larger number of competitors willing to offer a lower tariff to consumers and/or greater revenues to government.

We initially present the government guarantee mechanism in its most general form, with a non-specific "target function" for each partner in the concession. The paper then discusses in greater detail three specific mechanisms: ensuring an exchange rate level, with the government guaranteeing a specific inflation-adjusted exchange rate; a guarantee of the net present value at the beginning of the franchise; and a guarantee of equal returns among (national and non-national) partners, while not committing to a specific return. Note that in

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all three instances, the mechanism is presented as an option, in principle at zero cost: at the contractual outset of the concession, the investor decides whether she wants protection or not, and what share of the project is to be insured. The mechanism is also adaptable to countries with limited regulatory resources, insofar as the tests to establish if the partner supplying capital in foreign currency should enjoy additional concession time (the "test function") can be conducted in notional (or proxy) values, thereby reducing information requirements from the concession to the government, and thereby minimizing the probability of (bilateral) opportunistic behavior, while eventually facilitating court decisions.

Finally, empirical tests use data from a real world 25-year highway concession and exchange rates from 8 countries – Mexico, Peru, Chile, Brazil, South Africa, Mozambique, Indonesia and the Philippines – to show that the variations in concession length resulting from a hypothetical guarantee are relatively small under alternative functions. Given the baseline of 25 years, it is striking that the adjustment is less than 4 years (that is, 16% of the project's length) for all countries. It is equally noteworthy that in six of the sixteen simulations performed, the government would receive back excess value at the end of the concession period. The results are similar and consistent across countries at different levels of development and exchange rate trajectories, with countries sharing only a relatively free-floating exchange rate regime.

The mechanism proposed, formalized and tested in this paper, would be a valuable addition to the toolkit of emerging and developing countries which aim to attract larger volumes of long-term investment for economic and social infrastructure projects which could be modelled as public-private partnerships, with investors remunerated over a set period of years under a concession contract. Such long-term commitments are a burden for many governments facing fiscal restrictions, but also limitations on investing (and delivering services) on an efficient basis due to poor project governance and management. By removing what is generally regarded as a binding entry barrier for investors - currency risk - the guarantee mechanism enables a greater participation of private capital in such projects. And it does so with positive social welfare gains under general assumptions. Firm and government preferences suggest low (or even negative) opportunity cost to providing guarantees against long-term exchange rate movements as illustrated by relatively small variations in concession length. Finally, the paper also contributes to the literature by drawing upon the consequences of the exchange rate stochastic processes being approximated by a martingale to provision of currency hedge by government. It provides a general framework for thinking about guarantee mechanisms and their properties, suggesting a potential large menu of functional forms for target and test functions.

How to deal with exchange rate risk in infrastructure and other long-lived projects

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Abstract

Most developing economies rely on foreign capital to finance their infrastructure needs. These projects are usually structured as long-term (25-35 year) franchises that pay in local currency. If investors evaluate their returns in terms of foreign currency, exchange rate volatility introduces risk that may reduce the level of investment below what would be socially optimal. In this paper, we propose a mechanism with very general features that hedges exchange rate fluctuation by adjusting the concession period. Such mechanism does not imply additional costs to the government and could be offered as a zero-cost option to lenders and investors exposed to currency fluctuations. We illustrate the general mechanism with three alternative specifications and use data from a 25-year highway franchise to simulate how they would play out in eight different countries that exhibit diverse exchange rate trajectories.

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1 Introduction

In most developing and emerging economies there is excess demand for infrastructure investment. Meanwhile, the stock of "public capital" (defined as the stock of assets in water, power, transportation and eventually telecoms) is significantly below what is required to provide universal services (estimated at 60% of GDP).[1] Moreover, in order to expand infrastructure investment, the government of these countries often face significant fiscal restrictions in addition to limited execution capability. Such constraints tend to be even more critical in poorer economies, with less developed capital markets, weaker institutions and stronger dependency on external flows, both public and private. Nonetheless, foreign capital – for both equity and debt — is also quite relevant for larger and more sophisticated economies. Modernization of the physical infrastructure is not only capital (and technology) intensive, but dependent on investors willing to commit for relatively long periods (anywhere from 10 to over 30 years), and therefore the ability to attract "patient" (institutional and long-term oriented) capital.

Investing in infrastructure involves constraints and risks which sometimes distort capital allocation away from areas with highest social rates of return and where investment is most needed.[2] Investors are driven away by poor governance – the absence of a stable legal and regulatory framework, a planning process which provides visibility for investors, and transparency in project identification, tender and monitoring. They are also deterred by macroeconomic instability, demand-related risks and the specific operational and technical risks. Nonetheless, to the extent that governments attempt to improve the governance of infrastructure investment, reduce macroeconomic policy volatility, while ensuring a well understood PPP model and auction design, they contribute to present investors a more attractive — or less uncertain — environment.

In recent years, governments and multilateral institutions have been engaged in attempts to reduce perceived and actual risks facing infrastructure investors. However, some risks are exogenous to both the private investor and the contracting (government) party, while the costs to minimize them are not known ex-ante by either party. Among the main risks, project (construction cost), demand and exchange rate stand out. Countries deal with these risks through contractual and other means (Table 1).

[1] In Kamps (2006), the average level of net capital stock among 22 OECD countries decreased from 57.8% (20.6) of GDP in 1980 to 51.4% (17.1) of GDP in 2000 (with standard deviation in parentheses). In view of increased private sector participation in infrastructure investment since 1980. some of this slack was in all probability taken up by the private sector, although the author does not offer estimates to this effect. Frischtak and Mourão (2017b) and Frischtak and Mourão (2017a) estimate both the current level of capital stock and that which would bring about universal coverage of services in power, telecom, transportation, and water in Brazil, with the later calculated as 60.4% of GDP.

[2] See An (2017) on the nature of the risks associated with infrastructure projects and instruments for risk mitigation, either through contractual arrangements or through instruments (for credit enhancement, co-financing, among others). See also Verdouw et al. (2015).

Table 1: A sample of country approaches to mitigate risk

	Risk to be mitigated		
	Construction	Demand	Exchange Rate
Mexico	Government secures right of way, provides detailed engineering design and a construction subsidy (CAI) in one concession model		
Chile		Least Present Value bid by concession winner**; MIG - Minimum Income (or traffic) Guarantee covers up to 70% of costs, adjusted for excess traffic or IRR.	For highway concessions in Chile, the government has in the past provided an exchange rate guarantee for the hard currency-denominated financing component.
Peru*	RPI-CAO mechanism: government payment obligations ensuring the private party repayment for construction costs *		RPI-CAO denominated in hard currency; USD indexed and inflation-adjusted rates
Brazil			Lump-sum payments due to the government during the concession period may be used to compensate exchange rate variations as they affect debt principal repayment, for debt taken in the first 5 years of the concession***.

Source: Own elaboration. (*) RPI-CAO mechanism: government payment obligations ensuring the private party repayment for construction costs, and O&M if falling below revenues. (***) It is a symmetric arrangement, and compensation may be due government.

For many countries, exchange rate risk is an effective deterrent, not only to greater external equity inflows, but also foreign-exchange denominated debt [3]. The problem is exacerbated by

[3]Exchange rate risk has been identified by the World Economic Forum (WEF) as one of the main concerns of infrastructure investors. See Forum (2016). See as well Verdouw et al. (2015), which starts by noting that “In PPPs, an optimal risk allocation generally means that a risk should be allocated to the party that is best positioned to manage or bear that risk, or more specifically, the party that can accept the risk at the lowest costs. However, regarding currency risk in these markets, this optimal risk allocation may not be so straightforward. A typical private sector developer has no influence over the exchange rate. Although the central bank has some control

the cost and unavailability of hedging instruments (such as derivative contracts) [4]. Thus, either government assumes the implicit liability by providing guarantees (as in the case of Peru), and thereby increases their foreign exchange exposure -- in a few instances with some comfort from a multilateral institution -- or contents itself with a smaller inflow of foreign capital. A weak version of exchange rate volatility mitigation instrument (as the case in Brazil) appears to have a limited impact. The fact that countries which are most in need of infrastructure investment inflows typically face balance of payment problems and a high opportunity cost of foreign exchange makes this a non-trivial problem.

Is there a better way to address currency risk in infrastructure investments? In this paper we propose an alternative approach grounded on the notion that it would be more efficient for the government to provide a hedging or guarantee mechanism which adjusts the length of the concession through some publicized function defined ex-ante and dependent on the exchange rate (a "target function"). Not only the risk is symmetrical to both parties (government and the external provider of finance), insofar as an appreciation of the local currency leads to a shortened length of the concession and conversely, but more fundamentally, a risk-neutral government would be indifferent to the purely financial implications of the mechanism. Taking into account the likely increase in foreign investors interest for the countries' projects, the government might in fact benefit. We show this by considering very general social welfare function specifications. These and other desired properties make the mechanism proposed in this paper potentially attractive to governments which are struggling to attract larger volumes of foreign capital to infrastructure.

We initially present the government guarantee mechanism in its most general form, with a non-specific "target function" for each partner in the concession. In addition, there is a "test function", that is a proxy for the actual profits that the project generate.[5] The main idea is to adjust the concession period in order that the test function achieves the level specified by the target function. In other words, a satisfactory concession period is reached when the "test function" is at least as high as the "target function". Either there is no excess value at the end of the satisfactory concession period and the adjustment is purely among partners, or there is excess value that can be returned to the government. Both target and test functions may be regarded as "black boxes" in their degree of generality.

While the "black box" functions can accommodate any form, including functions unrelated

over the exchange rate through its monetary policies, the government's effective control of the exchange rate may be limited. As a result of the above, unhedged currency risk is largely unmanageable for the private sector and may be beyond the control of the government agency in charge of infrastructure development, which means that it may not be easily acceptable for either party. Given the inherent uncertainties in exchange rate risk and the lack of a predetermined logical risk allocation to either the government or the private sector, currency risk can be a difficult and sensitive topic in negotiations between the private sector developer and the government" (pp. 1-2).

[4]See, for example, Hub (2019). The report notes (p. 42) that "Exchange rate risks are more substantial in markets where exchange rates are more volatile or long-term debt or swap markets are more illiquid (such as in countries with less developed capital markets). In more mature markets, the risk of currency fluctuations is typically not substantial enough to require the Contracting Authority to provide support and exchange rates risks are addressed solely through the Private Partner's own hedging arrangements. Where the exchange rates are more volatile, access to long term hedging may be either unavailable or too expensive". [Furthermore] "the likelihood of debt being dominated in a foreign currency is more likely in markets where financing by multilateral or international banks may be required (e.g., in less mature markets where there is limited depth in the local debt capital markets) ".

[5]If the actual profits are easily observable, the test function can be just the profits. By referring to test functions, We just emphasize the flexibility of the mechanism to work with just an easily determined proxy for the profit.

to exchange rate volatility, the paper discusses in greater detail three specific exchange rate guarantee mechanisms: ensuring an exchange rate level, with the government guaranteeing a specific inflation-adjusted exchange rate; a guarantee of the net present value at the beginning of the franchise; and a guarantee of equal returns among (national and non-national) partners, while not committing to a specific return. Note that in all 3 instances, the mechanism is presented to the investor/developer as an option, in principle at zero cost and decided by the investor at the contractual outset of the concession if she wants protection or not.

We then use those target functions to examine the corresponding information requirements for the regulator under two test functions, using respectively notional and real values. For many countries with limited regulatory resources, employing such guarantee mechanisms –– however attractive —- would be predicated on restricting information demands on the regulator. Test functions using notional values that approximate the profits with easy to check and transparent numbers – instead of the actual profits – are not only preferable in those cases, but they also reduce the probability of bilateral opportunistic behavior.

Finally, the paper examines the actual performance of the guarantee mechanism for eight countries – and currencies - from simulations based on the parameters of a 25-year highway concession which ended in 2021. Results were obtained for two target functions discussed in this paper, for countries which differ in levels of income, infrastructure assets and geographical location, but characterized by limited intervention in currency markets. The relatively small adjustments in franchise times suggest limited or zero opportunity cost for government, and the economic viability of the mechanism. Table 2 summarizes the results in case of eight different currencies against the US dollar. Given the baseline of 25 years, it is striking that the adjustment is less than 2 years (that is, 8% of the project's length) for all but one country (Indonesia, and for the guaranteed return target function). It is equally noteworthy that if the government were to guarantee an ex-ante return in foreign currency, in 6 out of 8 instances, it would receive back excess values, while in the case of guaranteed equal returns among partners, the average extension was in fact less than a year (which in Table 2 we approximate in the upper bound of integer numbers).

Table 2: Adjustment time required by alternative guarantee commitments – 25-year highway concession

	Adjustment time (in years)	
	Guaranteed return in Net Present Value (NPV)	Guaranteed equal returns among partners
Brazil	-1	+2
Mexico	-1	+1
Indonesia	+4	0
Mozambique	+1	+2
Chile	-2	+1
Peru	-1	+1
Philippines	-1	+1
South Africa	-1	+2

Source: See Section 6 and Appendix B. Note: In the case of the guaranteed equal returns among partners mechanism, negative values are associated with compensation the foreign investor must pay to the government instead of shortening the actual franchise time.

The mechanism discussed in this paper is related to Least Present Value models which provides a hedge against the demand risk by adjusting the length of the concession, and which has been put to test successfully in Chile and the U.K to a lesser extent (see Engel et al. (2001), Engel et al. (2019) and Engel et al. (2021)). However, there are significant differences. First, the paper extends the concept of concession period adjustment to remove the exchange rate risk from a concession, up to now a quasi-intractable problem due to its cost to either private party or government. Second, and more important, the literature apparently did not draw the consequences of the link between exchange rate processes being approximated by a martingale to provision of currency hedge by government. Third, the paper provides a general framework for thinking about guarantee mechanisms and their properties, suggesting a potential large menu of functional forms for target and test functions. Moreover, the paper model both firm and government preferences and shows social welfare gains allowed by the mechanism under general assumptions. In particular, the paper suggests low (or even negative) opportunity cost to governments in providing guarantees against long-term exchange rate movements in the context of economic and social infrastructure projects which can be modelled as long-term concessions.

This paper is organized as follows. Section 2 sets the basic model of a concession for firms and government, and the stochastic process which underlies exchange rate movements. By explicitly modelling firms and government preferences, and connecting both through an auction in which firms consortia bid for the franchise, it is shown that the guarantee mechanism bring social welfare gains if government value both consumer welfare and firm efficiency. Section 3 details the general guarantee mechanism and its functionality, which might entail either transfers among partners which have benefited more – or less – from the government guaranteeing a satisfactory concession period depending on their particular exchange rate movement, or transfer to the government in case of excess value. Section 4 illustrates the general mechanism with 3 target functions, while Section 5 discusses the information requirements regulators face if the government decides to offer such guarantees under notional and real values test functions. Section 6 presents the simulation results for two of these mechanisms and a total of 8 countries, against a 25-year actual highway concession, suggesting the applicability of the guarantee mechanism against long-term exchange rate fluctuations. Section 7 concludes.

2 Model

In this section, we will describe the model and introduce the guarantee mechanism that this paper proposes. Before detailing the technical parts of the model, it is useful to understand the context for which this mechanism is conceived.

We assume that a country which fiscal (and other) restrictions wants to undertake an infrastructure project with private resources. The government tenders a concession for the services rendered by the project during a given period of time. The firm (or consortium of firms) that will explore the concession needs to commit resources and will charge tariffs for the use of the infrastructure.

We focus initially on the definition of the contract that the government offers to the consortium of firms. The basic idea is to offer a contract that is more beneficial (less risky), without creating additional burden to the government. We are specifically interested in providing protection against exchange rate risk for foreign investors. In doing so, the government will increase competition by allowing for greater participation of providers of equity and debt in foreign currency, thus improving both the likelihood that the project is developed and the quality of its terms. Furthermore, the outcome of the competitive process under the guarantee would allow for the government to receive a larger payment for the right to explore the concession and/or the consumers paying a lower tariff.

Firms can form consortia with different partners to compete for the right to explore the concession. Specifically, a consortium may be composed of $m + 1$ firms, each of a possibly different country and interested in protection for exchange risk in its own currency. The value of the project for firm $j \in \{0, 1, ..., m\}$ will be denoted $V^j(T)$, expressed in j's currency. Here, T is the ex-ante duration of the concession, which will be important in what follows. The value functions $V^j(T)$ will be explicitly defined below.

The general form of our guarantee mechanism consists of defining, for each $j \in \{0, 1, ..., m\}$, test (real or proxy) functions $\widehat{V}^j(T)$ and target functions $\overline{V}^j(T)$. The test functions $\widehat{V}^j(T)$ may be thought as just the true value functions $V^j(T)$. Since the true value function may be difficult to verify by the government, we allow the possibility to use for the test a proxy function $\widehat{V}^j(T)$, that should be understood as an approximation of $V^j(T)$. This test function $\widehat{V}^j(T)$ may be easier for the government (and courts) to calculate and verify than the actual value function $V^j(T)$. The main idea of the mechanism consists in guaranteeing that these proxy functions will meet the target function $\overline{V}^j(T)$, that may be conceived as a guarantee of value for the firms. In more formal notation, the main idea of the guarantee mechanism is trying to ensure that for all $j \in \{0, 1, ..., m\}$,

$$\widehat{V}^j(T) \geqslant \overline{V}^j(T). \tag{1}$$

In Section 4 we give different examples of target functions, while in Section 5 we illustrate test (proxy) functions. But just to fix ideas, for now the reader can assume that the test function is the true value of the project and the target function in each currency is the value invested plus a given return over investment. In this specific case, the government is excluding all risks of the project, since it guarantees that the partners will receive the fixed return for sure.[6]

Even if $\widehat{V}^j(T) = V^j(T)$, the left hand side of condition (1) is still subjected to currency risks. In this case, condition (1) may not be met in the initially planned period T. The second part of the mechanism is to find a termination period P for the project that guarantees that condition (1) is met. This effective termination period will be random and determined after the realization of the exchange rate process during the concession period. In other words, the guarantee of the mechanism is not exactly (1), but that the concession period will last a period P so that the value of the project at period P satisfies the target function, that is,

$$\widehat{V}^j(P) \geqslant \overline{V}^j(P). \tag{2}$$

In the rest of this section, we will define the value functions $V^j(T)$, the preferences of the firms, that determine how they deal with risk, and the objective function of the government. The more detailed description of the mechanism is postponed to Section 3. Examples of test and target functions, respectively $\widehat{V}^j$ and $\overline{V}^j$, are discussed in Sections 4 and 5.

2.1 Time periods and discount factor

Time will be divided in periods, that can be interpreted as years, although there is no formal reason for not using quarters or months, if that is convenient or necessary. The first period will be labeled period 0 and all the (present) values will be expressed with respect to this period. After that, we will have periods $t = 1, 2, \dots, T, \dots$, where T will denote the typical (or expected) length of the concession period. Although time is, therefore, denoted in discrete units, it will be convenient to allow continuous time in some parts of our development, namely, in the definition of the moment that the test function achieves the level of the target function.

Let $\frac{1}{1+r_t} \in (0, 1)$ be the discount factor for period t. In principle, the value of r_t may be constant, in which case we would write only r. However, we can accommodate nonconstant discount factors by defining as δ_t the discount factor to bring values to year 0, that is,

$$\delta_t \equiv \frac{1}{(1+r_1)\cdots(1+r_t)}. \tag{3}$$

If $r_t = r$ for some opportunity cost of capital $r \geqslant 0$, which is fixed, then $\delta_t = (1+r)^{-t}$.[7]

2.2 Investments, prices, costs and demand for each period

We treat all investments as undertaken in period 0 and denote them by I. Of course, the actual investments can be spread in many different periods or years. We choose to talk about

[6] Of course, we are not recommending that the government offers such guarantee, but just illustrating a possibility.

[7] Below, we will consider discount rates that might vary with different countries. In this case, we will use δ_t^j instead of δ_t to refer to the discount rate in currency j.

only the *present value* of the investment in period 0 for simplicity.[8] This reflects the notion that period 0 corresponds to the development phase of the project (which may take more than a year), during which the project renders no service and, therefore, no revenue accrues.

Let c_t be all the costs of the project and p_t, the tariff or price for the service in period t. Consumers value the project as $v_t > p_t$ for the services that the project provides. Consumption q_t may vary with period t, but for simplicity we will assume that it does not depend on p_t, that is, demand is perfectly inelastic. This is a common (and realistic) assumption in electricity markets, for instance, and is probably a reasonable approximation for infrastructure projects.[9] Therefore, the revenue in period t is $p_t q_t$.

In this way, the project has net benefit $B_t = p_t q_t - c_t$ and the total consumer surplus is $S_t = (v_t - p_t)q_t$ in period t. Of course this is expressed in the domestic currency, that is, the currency of the country where the project is located.

2.3 Present value of the project

With the previous definitions, one can see that the present value of the project at period 0 is given by:

$$V(T) \equiv -I + \sum_{t=1}^{T} \delta_t B_t = -I + \sum_{t=1}^{T} \delta_t (p_t q_t - c_t), \tag{4}$$

where T represents the total time of the concession, that is, the time that the investors will be allowed to explore the services that the project renders.

Many terms in the expression (4) are not known at the time of the evaluation of the project: p_t, q_t, c_t and, to some extent, even I and δ_t are just projections. Thus, in principle, we should consider the value $V(T)$ as uncertain, and write it in terms of expectation. However, the main focus of this paper is on exchange rate risks. Therefore, for simplicity we will assume that the mentioned values are known, and consider only the uncertainty with respect to the exchange rates. This will be further discussed below.

2.4 Partners from different countries

One of the main motivations of this paper is to deal with currency risk. To take this into account, we will assume that the project will have $m + 1$ partners or investors from different countries, where partner $j = 0$ is the domestic partner. Partner $j = 0, 1, \ldots, m$ faces exchange X_t^j in period t, denominated in units of currency j for each unit of currency 0, the domestic or local currency, so that $X_t^0 = 1$ for all t.[10] For example, if the project is in Brazil, so that the domestic currency is the Brazilian Real and the investor j is using US dollar, then $X_t^j = 0.179457$ USD/BRL, if the period t corresponds to end of the year 2021. If we assume that benefit $B_t = p_t q_t - c_t$ is converted each period, it amounts to $X_t^j B_t = X_t^j (p_t q_t - c_t)$ in j's

[8] If one wants to be explicit, we can define I_t to be the investment made in period t and the present value of the investment is thus $I = I_0 + \sum_{t=1}^{T} \delta_t I_t$. In this case, we omit the dependence of I with respect to T, for a matter of convenience.

[9] Relaxing this assumption only makes the statement of our results unnecessarily more complicated. In any case, we will discuss below how the relaxation of this assumption impacts the results.

[10] Of course, more than one partner can belong to the same country. For instance, if j and j' use the same currency, we would have $X_t^j = X_t^{j'}$ and this would not create any problem to our formalism.

currency. It will be useful to define the value of the project as if it is exclusively expressed in j's currency:

$$V^j(T) \equiv -X_0^j I + \sum_{t=1}^{T} \delta_t^j X_t^j B_t = -X_0^j I + \sum_{t=1}^{T} \delta_t^j X_t^j (p_t q_t - c_t), \tag{5}$$

where we have used discounting rates specific for each j. Notice that since the initial investment I is given in the local (domestic) $j = 0$ currency and is realized in period $t = 0$. Obviously we are assuming that each period result is converted into j's currency. Notice also that, since $X_t^0 = 1$ for all t, $V^0(T) = V(T)$.

It is important to realize that the $V^j(T)$ – and basically all the values discussed in this paper – refer to the value in date 0. As already mentioned, in (5) we have used δ_t^j, that is, the discount rate depends on the country j. However, the model obviously allows that we assume $\delta_t^j = \delta_t$ for all j, that is, the same discount rate is used for all currencies. This flexibility allows the model to be used for different purposes, such as:

- *planning, ex ante*: the firm has a constant opportunity cost of capital given by i, so that $\delta_t^j = \delta_t = (1+i)^{-t}$ for all j, as commented after (3);
- *posterior evaluation*: δ_t^j takes into account the inflation in country j and, therefore, is differentiated by country.

In other words, the model is agnostic on how the firm (or the government) define or calculate δ_t^j: it may take into account only the opportunity cost of capital or include inflation and, possibly, other factors. What is important is that δ_t^j allows to bring values in the currency j and time t to time 0.[11] In any case, whether $\delta_t^j = \delta_t$ for all j or not, what is important is that the value $V^j(T)$ in expressed in j currency at period $t = 0$'s value. Also, we will write δ_t instead of δ_t^0 and, more generally, whenever the superscript j is omitted, the value is supposed to refer to the domestic currency 0.

Obviously, partner j will have just a share α^j of the project, with $\sum_{j=0}^{m} \alpha^j = 1$. Since we can just drop partners with zero participation, we will assume that $\alpha^j > 0$ for all $j = 0, 1..., m$. Therefore, each period t partner j receives $X_t^j \alpha^j (p_t q_t - c_t)$ in its currency. This implies that the value for partner j is

$$-X_0^j \alpha^j I + \sum_{t=1}^{T} \delta_t^j X_t^j \alpha^j (p_t q_t - c_t) = \alpha^j \left[-X_0^j I + \sum_{t=1}^{T} \delta_t^j X_t^j (p_t q_t - c_t) \right] = \alpha^j V^j(T).$$

Therefore, $\alpha^j V^j(T)$ is the value for partner j of its participation in the project.

As observed before, the above values are uncertain, but for simplicity, we will focus our attention only on the uncertainty with respect to the exchange rate. Thus, we denote by $\mathbb{E}_t[\cdot]$ the expectation with respect to exchange rates (in the foreign currencies).

If the expectation with respect to the future value of some exchange rate is its current value, as it is usually assumed in many financial markets, then the expected value of $V^j(T)$ is just the value $V(T)$ converted to the j currency at the current exchange rate. In order to formalize this result, we need an assumption about the nature of the underlying stochastic process governing exchange rate dynamics. This will be done in Section 3.

[11] Figure 1 below and the discussion that follows it help to further understand the roles that δ_t^j and δ_t play in our model.

2.5 Firms' preference

We assume that firms are risk averse and deduce from expected values a risk premium $R^j[V^j(T)]$ that depends on the volatility of the value of the project $V^j(T)$. Therefore, the final value of the project utility in currency j is:

$$U^j \equiv \mathbb{E}\left[V^j(T)\right] - R^j[V^j(T)]. \tag{6}$$

Of course partner j with fraction α^j of the project will evaluate its stake at $\alpha^j U^j$. From this, we can define the value of the project for the consortium of firms, denoted just u, as:

$$u = \sum_{j=0}^{m} \alpha^j \frac{U^j}{X_0^j}, \tag{7}$$

where we have omitted the dependence on T for simplicity and converted all values to the domestic currency.

For most of the paper, we will focus on only one consortium of firms that will execute the project. Thus, for most of the paper we do not need to make a special notation on the value of the project for the consortium, as in (7). However, to discuss the competition for the government concession, we will need to introduce a notation differentiating the values for each consortium. In this case, the value u that appears in (7) will appear with a subscript, as explained next.

2.6 The competition of different consortia for the project

In most of the paper we will consider only one consortium of firms building the infrastructure the project and exploring its service. Although this is convenient for most of the results of the paper, the analysis of the bidding process requires that we model how different consortia compete for the right to build the infrastructure and explore its service.

We will model this competition as an auction among K consortia. The consortium $k \in \{1, ..., K\}$ submits a bid b_k after learning/estimating value u_k for the project, where the reader can think of the value u_k coming from an aggregation of the different values for the firms which form the consortium k as in (7). We will assume that the the distribution of the values of the consortia are independent and identically distributed according to cumulative distributive functions (c.d.f.) $F : \mathbb{R} \to [0, 1]$, that is,[12]

$$\Pr[u_k \leqslant x] = F(x), \text{ for all } k \in \{1, ..., K\}. \tag{8}$$

The difference in values for the consortia arises from possible difference in costs and managerial expertise. The symmetry assumption is not extremely important for our results, that might hold under asymmetric distributions as well. However, symmetry allows us to use many convenient results in auction theory. Moreover, this may be a reasonable assumption at this level because, although the firms in different countries will have values that are very different, they can participate in consortia of different composition. Therefore, taking into

[12] In fact, later we will consider two c.d.f.s: F^0 when the guarantee mechanism is not used, and F^1 if the guarantee mechanism is adopted. This will be further explained in Subsection 3.4 below.

account the possibility of consortia made with firms of different countries, each consortium can be viewed, ex ante, more or less symmetrically with respect to other ones.

2.7 Government's preference

We assume that the firms competing for the project make a transfer τ to the government.[13] This can be done at the beginning of the concession period or be the present value of transfers made during the concession period.[14]

We assume that the government's preference is based on the consumer surplus $S(T) = \sum_{t=1}^{T} \delta_t(v_t - p_t)q_t$, the value $V(T)$ of the project (with a weight factor $\mu \in [0,1]$ further discussed below) and the transfer τ:

$$G(T) = S(T) + \mu V(T) + \tau = \left[\sum_{t=1}^{T} \delta_t(v_t - p_t)q_t\right] + \mu\left[-I + \sum_{t=1}^{T} \delta_t(p_t q_t - c_t)\right] + \tau.$$

Recall that τ includes the (present value) of the fees paid to the government for the concession rights (which may be zero) or all subsidies offered by the government (which will enter with a negative sign).This total transfer can be used by the government for promoting other social or public goals. The factor μ represents the weight that the government puts on the benefits obtained by the project. If $\mu = 1$, the government values the benefits on par with the consumer surplus, thus pursuing efficiency. If $\mu = 0$, the government is interested only on the consumer surplus and the transfers τ, and disregards the benefits accrued to the firms. In any case, notice that the government takes into account the total net benefit of the project $V(T)$ and not the firms' expected utility U^j introduced above. This is justified for two reasons. First, taking into account U^j would make the government objective vary with the composition of the consortium of firms and their shares in the project. Second, U^j deducts a risk premium that is faced by the firms but is not relevant for the government. Indeed, we assume that the government is risk neutral. This last assumption is justified by Arrow and Lind (1970), who argue that the government can spread risk better than firms. At this point the reader may ask: but $G(T)$ does not depend on the exchange rates and, therefore, will not be subjected to the uncertainty that we have previously specified. Although that is true, the guarantee mechanism that this paper proposes makes the actual concession length T dependent on the exchange rates. Therefore, rather than a certain period T, we will work with a random variable $\tilde{T}$, that depends on the exchange risks. In this case, by saying that the government is risk neutral, we are assuming that it is trying to maximize $\mathbb{E}[G(\tilde{T})]$.

We assume that the government may choose p_t and τ, subject to the incentive constraints defined by the preferences of the firms. Indeed, by reducing p_t (considering the demand q_t inelastic) and/or increasing τ, the government improves $G(T)$, but decreases U^j, making the project less attractive to firms. In the limit, no firm would undertake the project. Alternatively, by reducing τ, the government can expect firms to reduce the price p_t that firms require to participate in the project.

[13]We can model the situation in which the government pays a *subsidy* to the firm for completing the project by assuming that $\tau < 0$.

[14]In principle, this could include taxes as well. However, since taxes can come in different forms and be complex, we avoid discussing them in detail here and focus only on transfers directly related to the concession contract.

More specifically, we will consider two polar cases, both of which have occurred in the real world. In the *price competition* case, the government fixes τ (for instance, $\tau = 0$) and organize the bidding competition around the price p_t (usually taking at a fixed real value, that is, $\delta_t p_t = p$ for all t). The firm that offers the lowest price wins. In the *concession competition* case, the government fixes prices p_t and organizes an auction for the concession right. The firm that offers the highest payment τ wins the competition. The specific models of these two competition cases will be further discussed below.

3 The general guarantee mechanism

In this section, we develop mechanism of guarantees and explain in more detail how it works. Recall that the main idea was described at the beginning of Section 2. It consist of adjusting the concession period in order to provide the guarantee to partners. This guarantee will take the form of a target function $\overline{V}^j(T)$ for each partner j, that is, the purpose is to guarantee that a proxy for the value for partner j, $\widehat{V}^j(T)$, is at least as high as the target function $\overline{V}^j(T)$. In this section, we will simplify notation and denote the proxy functions $\widehat{V}^j(T)$ simply as the value functions $V^j(T)$. This will create no restriction since we will not make use of any specific properties of $V^j(T)$ nor its particular definition given in (5). Since an important case that interests us is exactly when the proxy functions are exactly equal to the value functions, $\widehat{V}^j(T) = V^j(T)$, this simplification has the advantage of helping understanding.

As explained in Section 2, the method requires finding the period P such that

$$V^j(P) \geqslant \overline{V}^j(P). \tag{9}$$

Obviously, partner j that owns share α^j of the total project, will receive only $\alpha^j V^j(P)$. In any case, (9) guarantees that this partner receives the corrresponding fraction of the target function, that is, $\alpha^j V^j(P) \geqslant \alpha^j \overline{V}^j(P)$.

Since different currencies will lead to different values for different partners, and therefore, different target functions, the condition expressed by (9) might in principle lead to different end dates for the project. This is obviously not convenient, since the project needs a definitive time for the concession to end. A solution could be to use the longer P, that makes all restrictions (9) satisfied. While this guarantees that all partners achieve their target values, it also implies that some partners will have a large advantage over others. Instead, we propose to introduce transfers (that we will call adjustments) between partners. It will be established that the project produces enough resources in order to meet the guarantees made for all partners, but there will be adjustments between them to guarantee that all targets are satisfied at the earliest possible concession period P. We discuss the transfers among partners in more detail in the next subsection.

3.1 The definition of the concession period

Our main task is to show that the proposed mechanism can work under certain conditions. For this, it is of central importance to discuss the transfers among partners. As anticipated above, those transfers are what allow the mechanism to be practical. In fact, the definition of the concession period, which we name a "satisfactory concession period" needs to be introduced together with the transfers (adjustments) among partners. This is accomplished by the following:

Definition 1. *We say that* $P \in \mathbb{R}_{++}$ *is a satisfactory concession period if there exist adjustments* $A^j \in \mathbb{R}$, *for* $j = 0, 1..., m$, *such that:*[15]

1. *the adjustments are just transfers between the partners, that is,*

$$\sum_{j=0}^{m} \alpha^j A^j = 0, \tag{10}$$

   and

2. *for all* $j = 0, 1..., m$,[16]

$$V^j(P) + X_0^j A^j \geqslant \overline{V}^j(P). \tag{11}$$

Recall that all $V^j(P)$ are given at period $t = 0$ values. Thus, the adjustments are also expressed in those values.[17] More specifically, all the adjustments are given in the domestic currency at period 0 values. This allows us to add them with respect to the shares that each partner has, as (10) requires. Notice also that the definition of satisfactory concession period P modifies (9) to include the adjustments (transfers) converted to j currency and changes it to (11).

The main question is under what conditions we can find a satisfactory concession period that guarantees, as Definition 1 requires, that all partners receive at least the value assigned by their target functions. The conditions are in fact simple and are given by the following:

Theorem 1. *Assume that the functions* $V^j, \overline{V}^j, X^j : [P_0, P_1] \to \mathbb{R}_{++}$ *are continuous and*

$$V^j(P_1) \geqslant \overline{V}^j(P_1), \forall j = 0, 1, ..., m, \tag{12}$$

that is, the value of the target functions are necessarily satisfied at the end of maximum concession period P_1. *Then there exists the lowest satisfactory concession period, that is, there exists* $P \in [P_0, P_1]$ *such that* P *is a satisfactory concession period and if* $P' \in [P_0, P_1]$ *is also a satisfactory concession period, then* $P \leqslant P'$.

Remark 2. *As already mentioned at the beginning of this section, in Theorem 1 we do not use the specific formula* (5) *that defines the value functions* $V^j(P)$. *That is, those functions are treated as "black box" functions without specific form. Therefore,* $V^j(P)$ *could be any "real" or "proxy" test function* $\widehat{V}^j$ *against which the (also arbitrary) target function* $\overline{V}^j(P)$ *is compared. This will allow us to work with different specifications of the mechanism illustrated in Sections 4 and 5.*

[15] Note that we do not restrict the satisfactory concession period to be restricted to integers, as previously commented in Subsection 2.1.

[16] In this section, it will be convenient to maintain the time explicitly denoted in the notation of $X^j(T)$, instead of just X_T^j.

[17] In real world implementations of our guarantee mechanisms, it will be natural to convert them to period P values. This is, of course, straightforward, but would complicate our notation and we refrain from doing this.

Theorem 1 is proved in Appendix A. The proof also establishes the specific value of the adjustments among partners that should be implemented. Since these adjustments may be important for the illustrations of specific mechanisms in Section 4, we reproduce those formulas here. For $j = 1, \ldots, m$ define the adjustment

$$A^j \equiv \frac{\overline{V}^j(P) - V^j(P)}{X_0^j}, \tag{13}$$

and for $j = 0$, define

$$A^0 \equiv \frac{1}{\alpha^0} \sum_{j=1}^{m} \alpha^j \frac{V^j(P) - \overline{V}^j(P)}{X_0^j}. \tag{14}$$

If all the conditions (11) are to hold with equality for the satisfactory concession period P, the above adjustments are unique. If one or some of the inequalities in (11) are allowed to be strict, then we would have an infinity of possible adjustments.

Remark 3. *Condition* (12) *in Theorem 1 is in fact stronger than what is actually necessary for the theorem to hold. In the proof of Theorem 1 in the appendix, we show that the following condition is sufficient for its conclusion:*

$$V^\alpha(P_1) \equiv \sum_{j=0}^{m} \alpha^j \frac{V^j(P_1)}{X_0^j} \geqslant \sum_{j=0}^{m} \alpha^j \frac{\overline{V}^j(P_1)}{X_0^j} \equiv \overline{V}^\alpha(P_1), \tag{15}$$

where $\alpha = (\alpha^0, \alpha^1, \ldots, \alpha^m)$. *That is, in the appendix we show that* (12) *implies* (15)*, which means that* (15) *is a weaker condition than the (simpler one) stated in Theorem 1. Notice that* (15) *is just the requirement that the weighted value of the project (reflecting the participation of each partner) is above the weighted value of the target value. In this way it is easy to understand why* (15) *is weaker than* (15)*: this last condition could fail for some partner* j*, but the overall weighted value still be above the target because the lack of value for partner* j *is compensated by the excess of a different partner.*

In fact, it is useful to formally state the following result:

Lemma 3.1. *If* $V^\alpha(P) \geqslant \overline{V}^\alpha(P)$ *then* P *is a satisfactory concession period.*

In other words, the condition on weighted value functions (15) is sufficient for having a satisfactory concession period.

3.2 Transfer to the government in case of excess value

The difference $V^\alpha(P) - \overline{V}^\alpha(P)$ defines the excess value that is left to the partners at the end of the satisfactory concession period. This is nonnegative and it can be strictly positive. In the latter case, we could stipulate that the excess value had to be transferred to the government. This is not necessary for the general mechanism, but it might seem advisable as an additional rule. Therefore, we discuss this in more detail in this subsection.

Instead of leaving the possible excess value to the participants, the government should require that this excess value is returned to it, in order to insure symmetrical conditions.

Thus, if

$$V^{\alpha}(P_0) > \overline{V}^{\alpha}(P_0), \tag{16}$$

that is, if the project gives a combined value for the partners that is above the target at the first possible concession deadline P_0, then we would require that the partners make the transfer of the excessive amount

$$V^{\alpha}(P_0) - \overline{V}^{\alpha}(P_0) > 0 \tag{17}$$

to the government. On the other hand, if $V^{\alpha}(P_0) - \overline{V}^{\alpha}(P_0) \leqslant 0$, then Theorem 1's proof guarantees that $V^{\alpha}(P) = \overline{V}^{\alpha}(P)$ at the satisfactory concession period P. In this case, no transfer to the government would be necessary (just transfers or adjustments among the partners).

If we adopt this rule, then we obtain two consequences. First, each partner receives exactly $\alpha^j V^j(P) = \alpha^j \overline{V}^j(P)$ at the end of the satisfactory concession period P. Second, the adjustments A^j defined by (13) and (14) are the unique possible adjustments, which gives the extra benefit of avoiding possible disputes among partners over the division of the excess value.

It is important to notice, however, that the adoption of this rule *does not* imply that the partners do not face risks or receive a predetermined value. Indeed, the functions $\overline{V}^j(P)$ may not be constant (as in the case the government guarantees a net present value) and may be defined only at period P (in view of demand, construction, and other risks other than exchange rate). That is, they can be unknown and uncertain ex ante. In fact, we could even have $\overline{V}^j(P) = V^j(P)$, in which case no guarantee is provided. What determines the level of certainty that is provided for the partners is defined exactly by the specific format that the functions $\overline{V}^j(P)$ assume. We next evaluate the proposed general mechanism from the point of view of its consequences to the bidding process and the government objectives. Section 4 will discuss specific guarantee mechanism and corresponding target functions. But before this, we will discuss the source of difference between partners in different countries and give conditions to show that those differences are related only to risk aversion with respect to exchange rate.

3.3 Foreign partners and their values

We want to highlight that the guarantee mechanism does not introduce any special advantages to the foreign partners. Indeed, it only corrects for the exchange rate risks that foreign partners face. In other words, if all partners were risk neutral with respect to currency risks, they would be on an equal footing in evaluating the project. In order to establish this, we will introduce the assumption that the exchange rates form a martingale. This is supported by recent economic literature, such as Phillips and Jin (2014) and Fong et al. (1997), among others.

Let us assume for a moment that $\delta_t^j = \delta_t$ for each j, that is, the discount rate is the same for all currencies. In this case, our main assumption takes the following form:

$$\mathbb{E}_t\left[X_t^j\right] = X_0^j, \tag{18}$$

which is just the standard formulation of a martingale assumption, that is, the expected value of the exchange rate is just the current value.

In order to take into account the possibility that the discount rates are different, let us assume for a moment that the exchange rates are known. Consider Figure 1. In this figure, point A corresponds to values in the currency j in time $t = 0$; point B corresponds to values in currency j in time t; point C corresponds to values in the domestic currency (0) in time t; point D corresponds to values in the domestic currency in time $t = 0$.

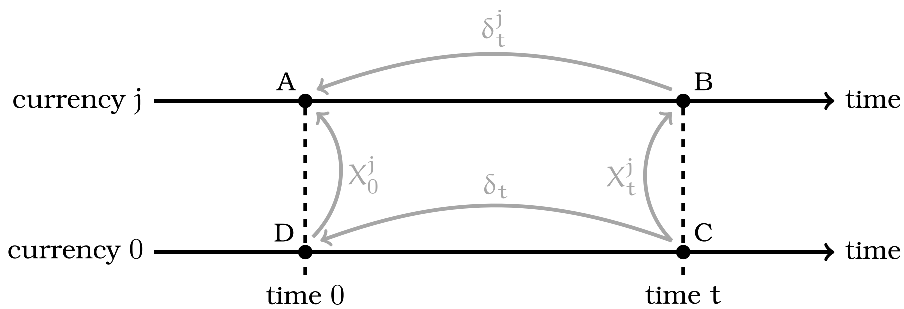


Figure 1: Exchange and discount rates in times 0 and t

To bring values in point B to point A, we must use the discount rate in currency j from time t (to time 0), which is denoted δ_t^j. To bring values from point C to point B, we must apply the exchange rate X_t^j. To bring values from point C to point D, we must depreciate the domestic currency using the depreciation factor δ_t. To bring values from point D to point A, we must apply the exchange rate X_0^j. Now, if we want to convert values in point C to point A, there are two possible pathways: 1) we can convert from C to B and then from B to A; or 2) we can convert from C to D and then from D to A. In the first pathway, we obtain the convergence coefficient $\delta_t^j X_t^j$; in the second, we obtain $\delta_t X_0^j$. If there is no uncertainty, an arbitrage condition would impose that the two coefficients must be the same, otherwise an investor would have an arbitrage opportunity. Therefore, the following relation must hold if there is no uncertainty:

$$\delta_t^j X_t^j = \delta_t X_0^j. \tag{19}$$

Our main assumption just introduces back the uncertainty into (19). Indeed, if only the *expectation* of X_t^j is known, then it is natural to assume:

Assumption 1 (Main Assumption)**.** *For any* j,

$$\delta_t^j \mathbb{E}\left[X_t^j\right] = \delta_t X_0^j. \tag{20}$$

Notice that if we assume that $\delta_t^j = \delta_t$, then (20) simplifies to (18). That is, if the depreciation in all currencies are the same, then Assumption 1 just requires that the exchange rates are martingales. On the other hand, the only difference between (19) and Assumption 1 is the uncertainty in the future exchange rates. Therefore, we can say that Assumption 1 is just a martingale condition, adjusted for a possible difference in depreciation.

This assumption allows us to show that the *expected* value of the project is the same for all currencies, as formalized by the following:[18]

[18]In any case, it should be highlighted that the mechanism proposed in this paper do not depend on this assumption and may be considered even if it is not satisfied.

Lemma 3.2. *Let Assumption 1 hold. Then*

$$\mathbb{E}\left[V^j(T)\right] = X_0^j V(T).$$

Notice that this result suggests that the value of the project for all partners will be unaffected by the currency used. This might induce the reader to think that risk neutral firms will not appreciate the kind of exchange rate protection that we are considering in this paper. However, even risk neutral firms may benefit from this mechanism because they may lack relevant information regarding the underlying exchange rate dynamics. In any case, in our experience, most investors express strong concerns regarding the exchange rate uncertainty and would like to have mechanisms to curb such risks. Moreover, firms are generally not risk neutral for large stakes. As discussed in Subsection 2.5, it is natural to assume that foreign firms deduce a risk premium from the expected values that they perceive in the projects. The guarantee mechanism affects (reduces) the risk premium $R^j[V^j(T)]$ that appears in (6), but it does not necessarily improve the value function for the partners. In particular, the guarantee mechanism does not favor any particular firm.[19] This observation leads to an important remark with respect to the adoption of this mechanism.

Remark 4. *Throughout the paper we have been referring to foreign firms as participating in the consortium as partners, thereby supplying equity in foreign currency. However, this is not necessarily the case as domestic firms can procure equity in different countries and may be interested in protecting these values. Even if the consortium is made only of domestic firms, they may decide to protect a fraction of the project in various currencies, in order to hedge themselves with respect to credit that they have taken. In particular, the consortium may define the fractions α^j protected for each currency in a completely arbitrary way.*

3.4 Impact of the mechanism on the bidding process

Recall from section 2.6 that there are K consortia competing for the project and consortium $k \in \{1, ..., K\}$ values the project at u_k. All u_k are independent and identically distributed. As anticipated in footnote 12, we consider two cumulative distributive functions (c.d.f.) for the values u_k: when there is *no* guarantee mechanism, the c.d.f. is $F^0 : \mathbb{R} \to [0, 1]$ and when the guarantee mechanism is in place, the c.d.f. is $F^1 : \mathbb{R} \to [0, 1]$.

Let us remember from (7) that the values u_k are obtained as the weighted sum of values U^j of the partners of consortium $k \in \{1, ..., K\}$.[20] In turn, the values U^j are given by expression (6), which takes into account a risk premium R^j. It is natural to assume that the introduction of the guarantee mechanism reduces the risks for each partner j and, therefore, increases the value U^j. As a result, the values u_k are likely to be higher if it exists at least one partner supplying equity in foreign currency in consortium k. We formalize this intuitive and natural condition as the following:

[19] Some specifications of the mechanism *may* indeed favor some firms. For this, the guarantee $\bar{V}^j$ may be high or very generous for some j and not for others. This is formally allowed in the specification above. However, in principle the guarantee mechanism does not need to favor any currency. This can be accomplished by requiring that the target function (adjusted for the exchange rate) is the same for all partners, that is, $\bar{V}^j / X_0^j = \bar{V}^0$.

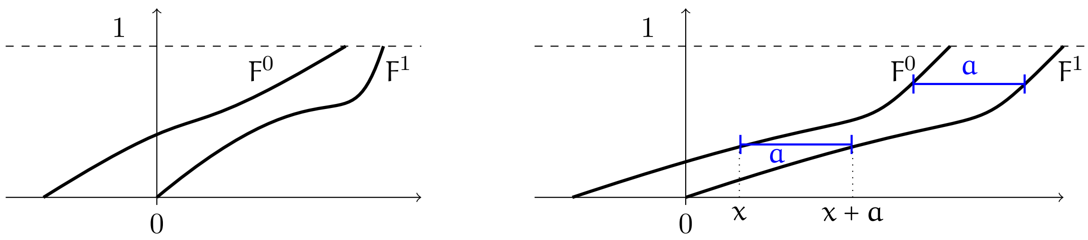
[20] It would be more correct to talk about values $U^{j,k}$ of partner j of consortium k. We avoid this notation for simplicity.

Assumption 2. F^1 *first-order-stochastically dominates* F^0*, that is,* $F^1(x) \leqslant F^0(x)$*, for all* $x \in \mathbb{R}$.

A particular case of interest is when F^1 is just a location shift of F^0, that is,

$$\exists a > 0 \text{ such that } F^1(x + a) = F^0(x), \forall x \in \mathbb{R}. \tag{21}$$

Of course, (21) implies Assumption 2.



(a) F^0 and F^1 satisfy Assumption 2 (b) F^0 and F^1 satisfy (21)

Figure 2: Illustration of Assumption 2

We will use this assumption to show the benefits of the introduction of the guarantee mechanism in the following:

Proposition 3.3. *Let Assumption 2 hold. Then the introduction of the guarantee mechanism increases the expected revenue of the bidding process.*

In the particular case in which (21) holds, we can have a very clear expression by how much the revenue increases, as the following result establishes.

Lemma 3.4. *Let (21) holds. Then, the expected revenue with the guarantee mechanism is increased by* a*, where* $a > 0$ *is the number that makes (21) hold.*

3.5 Impact of the mechanism on the government's objective

The mechanism proposed above converts the concession period of the project, initially projected to be T, into a random variable $\tilde{T}$ that depends on the realization of the exchange rates X_t^j, for $j = 1, \ldots, m$ and the satisfaction of the condition (9), that is, that the test functions are at least as large as the target functions.

Recall from Section 2.7 that the government is interested in maximizing $G(T)$. Since T is now a random variable $\tilde{T}$, then the objective function of the government is, naturally, $\mathbb{E}[G(\tilde{T})]$.[21] From the comments above, it is clear that the distribution of $\tilde{T} \in [P_0, P_1]$ depends not only on the exchange rate process for all currencies, but also on the test and target functions. Since we are not making particular assumptions on the test and target functions in this section, we will assume directly something about the random variable $\tilde{T}$, namely:

Assumption 3. *The expectation of* $\tilde{T}$ *is equal to the original predefined concession period* $T \in [P_0, P_1]$*, that is,* $T = \mathbb{E}[\tilde{T}]$.

[21]Recall that in Section 2.7 we have argued, from Arrow and Lind (1970), that the government should be concerned with expected values.

Assumption 3 requires that if the government wants to achieve a particular concession period, it might adjust the test and targets functions so that the expected value of the concession is just this target period. Thus, we can interpret $T = \mathbb{E}[\tilde{T}]$ as the period that the government is aiming at. Another interpretation of Assumption 3 is that the government wants to implement the expectation of $\tilde{T}$, which is defined as T.

We want to provide sufficient conditions to show that, holding T constant, the expectation $\mathbb{E}[G(\tilde{T})]$ is just the initial planned valued $G(T)$, where the proposed mechanism was not in place. This means that the introduction of the mechanism does not modify the expected value for the government. For this, we introduce the following:

Assumption 4. *For all* t, $\delta_t p_t = p$; $\delta_t c_t = c$; $\delta_t v_t = v$ *and* $q_t = q$.

From an *ex ante* point of view, it is a normal practice to estimate the future values of p_t, q_t, c_t and v_t at a constant, present value. Assumption 4 just formalize this standard practice. Therefore, it can be considered a mild assumption. With this, we have following:

Proposition 3.5. *Let Assumptions 1, 3 and 4 hold. Then the government payoff expectation is the same as the originally planned value, that is,* $\mathbb{E}[G(\tilde{T})] = G(T)$.

This result has the important consequence that the mechanism, by itself, does not have a direct consequence on the government's objective, that is, it does not create a direct burden on the government. We will argue now that the mechanism can actually lead to a gain by the government.

We can rearrange $G(T)$ to write it in a more convenient way as follows:

$$G(T) = \underbrace{\left[\sum_{t=1}^{T} \delta_t v_t q_t - \mu\left(I + \sum_{t=1}^{T} \delta_t c_t\right)\right]}_{\text{(weighted) net social benefit}} - \underbrace{(1-\mu)\left(\sum_{t=1}^{T} \delta_t p_t q_t\right)}_{\text{revenue penalty}} + \underbrace{\tau}_{\text{transfers (to gov.)}}. \tag{22}$$

The first term in (22), the (weighted) net social benefit, is equal to the project's net social benefit if $\mu = 1$. If $\mu < 1$, the costs of the project are de-emphasized. Notice, however, that this term is not affected by the choices of p_t and τ, which are government's control variables, since we have assumed in Section 2.2 that demand q_t is inelastic. Therefore, the first term in (22) is a constant with respect to the parameters of choice for the government and we can disregard it.[22]

Now let us analyze the revenue penalty. First, notice that if $\mu = 1$, that is, if the government is interested in efficiency and does not discriminate against the firm, then there is no revenue penalty (or this is just zero). In other words, the revenue penalty only plays a role when the government favors consumers ($\mu < 1$). In the extreme case in which $\mu = 0$, that is, when the government only cares about consumers well-being and not efficiency, then the revenue penalty has the highest weight.

In order to better analyze this, let us use again Assumption 4. In this case, $G(T)$ can be

[22] As we emphasized in Section 2.2, the assumption that q_t is not affected by p_t is adopted for simplicity, exactly to allow us to disregard the first term. If demand is elastic, reducing p_t will increase q_t and the (weighted) net social benefit, thus increasing the attractiveness of reducing p_t that we analyze next.

written as

$$G(T) = \underbrace{[Tvq - \mu(I + Tc)]}_{\text{(weighted) net social benefit}} - \underbrace{(1-\mu)Tpq}_{\text{revenue penalty}} + \underbrace{\tau}_{\text{transfers (to gov.)}}. \tag{23}$$

Let us assume that the price is increased from p to $p + \epsilon$. Then, the value for the firms is likely to increase by $a \equiv T\epsilon q$. In this case, it is natural to assume that (21) holds with $a \equiv T\epsilon q$. By Lemma 3.4, this implies that the revenue τ is increased by $a = T\epsilon q$. Therefore, the goverment objective function changes from G to G^{ϵ}, such that

$$G^{\epsilon}(T) - G(T) = -(1-\mu)T\epsilon q + T\epsilon q = \mu T\epsilon q.$$

Thus, unless $\mu = 0$ (the case in which the government puts zero weight on the firms profits), then it is better for the government to put the price as high as reasonable, in order to maximize the revenue τ. This holds because the gain from doing that is positive, as shown above. In order words, the above suggests that the government should structure a competitive process for firms to compete for the project, thus receiving a higher payment τ, for a given price p .

In our model, there is a limit to the price that can be charged: v, the reserve value of consumers. Any price above v would lead to zero revenue. In fact, this is the price that maximizes revenue in this model and it would be the optimal price to be charged. If we allow demand to be elastic, then this analysis suggests that the government should allow firms to choose prices to maximize revenue. However, in this case one has to take into account the effect of prices on the net social benefit. Indeed, assuming that consumers, instead of having a fixed value v, their values is variable with quantity, given by the inverse demand $p(q)$, and taking into account the result of Lemma 3.4, that τ is equal to the revenue plus an adjustment term, then we can rewrite (23) as:

$$\begin{aligned} G &= \underbrace{[Tp(q)q - \mu(I + Tc)]}_{\text{(weighted) net social benefit}} - \underbrace{(1-\mu)Tp(q)q}_{\text{revenue penalty}} + \underbrace{Tp(q)q + \text{some constant}}_{\text{transfers (to gov.)}} \\ &= (1+\mu)Tp(q)q + \text{other constant} \end{aligned}$$

It is easy to see that G is maximized by maximizing the revenue.

Remark 5. *Corruption: If the government itself is benevolent, but knows that a corrupt bureaucracy will capture a fraction λ of τ, then (23) becomes:*

$$\begin{aligned} G(T) &= \underbrace{[Tvq - \mu(I + Tc)]}_{\text{(weighted) net social benefit}} - \underbrace{(1-\mu)Tpq}_{\text{revenue penalty}} + \underbrace{(1-\lambda)\tau}_{\text{where } \tau = Tpq + \text{constant}} \\ &= (\mu - \lambda)Tpq + \textit{some constant} \end{aligned}$$

Thus, as long as $\mu > \lambda$, it is still better for the government to set a revenue maximizing price and collect the corresponding τ. Only if $\lambda > \mu$ then the government should avoid maximizing the revenue from the competitive bidding. Notice that this result just expands the previous analysis, where $\lambda = 0$. Without corruption, we have seen that the government should prefer to maximize the competitive bidding revenue as long as $\mu > 0$.

4 Specific guarantee mechanisms: target functions

Theorem 1 describes a general guarantee mechanism based on a test function $V^j(P)$ and a target function $\overline{V}^j(P)$. As Remark 2 emphasizes, the form of those functions is arbitrary in Theorem 1. Depending on what we decide to guarantee, we will have many choices for those functions. This section illustrates some possibilities.

We begin in Subsection 4.1 by the guarantee of the exchange rate. Then, 4.2 considers a fixed return for the project that is defined at the beginning of the concession period. In 4.3, the return at the final period of the concession P_1 is defined after the initial period P_0 based on that obtained by the domestic partner. The guarantee in this case is that the partners supplying equity in foreign currency (foreign partner) receive exactly the same return in their currency as the domestic partner does in domestic currency.

4.1 Guaranteed exchange rate level

Let us consider the case in which the government guarantees a specific domestic inflation-adjusted exchange rate g^j. More specifically, the target function is defined for $P \geqslant T = P_0$ by:

$$\overline{V}^j(P) = \sum_{t=1}^{T} \delta_t^j g^j (p_t q_t - c_t). \tag{24}$$

Notice that the right-hand expression above *does not* depend on P, that is, $\overline{V}^j(P)$ is constant for all $P \geqslant P_0$. In this way, the exchange rate g^j is guaranteed at the time $T = P_0$. If the exchange rate is favorable for partner j, then this partner will pay the government an adjustment to bring it back to the level g^j if the rule discussed in Subsection 3.2 is in place, or keep this value if not. On the other hand, if the exchange rate is not favorable, the concession may be extended. Our task now is to provide sufficient conditions for the assumptions in Theorem 1 when $\overline{V}^j(P)$ is defined by (24). We need to bound the values of the exchange rates (they cannot become arbitrarily large or low), that is:

Assumption 5. *Assume that for each* j, *there exist* $\underline{e}^j, \overline{e}^j$ *such that* $\overline{e}^j \geqslant \underline{e}^j > 0$ *and* $X_0^j \in \left[\underline{e}^j, \overline{e}^j\right]$ *for all* $P \leqslant P_1$.

If the guaranteed exchange is sufficiently low, given the values of P_0, P_1 and the minimal and maximal benefits that the project produce, then the assumption (12) holds. More formally, we have the following:

Lemma 4.1. *Let Assumption 5 hold. Assume that* $B_t = p_t q_t - c_t \in \left[\underline{B}, \overline{B}\right]$, *with* $\overline{B} \geqslant \underline{B} > 0$ *and* $\delta_t = 1$ *for all* t. *If* $\widehat{V}^j(T) = V^j(T)$ *and*

$$g^j \leqslant \frac{P_1 \underline{e}^j \underline{B}}{P_0 \overline{B}} \tag{25}$$

then Theorem 1 holds.

4.2 Guaranteed return

Let us assume that the guarantee is a given net present value N set at the beginning of the franchise. For simplicity, let us assume in this subsection that there are just two partners in the consortium, that is, $m = 1$. The target function in this case is:

$$\overline{V}^0(P) = N$$
$$\overline{V}^1(P) = X_0^j N$$

That is, the target is a previously agreed upon present value that has to be reached by both the domestic and foreign partner in their respective currencies in order for the franchise to end. This particular value can be set through a least present value bidding mechanism. We will who below that the following assumption is sufficient for the conclusion of Theorem 1.

Assumption 6. *There exist* $a, b \in \mathbb{R}_{++}$ *such that* $X_P^1 \geqslant a$ *and* $B_P \geqslant b$, $\forall P \in [P_0, P_1]$, *and:*

$$-I + b\left(\frac{\alpha a + 1 - \alpha}{\alpha X_0 + 1 - \alpha}\right)\sum_{t=1}^{P_1} \delta_t \geqslant N \tag{26}$$

The mathematical condition required by Assumption 6 is numerically illustrated below, since the complexity of (26) does not allow a straightforward interpretation. Its motivation, however, should be clear: it sets a lower bound on the exchange rate and the net benefits for a given maximum duration of the project. Indeed, (26) requires that the P_1 is sufficiently large such that, even if the benefits are very low in terms of foreign currency and the exchange rate is very unfavorable to the external investor, there will be enough time for him to achieve the previously guaranteed return.

Lemma 4.2. *Assumption 6 implies that* $V^\alpha(P_1) \geqslant \overline{V}^\alpha(P_1)$, *that is, the conditions of Theorem 1 are satisfied.*

4.3 Guaranteed equal returns among partners

A modification of the above example is that the target function for the foreign partner be defined by the return obtained by the domestic partner at period T. As in the previous subsection, let us assume for simplicity that there are just two partners, the domestic $j = 0$ and the foreigner, $j = 1$. An endogenous target function based on franchise parameters can be particularly useful for countries that auction their infrastructure concessions by the lowest tariff or highest bestowal. In order to ensure a level playing field for all investors, the government may be interested in providing an insurance against exchange-rate fluctuation without necessarily guaranteeing a fixed rate of return — that is, eliminating exchange-rate risk without eliminating demand and construction risk. This can be accomplished by letting the return r^0 to be determined as the return of the project in domestic currency at period T:

$$(1 + r_T^0)^T = \frac{\sum_{t=1}^T \delta_t^j B_t}{I} = \frac{\sum_{t=1}^T \delta_t^j (p_t q_t - c_t)}{I}. \tag{27}$$

With this, we can define:

$$\overline{V}^0(P) = V^0(T)$$
$$\overline{V}^1(P) = \left(1 + r^0\right)^T X_0^j I$$

Let us adopt the rule discussed in Subsection 3.2, that is, if at P_0, $V^1(P_0) > \overline{V}^1(P_0)$, the difference $V^1(P_0) - \overline{V}^1(P_0)$ is returned to the government. From (5) and (27), we can calculate this difference as:

$$V^1(T) - \overline{V}^1(T) = \alpha I \left(\frac{\sum_{t=1}^T \delta_t X_t B_t - X_0^j \sum_{t=1}^T \delta_t B_t}{\sum_{t=1}^T \delta_t B_t} \right). \tag{28}$$

After period T, the domestic partner transfers all the benefits of the project to the foreigner partner. Therefore, the totality of the project's additional benefits in periods $T+1, T+2, \ldots, P$ are added to the foreign investor's previous benefits.

$$V^1(P) \equiv -X_0^1 I + \alpha \sum_{t=1}^{P} \delta_t^1 X_t^1 B_t + \sum_{t=T+1}^{P} X_0^1 X_t^1 A^1(t) = V^1(T) + \sum_{t=T+1}^{P} \delta_t^1 X_t^1 B_t \tag{29}$$

Since the target function and test function for the domestic investor are equal by default, the validity of the assumption of Theorem 1 depends only on the foreign investor. Similar to Subsection 4.2, the return of the foreign investor relies on the range of the exchange rate and the benefits, with the distinction that since the target function is endogenously defined by the project in period T, there are only fluctuation restrictions after T.

Assumption 7. *There exist* $a, b \in \mathbb{R}_{++}$ *such that* $X_P \geqslant a$ *and* $B_P \geqslant b$, $\forall P \in [T+1, P_1]$, *and:*

$$ab \sum_{t=T+1}^{P_1} \delta_t \geqslant (1 + r_T^0)^T X_0 I - V^1(T) \tag{30}$$

Similar to Subsection 4.2, we set minimum values for the exchange-rate and benefits for a minimum period P_1. However, unlike Subsection 4.2, the boundaries also depend on endogenous values r_T^0 and $V^1(T)$, that is, the difference between the present value of the project for the domestic and foreign investors in T. If the exchange rate is unfavorable to the foreign partner before T, the maximum period P_1 must be greater, even more so if the exchange rate continues to be unfavorable after the extension.

Lemma 4.3. *Assumption 7 implies that* $V^1(P_1) \geqslant \overline{V}^1(P_1)$.

In these illustrative target functions, clearly a favorable exchange rate does not create a problem for the supplier of equity in foreign currency. She simply returns the excess to the government. However, if the exchange rate conditions are unfavorable, we establish fluctuation boundaries such that an extension of the concession period satisfies the target commited or guaranteed by the government. Of course, the mechanism guarantees only exchange rate within the boundaries assumed above. If the level of depreciation falls outside of those boundaries, then the guarantee would be honored only up to that limits previously defined.[23]

5 Specific guarantee mechanisms: test functions

In this section, we compare the informational requirements for the regulator of alternative test functions characterized respectively by notional (or proxy) and real values, and for different guarantee mechanisms discussed in Section 4. Notional (or proxy) values might be particularly useful for countries with limited regulatory resources and as a means to deal with the growing regulatory burden associated with private capital to infrastructure. Indeed, countries suffer with a lack of specialized and experienced human capital to staff agencies, and their eventual use as political currency affected the quality of regulation, the credibility of the agencies and the perception of society that decisions are sometimes not technically grounded or less than transparent.

Several countries facing such constraints, have sought simpler or less costly alternatives – a multisector agency, thereby avoiding fragmentation of resources and being guided by OECD "Best Practice" recommendations;[24] regulation by contract, where the burden lies in spelling out in the greatest details the rights and obligations of the regulated entity; and contracting out regulatory reviews to independent third parties. Any such alternative faces problems in attending the objectives of ringfencing from undue political interference, reducing the regulatory risk premium, improving regulatory predictability, and avoiding contract renegotiation.

One recurrent problem in regulation is bilateral opportunistic behavior whereas governments have an incentive for expropriating private investors and the latter to gouge consumers, in face of incomplete contracts and information asymmetry respectively, even if it were rational to avoid such behavior from a longer-term perspective. Often, it is the perception in government that the private sector is hiding or distorting information that generates undue friction. To address this issue, one possible criterion for choosing the guarantee mechanism proposed in this paper is the volume of information required, on the presumption that minimizing such requirements decreases information asymmetry between the government and the operator. This is most relevant for governments which face problems accessing accurate operator information and adequately processing such information.[25]

5.1 Notional costs

In general, it might be very difficult for governments to verify the costs of a firm. This opens the possibility of misrepresentation of costs in order to manipulate the application of the guarantee mechanism. One way to avoid this is to consider notional costs, previously defined, instead of true or realized/reported costs. For this, it would be necessary to define a sequence of

[23] It is important that those limits are explicitly defined before the competitive bidding for the project, since they influence the value of the project for the partners.

[24] The OECD has advanced 7 principles of best practice: role clarity; preventing undue influence and maintaining trust; independence for decision-making, based on solid governance; accountability and transparency; engagement with stakeholders; adequate funding; and performance evaluation.

[25] According to a major World Bank report, "most developing and transition economies do not have well-established cost accounting and auditing systems. And as noted, they often lack regulatory expertise. Thus the information and human capital requirements of different regulatory mechanisms are important." See Kessides et al. (2004) (p. 122 and passim.)

predefined costs $\bar{c}_t$.[26] We will comment in a moment about options to define these predefined costs. In fact, we could also consider a notional value for investments, $\widehat{I}$. In this case, the test functions with notional costs can be defined as follows:

$$\widehat{V}^j(P) = -X_0^j\widehat{I} + \sum_{t=1}^{P} \delta_t^j X_t^j (p_t q_t - \widehat{c}_t).$$

Perhaps the simplest possibility for the definition of the notional costs is to define them as constant, that is, $\widehat{c}_t = c$ for all t. They can also be defined as constant in real terms, that is, $\widehat{c}_t = \frac{c}{\delta_t}$, so that $\delta_t\widehat{c}_t = c$. However, more complex schemes can be conceived. For example, it can be stipulated that the notional costs will be defined by a particular index of prices i_t, widely available and not manipulable. For instance, this index could be international oil prices. Maybe some price indices can be found that are directly linked with the costs of the particular industry in question. In this case, one can define $\widehat{c}_t = i_t c$.[27] Similar comments can be applied to the investment. In any case, as the reader can see, the definition is very flexible.

5.2 Notional demand

Another topic that may be difficult to verify in certain cases is demand. As in the previous case, one can define notional demand $\widehat{q}_t$. In this case, the test (proxy) function can be defined as:

$$\widehat{V}^j(P) = -X_0^j I + \sum_{t=1}^{P} \delta_t^j X_t^j (p_t \widehat{q}_t - c_t).$$

The notional demand can be defined by a fixed quantity, that is, $\widehat{q}_t = q$. Alternatively, it can be defined as growing following a given index i_t, such as population growth or economic growth. In this case, $\widehat{q}_t = i_t q$.

5.3 Notional prices

Prices are, in general, easier to verify and it might not be important to use notional values for it. However, as we commented in Subsection 3.5, the government might be interested in letting the consortium to define the price in order to maximize revenue. In this case, since the price will be freely determined by the consortium, it might be convenient to fix a notional price for the purpose of the guarantee mechanism. In this case, the government may define a notional price $\widehat{p}_t$ and the corresponding test (proxy) function is:

$$\widehat{V}^j(P) = -X_0^j I + \sum_{t=1}^{P} \delta_t^j X_t^j (\widehat{p}_t q_t - c_t).$$

As before, the notional price may be fixed, $\widehat{p}_t = p$ or adjusted by some index i_t, that is, $\widehat{p}_t = i_t p$.

[26] As the following discussion makes clear, what needs to be predefined is not exactly the value of $\bar{c}_t$, but the rule that defines it. If the government tries to define $\bar{c}_t$ a posteriori, then there will be opportunities for manipulation and the guarantee mechanism would not guarantee anything.

[27] Here, we are obviously denoting by i_t the accumulated adjustment from time 0 to time t.

5.4 Notional and real values

Of course we can combine all of the previous cases, by defining notional values $\widehat{p}_t$, $\widehat{q}_t$ and $\widehat{c}_t$. In fact, the logic may be extended also to the investment I, whose notional value may be denoted by $\widehat{I}$ and even to the discount rates $\widehat{\delta}^j_t$. The form of these notional values is, to some extent, arbitrary. A particular case of interest is, of course, that they are the real values.

In any case, giving the target values above, the general test (proxy) functions $\widehat{V}^j(P)$ can be defined by

$$\widehat{V}^j(P) = -X^j_0\widehat{I} + \sum_{t=1}^{P} \widehat{\delta}^j_t X^j_t(\widehat{p}_t\widehat{q}_t - \widehat{c}_t).$$

The choice of which notional or real values to use will depend, of course, in the special cases. An important aspect to consider is the institutional maturity and stability of the country. Using notional values may be better in a situation of poor institutional development. However, the use of notional values increase the risks for companies, since it introduces the possibility that the test or proxy functions do not approximate well their real profits. An important question to ask in this context is how much risk should be borne by the firms and how much by the government.[28] On the other hand, if the test function is the true value and the target functions are guaranteed returns, the government is eliminating all risks for the firms, thus absorbing more than its fair share. Insofar as government may also be in a weak position to manage effectively risks of certain nature or exogenous shocks, this should be avoided. In our view, a good starting point can be that the private party absorbs risks associated with demand and costs, on the presumption that price (tariff of the service) is preset at the beginning of the period and corrected by a certain rule established in contract. That is, the contract establishes only notional values for demand and costs, $\widehat{q}_t$ and $\widehat{c}_t$, respectively, but use real values for $\widehat{p}_t$.

5.5 Information requirements under different target functions and alternative test functions

Below we summarize the information requirements for regulators of the 3 guarantee mechanisms discussed in this paper, and under two types of test functions, respectively with notional and real values. In all cases, the exchange rate of interest to partner j at time t X^j_t is directly observable in markets; by the same token, the discount factor δ_i is also market observable; the tariff p is set and publicized; and the share of α^j of partner j is contractually observable.

Table 3 indicates that for countries with limited regulatory resources, the use of notional or proxy test functions would be preferable, though service levels would still need to be observed and assessed independently by the regulator or third party to ensure the provider is fulfilling its obligations. In terms of real values, target functions in 4.2 and 4.3 would be preferable insofar as they demand or can do with less information from the private party.

[28] Although we have argued that the government should be risk neutral, it is probably better to avoid widespread protection, not exactly because of risk aversion, but because moral hazard. Indeed, due to informational asymmetries, the government may be put in disadvantage in certain contractual arrangements.

Table 3: Information requirements for regulators under alternative guarantee mechanisms

<table>
<tr><th></th><th colspan="3">Target Functions</th></tr>
<tr><th></th><th>Guarantee an Exchange Rate (4.1)</th><th>Guarantee a Return (4.2)</th><th>Guarantee Equal Return Among Partners (4.3)</th></tr>
<tr><td>Notional (or proxy) Values (for I, q, and c fixed and known in advance)</td><td colspan="3">For the three mechanisms, all information required by regulators is directly observable, contractually set or gleaned from the market at minimal cost; conversely, none needs to be supplied by the regulated entity, which minimizes renegotiations and (bi-lateral) opportunistic behavior. The use of notional values would only require an underlying model verified by both regulator and investor that relates investment, costs, tariff and quantity.</td></tr>
<tr><td>Real Values (for I, q and c)</td><td>In order to calculate excess benefit caused by the exchange rate guarantee and therefore determine the extension of T, the regulator would require real values for I, q and c for each period.</td><td colspan="2">The regulator must be able to calculate the real I and real net Benefit B for each period. The government does not need to observe individual q and c values directly in every period, even though doing so may reduce opportunistic behavior by the incumbent. If the regulator faces information restrictions, it can audit the incumbent on a random basis.</td></tr>
</table>

6 Simulating alternative guarantee mechanisms in a real life highway concession

In this section, we investigate the behavior of the proposed mechanisms using data from CCR NovaDutra, a highway franchise taken place in Brazil from 1996 to 2020. We test target functions from Subsections 4.2 and 4.3 with profit functions constructed under a broad array of different exchange rate scenarios.[29] For that, we suppose the same project have taken in eight developing countries characterized by limited currency market interventions between 1996 and 2020: Brazil, Mexico, Indonesia, Mozambique, Chile, South Africa, Peru and the Philippines. Table 4 compares selected indicators among these countries and shows they are diverse in terms of income, export composition, population and private investment in infrastructure.

[29] For detailed information about the project, see Appendix B. Due to lack of data regarding prices and demand, it was not possible to simulate the target function discussed in Subsection 4.1 nor propose a test function based on notional values that would be consistent with the project's cost structure.

Table 4: Selected indicators

Country	GDP per capita[1]	Manufacture (% of exports)[2]	Population[3]	PPI per capita[4]
Brazil	14,835	25	212.6	35.624
Mexico	18,444	77	128.9	33.847
Chile	25,110	3	19.1	N/A
Indonesia	12,072	47	273.5	11.976
South Africa	13,360	38	59,3	73.481
Peru	11,878	9	33,0	13.660
Philippines	8,389	80	109,6	17.459
Mozambique	1,297	6	31,3	13.614

Notes: [1]Purchase power parity (PPP) in current USD (2021). Source: International Comparison Program, World Bank; [2]Manufactures goods as a % of merchandise exports (2021). Source: World Bank using data from the United Nations.; [3]Total population (in million) (2021). Source: United Nations Population Division; [4]Investment in infrastructure (water and sanitation, energy and transport) with private participation (PPI) per capita in current USD (2021). Source: Private Participation in Infrastructure Project, World Bank.

For simplification purposes, we consider all simulations in this section are composed of two partners – one domestic and one foreign with benefits indexed in USD – and with $\alpha = 0.5$. Furthermore, we use the Consumer Price Index (CPI) inflation rate for each country as the discount rate i_t and the real exchange rate between the local currency and USD adjusted by price rates as the exchange rate X_t, considering the project's investment and benefit levels as denominated in each country's local currency. Figure 3 shows the exchange rate variations across the eight countries in the period (1996 = 100).

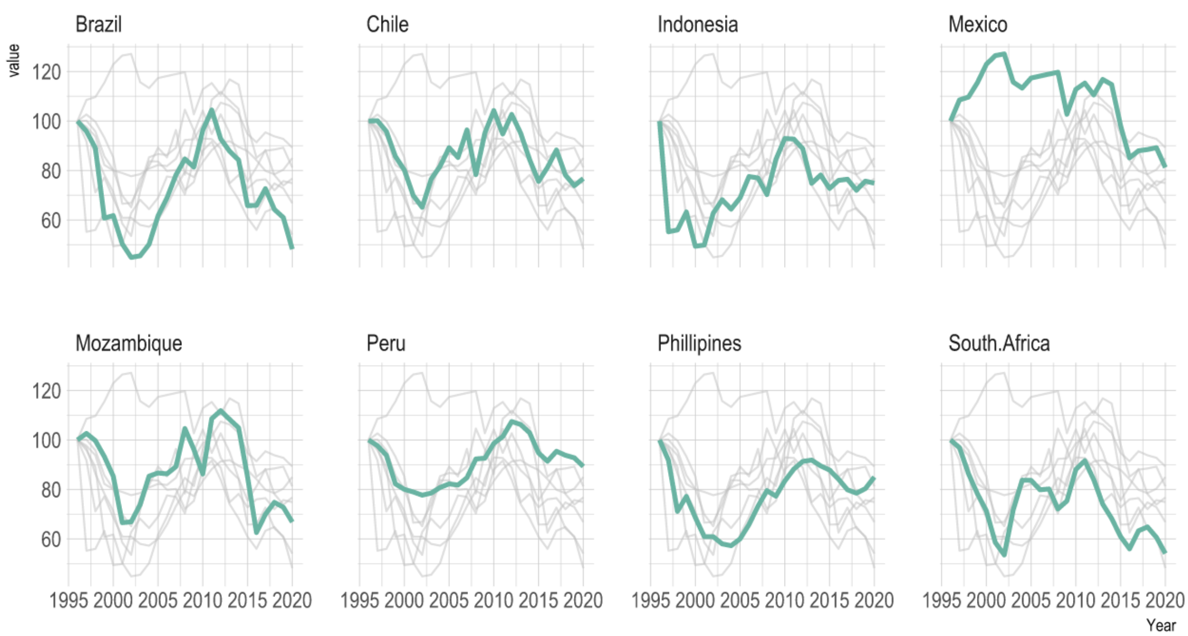


Figure 3: Real Effective exchange rate (based on the CPI) between local currency and US Dollar by country (1996 = 100)
Source: Own elaboration with data from the International Financial Statistics (IMF).

6.1 Guaranteed return

For this Subsection, we will use the target functions outlined in 4.2, that is:

$$\overline{V}^{0}(P) = N$$
$$\overline{V}^{j}(P) = X_0^j N$$

We defined N by applying a 5.88% compound rate for 25 periods on the total investment denominated in each local currency.[30] It is important to note, however, that this value can be determined in different ways for distinct countries, including a least present value auction that captures the market's expected return for a given project in a country with certain characteristics. Therefore, the franchise will be terminated when the value in mixed currency is at least as high as the target function in mixed currency (that is, $V^{\alpha} \geqslant \overline{V}^{\alpha}$ as defined in (15). Results for the simulations are shown in Table 5.

[30]This is the internal rate of return of the project when $\alpha = 0$, obtained using data from the project and Brazil's CPI inflation as the discount factor. This is further clarified in Appendix B.

Table 5: V^{α} (by year) and $\overline{V}^{\alpha}$ for each country (in billion LCU)

Year	Brazil	Mexico	Indonesia	Mozambique	Chile	Peru	Philippines	South Africa
1996	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
1997	0.22	0.20	0.24	0.21	0.21	0.21	0.22	0.22
1998	0.39	0.32	0.32	0.37	0.37	0.36	0.40	0.38
1999	0.58	0.39	0.36	0.49	0.50	0.49	0.48	0.50
2000	0.73	0.50	0.50	0.67	0.68	0.65	0.66	0.68
2001	0.98	0.60	0.58	0.91	0.90	0.81	0.86	0.91
2002	1.18	0.71	0.59	1.03	1.09	0.97	1.00	1.09
2003	1.37	0.91	0.68	1.16	1.26	1.22	1.26	1.15
2004	1.53	1.13	0.84	1.28	1.54	1.51	1.56	1.34
2005	1.64	1.34	0.95	1.49	1.84	1.85	1.83	1.66
2006	1.83	1.58	1.04	1.71	2.27	2.24	2.06	2.03
2007	2.01	1.83	1.19	1.89	2.53	2.60	2.30	2.36
2008	2.25	2.10	1.39	1.97	3.21	2.93	2.57	2.83
2009	2.57	2.52	1.41	2.24	3.32	3.33	2.92	3.08
2010	2.74	2.72	1.52	2.59	3.70	3.75	3.23	3.25
2011	3.01	3.02	1.70	2.55	4.41	4.26	3.59	3.60
2012	3.54	3.43	1.92	2.75	4.82	4.72	3.98	4.16
2013	4.00	3.69	2.26	3.03	5.59	5.34	4.44	4.86
2014	4.44	4.08	2.39	3.32	6.53	6.02	4.96	5.48
2015	5.37	4.78	2.65	3.92	7.49	6.84	5.47	6.23
2016	5.68	5.52	2.76	4.87	7.78	7.55	6.07	6.92
2017	5.73	5.77	2.93	4.77	8.07	8.00	6.73	6.86
2018	6.45	6.11	3.20	4.79	**9,20**	8.70	7.29	7.17
2019	**6.95**	**6.41**	3.29	5.04	-	**9.34**	**7.68**	**7.80**
2020	-	-	3.47	5.47	-	-	-	-
$\overline{V}^{\alpha}$	6.89	6.18	4.06	5.60	9.07	9.04	7.85	7.41

Note: Bold values indicate that the test function is at least as high as the target function (shown in the last line) and therefore the franchise is terminated.

These results mean the franchise would end in 2018 in Chile, and in 2019 in Mexico, Brazil, Peru, Philippines and South Africa. However, the project in Indonesia and Mozambique would be extended beyond 25 years, until they generated additional 590,772 USD in Indonesia and 128,108 USD in Mozambique. By contrasting these values with 2020 benefits (178,979 USD for Indonesia and 423,363 USD for Mozambique), we can see that if the benefits and exchange rate from 2020 remained constant, the project would be extended by less than one year in Mozambique and less than four years in Indonesia.

6.2 Guaranteed equal returns among partners

In this Subsection, we will use the target function outlined in Subsection 4.3, that is:

$$\overline{V}^0(P) = V^0(T)$$
$$\overline{V}^1(P) = \left(1 + r^0\right)^T X_0^j I$$

Using the project data and each discount factor, it becomes possible to compute the project's effective return rate in each country after the 25 years of the franchise ($T = 25$). For that, we use Equation (27).

Table 6: Yearly return rate for the domestic investor at T=25

Country	r_T^0
Brazil	5.88%
Mexico	6.09%
Indonesia	5.45%
Mozambique	5.38%
Chile	6.47%
South Africa	6.11%
Peru	6.52%
Philippines	6.31%

Using these values, we calculate the value of the target functions for the foreign partners in each country and compare it with test functions. Results are shown in Table 7.

Table 7: Target and Test function at T=25 for each country.

Country	$V^1(T)$	$\overline{V}^1(T)$	$V^1(T) - \overline{V}^1(T)$
Brazil	2583574	2402383	181191
Mexico	441460	427495	13965
Indonesia	531	543.5	-12.6
Mozambique	195085	184045	11039
Chile	10332.7	9945.8	387
South Africa	676789	626224	50565
Peru	1885423	1875655	9768
Philippines	125239	123940	1299

The values of $V^1(T)$ and $\overline{V}^1(T)$ are vastly different between the countries and don't mean anything on their own as we are considering equal I_t and B_t in local currency across vastly different exchange rates. Therefore, it is useful to compare the magnitude of the adjust compared to the revenue in USD for the last year of the franchise (2020) in each country.

Table 8: Extension of the concession period by country

Country	$V^1(T) - \overline{V}^1(T)$	$\delta X_{2020} B_{2020}$	# of years of extension
Brazil	181191	138333	1.3
Mexico	13965	33102	0.4
Chile	387	978.4	0.4
Indonesia	-12.5	50.6	-0.2
South Africa	50565	43243	1.2
Peru	9768	204071	0.1
Philippines	1299	14372	0.1
Mozambique	195085	184045	1.1

Table 8 shows that if the transfer rules from Subsection 3.2 were adopted, the compensation the foreign investor would have to pay to the government in Indonesia is equal to 20% of the benefit of the project in 2020. Moreover, if the benefit of the project stayed the same as in 2020 for the subsequent years, the extension of the project in Mexico, Chile and Philippines would last a fraction of one year, and in Mozambique, South Africa and Brazil less than two years.

7 Conclusion

Currency risk remains a significant deterrent to foreign investment flows to developing and emerging economies, particularly for long-duration infrastructure investments. Given the nature of such long-term investments, that take place in non-tradable sectors for which natural hedge is unlikely, hedging instruments are frequently costly, if at all available. Therefore, the search for hedging substitutes is a policy priority for countries in need to build or modernize their infrastructure.

This paper shows that a mechanism which adjusts the length of a concession contract as a means to protect investors supplying equity (and debt) denominated in foreign currency would not only be feasible but provide the government with a gain measured against a social welfare function which values both consumer (user) surplus and the efficiency of firms providing the service, even if the government were not to charge for providing hedge. This would be self-intuitive to the extent that by providing hedge to suppliers of foreign capital, governments attract a larger number of competitors which bring a lower tariff to consumers or greater revenues to government, or both.

Two additional features should be underlined. The mechanism is symmetric. For those that opt for it would either benefit from an extension in the concession contract – if the currency moves against them – or conversely return the excess gains to the government, without altering the original concession length. The mechanism is adaptable to countries with limited regulatory resources insofar as the tests to establish if the partner supplying capital in foreign currency should return excess gains to the government or enjoy additional concession time can be conducted in notional or proxy values, thereby minimizing the probability of (bilateral) opportunistic behavior, while eventually facilitating court decisions.

Finally, the empirical tests provide evidence that using the numbers of a real world 25-

year highway concession, the variation in concession length resulting from the government of 8 countries hypothetically providing a guarantee mechanism under alternative formats or functions are relatively small. It is striking that the mechanism works across countries at different levels of development and with 8 currencies which the only common denominator is the exchange rate regime, namely, they operate with very limited Central Bank intervention.

We conclude that the mechanism proposed, modelled and tested in this paper, would be a valuable addition to the toolkit of emerging and developing countries which have as a policy objective the provision of better infrastructure services to the population and firms. Such services more often than not depend on significant capital and operational expenditures over many years, and outside the reach for many countries facing fiscal restrictions. Even if this were not the case, private capital and operators normally bring about significant efficient gains due to superior governance and management. The guarantee mechanism reduces what is generally regarded as a binding entry barrier for many investors: currency risk. And it is shown that it does with relatively small variations in concession length and positive gains in social welfare.

A Proofs

A.1 Proof of of Theorem 1

In this appendix we provide a proof of Theorem 1. To establish this result, it will be convenient to define the following function:

$$\overline{V}^{\alpha}(T) = \sum_{j=0}^{m} \alpha^j \frac{\overline{V}^j(T)}{X_T^j}. \tag{31}$$

Also, it will be important to convert back the value for partner j, $\alpha^j V^j(T)$, to the domestic currency at period T, that is just $\left(X_T^j\right)^{-1} \alpha^j V^j(T)$. The collection of those values define a virtual value for the project that depends on the vector of shares $\alpha = (\alpha_0, \ldots, \alpha_m)$ and it is given by:

$$V^{\alpha}(T) = \sum_{j=0}^{m} \alpha^j \frac{V^j(T)}{X_T^j}. \tag{32}$$

This "mixed" value of the project will be used to prove Theorem 1. Indeed, we have the following:

The problem will be to find the period P such that $V^{\alpha}(P) = \overline{V}^{\alpha}(P)$.

Lemma A.1. *If there is $P \in [P_0, P_1]$ such that*

$$V^{\alpha}(P) \geqslant \overline{V}^{\alpha}(P) \tag{33}$$

and

$$P' \in [P_0, P_1], P' < P \Rightarrow V^{\alpha}(P) < \overline{V}^{\alpha}(P), \tag{34}$$

then P satisfies the conditions of Theorem 1.

Proof. Let P satisfy the assumptions above. We have to show two things: 1) P is a satisfactory concession period; and 2) if P' is also a satisfactory period, then $P \leqslant P'$.

For $j = 1, ..., m$ define the adjustment

$$A^j \equiv \frac{\overline{V}^j(P) - V^j(P)}{X_0^j},$$

and for $j = 0$, define

$$A^0 \equiv \frac{1}{\alpha^0} \sum_{j=1}^{m} \alpha^j \frac{V^j(P) - \overline{V}^j(P)}{X_0^j}.$$

Therefore,

$$\sum_{j=0}^{m} \alpha^j A^j = \alpha^0 A^0 + \sum_{j=1}^{m} \alpha^j A^j = \sum_{j=1}^{m} \alpha^j \frac{V^j(P) - \overline{V}^j(P)}{X_0^j} + \sum_{j=1}^{m} \alpha^j \frac{\overline{V}^j(P) - V^j(P)}{X_0^j} = 0,$$

which establishes (10). Now we will verify (11) for $j = 1, ..., m$. By definition,

$$V^j(P) + X_0^j A^j = \overline{V}^j(P),$$

that is, (11) is satisfied with equality. Now, for $j = 0$, we have:

$$\begin{aligned}
V^0(P) + X^0(P)A^0 &= V^0(P) + X^0(P) \frac{1}{\alpha^0} \sum_{j=1}^{m} \alpha^j \frac{V^j(P) - \overline{V}^j(P)}{X_0^j} \\
&= \frac{X^0(P)}{\alpha^0} \left[\alpha^0 \frac{V^0(P)}{X^0(P)} + \sum_{j=1}^{m} \alpha^j \frac{V^j(P)}{X_0^j} - \sum_{j=1}^{m} \alpha^j \frac{\overline{V}^j(P)}{X_0^j} \right] \\
&= \frac{X^0(P)}{\alpha^0} \left[\sum_{j=0}^{m} \alpha^j \frac{V^j(P)}{X_0^j} - \sum_{j=0}^{m} \alpha^j \frac{\overline{V}^j(P)}{X_0^j} + \alpha^0 \frac{\overline{V}^0(P)}{X^0(P)} \right] \\
&= \frac{X^0(P)}{\alpha^0} \left[V^\alpha(P) - \overline{V}^\alpha(P) \right] + \overline{V}^0(P) \\
&\leqslant \overline{V}^0(P),
\end{aligned}$$

where the last inequality comes from (33). This shows that (11) is satisfied for all $j = 0, 1, ..., m$. This shows that P is a satisfactory concession period.

Now, we will show the second claim by contradiction. That is, assume that $P' \in [P_0, P_1]$ is also a satisfactory concession period and $P' < P$. Then (34) implies that $V^\alpha(P) < \overline{V}^\alpha(P)$. Since P' is also a satisfactory concession period, there exists adjustments A^j such that (10) and (11) hold. From (11), we have

$$A^j \geqslant \frac{\overline{V}^j(P) - V^j(P)}{X_0^j}$$

for all $j = 0, 1, ..., m$. From (11), we have

$$0 = \sum_{j=0}^{m} \alpha^j A^j \geqslant \sum_{j=0}^{m} \alpha^j \frac{\overline{V}^j(P) - V^j(P)}{X_0^j} = \sum_{j=0}^{m} \alpha^j \frac{\overline{V}^j(P)}{X_0^j} - \sum_{j=0}^{m} \alpha^j \frac{V^j(P)}{X_0^j} = \overline{V}^\alpha(P) - V^\alpha(P),$$

which contradicts $V^\alpha(P) < \overline{V}^\alpha(P)$. The contradiction concludes the proof. □

Therefore, we can conclude the proof of Theorem 1 as follows.

Proof. We want to show that there exists $P \in [P_0, P_1]$ satisfying the conditions of Lemma A.1. Note that if $V^\alpha(P_0) \geqslant \overline{V}^\alpha(P_0)$ then $P = P_0$ satisfies those conditions. Therefore, assume that $V^\alpha(P_0) < \overline{V}^\alpha(P_0)$. From the assumptions,

$$d(P) \equiv V^\alpha(P) - \overline{V}^\alpha(P)$$

is continuous in P and $d(P_0) < 0$. If we can show that $d(P_1) \geqslant 0$, there exists P such $d(P) = 0$ and $P' \in [P_0, P_1]$, $P' < P$ implies $d(P') < 0$, that is, P is the smallest zero of the function $P \mapsto d(P)$. This P would satisfy therefore (33) and (34). Thus, it is sufficient to show that $d(P_1) \geqslant 0$. Notice that

$$d(P_1) = V^\alpha(P_1) - \overline{V}^\alpha(P_1) = \sum_{j=0}^{m} \alpha^j \frac{V^j(P_1)}{X^j(P_1)} - \sum_{j=0}^{m} \alpha^j \frac{\overline{V}^j(P_1)}{X^j(P_1)} = \sum_{j=0}^{m} \alpha^j \frac{V^j(P_1) - \overline{V}^j(P_1)}{X^j(P_1)} \geqslant 0,$$

where the inequality at the end holds because $V^j(P_1) \geqslant \overline{V}^j(P_1)$ for every j. □

A.2 Other proofs

Proof of Lemma 3.1. Let the adjustments A^j be defined by (13) and (14). Then,

$$\sum_{j=0}^{m} \alpha^j A^j = \alpha^0 \left(\frac{1}{\alpha^0} \sum_{j=1}^{m} \alpha^j \frac{V^j(P) - \overline{V}^j(P)}{X_0^j} \right) + \sum_{j=1}^{m} \alpha^j \frac{\overline{V}^j(P) - V^j(P)}{X_0^j} = 0,$$

which establishes condition 1 of Definition 1. For condition 2, let $j \in \{1, \dots, m\}$. Then,

$$V^j(P) + X_0^j A^j = V^j(P) + X_0^j \left(\frac{\overline{V}^j(P) - V^j(P)}{X_0^j} \right) = V^j(P) + \overline{V}^j(P) - V^j(P) = \overline{V}^j(P).$$

For $j = 0$, recall that $X_0^0 = X_0 = 1$. It is sufficient to show that $\alpha^0 \left[V^0(P) + A^0 - \overline{V}^0(P) \right] \geqslant 0$.

$$\begin{aligned} \alpha^0 \left[V^0(P) + A^0 - \overline{V}^0(P) \right] &= \alpha^0 \left[V^0(P) - \overline{V}^0(P) + \frac{1}{\alpha^0} \sum_{j=1}^{m} \alpha^j \frac{V^j(P) - \overline{V}^j(P)}{X_0^j} \right] \\ &= \sum_{j=0}^{m} \alpha^j \frac{V^j(P)}{X_0^j} - \sum_{j=0}^{m} \alpha^j \frac{\overline{V}^j(P)}{X_0^j} \\ &= V^\alpha(P) - \overline{V}^\alpha(P) \geqslant 0. \end{aligned}$$

Therefore condition 2 of Definition 1 holds for all $j \in \{0, 1, \dots, m\}$, which concludes the proof. □

Proof of Lemma 3.2. This is an easy application of the properties of the expectation:

$$\begin{aligned}\mathbb{E}\left[V^j(T)\right] &= \mathbb{E}\left[-X_0^j I + \sum_{t=1}^{T} \delta_t^j X_t^j (p_t q_t - c_t)\right] \\ &= -X_0^j I + \sum_{t=1}^{T} \delta_t^j \mathbb{E}_t\left[X_t^j\right](p_t q_t - c_t) \\ &= -X_0^j I + \sum_{t=1}^{T} \delta_t X_0^j (p_t q_t - c_t) \\ &= X_0^j\left[-I + \sum_{t=1}^{T} \delta_t (p_t q_t - c_t)\right] = X_0^j V(T)\end{aligned}$$

□

Proof of Proposition 3.3. Since we are assuming symmetry (the distribution for all bidders is the same), independence and risk neutrality with respect to the actual outcomes of the auction,[31] the Revenue Equivalence Theorem applies and the auction format is not relevant to determine the expected revenue from the auction.[32] In fact, in this private value setting, the expected revenue is just the expected value of the second highest valuation. The probability that the second highest valuation is less than x is given by $G^i(x) = F^i(x)^K + KF^i(x)^{K-1}[1 - F^i(x)]$,[33] for $i = 0, 1$, depending whether the guarantee mechanism is in place or not. We will show that $G^1(x) \leqslant G^0(x)$ for all $x \in \mathbb{R}$, that is, G^1 first order stochastically dominates G^0. This is sufficient for the conclusion of the theorem since, as it is well known, it implies that the expectation with respect to G^1 is higher than the expectation with respect to G^0. The proof of the dominance of G^1 is elementary, but included for reader's convenience.

Fix $x \in \mathbb{R}$ and define $t = F^1(x) \in [0, 1]$ and $s = F^0(x) - F^1(x) \in [0, 1]$, with $t + s \in [0, 1]$. Then,

$$\begin{aligned}G^0(x) - G^1(x) &= (t + s)^K + K(t + s)^{K-1}(1 - t - s) - t^K - Kt^{K-1}(1 - t) \\ &= (t + s)^{K-1}\left[K - (K - 1)(t + s)\right] - t^{K-1}\left[K - (K - 1)t\right] \\ &= \left[K - (K - 1)(t + s)\right]\left[(t + s)^{K-1} - t^{K-1}\right] - t^{K-1}(K - 1)s. \end{aligned} \tag{35}$$

Since $t + s \in [0, 1]$, $K - (K - 1)(t + s) \geqslant 1$ and from the binomial formula and $t, s \geqslant 0$, we have $(t + s)^{K-1} - t^{K-1} \geqslant (K - 1)t^{K-2}s$. Using these two inequalities in (35), we obtain

$$G^0(x) - G^1(x) \geqslant 1 \cdot (K - 1)t^{K-2}s - t^{K-1}(K - 1)s = st^{K-2}(K - 1)(1 - t) \geqslant 0,$$

that is, $G^0(x) \geqslant G^1(x)$, which is what we needed to show. □

[31] In other words, the participants may have risk aversion with respect to exchange rate, for instance. The assumption of risk neutrality here is restricted to the auction itself.

[32] See Menezes and Monteiro (2004) for the Revenue Equivalence Theorem.

[33] See, for instance, Menezes and Monteiro (2004, p. 19).

Proof of Lemma 3.4. Using the notation in the proof of Proposition 3.3, the revenue in situation $i = 0, 1$ is $\int_{\mathbb{R}} x dG^i(x)$. Since $F^1(x + a) = F^0(x)$, we have

$$G^1(x+a) = F^1(x+a)^K + KF^1(x+a)^{K-1}[1 - F^1(x+a)] = F^0(x)^K + KF^0(x)^{K-1}[1 - F^0(x)] = G^0(x).$$

Therefore,

$$\begin{aligned}\int_{\mathbb{R}} x dG^1(x) &= \int_{\mathbb{R}} (x+a) dG^1(x+a) \\ &= \int_{\mathbb{R}} (x+a) dG^0(x) = \int_{\mathbb{R}} x dG^0(x) + \int_{\mathbb{R}} a dG^0(x) = \int_{\mathbb{R}} x dG^0(x) + a.\end{aligned}$$

This establishes the result. □

Proof of Proposition 3.5. Under the stated assumptions, $G(T)$ is given by:

$$\begin{aligned}G(T) &= \left[\sum_{t=1}^{T} \delta_t (v_t - p_t) q_t\right] + \mu \left[-I + \sum_{t=1}^{T} \delta_t (p_t q_t - c_t)\right] + \tau \\ &= T(v - p)q + \mu[-I + T(pq - c)] + \tau \\ &= T[vq - (1-\mu)pq - \mu c] - \mu I + \tau.\end{aligned}$$

Where $\mu \in \mathbb{R}^+$ is how the government values the social benefit brought about by the firm and τ is the transfers to the governments. Therefore, $G(T)$ is an affine function of T and $\mathbb{E}[G(\tilde{T})] = G(\mathbb{E}[\tilde{T}])$. Since by assumption $\mathbb{E}[\tilde{T}] = T$, we conclude that $\mathbb{E}[G(\tilde{T})] = G(T)$, as we wanted to show. □

Proof of Lemma 4.1. We will first observe that (12) holds, that is, $V^j(P_1) \geqslant \overline{V}^j(P_1)$. Indeed,

$$\begin{aligned}V^j(P_1) - \overline{V}^j(P_1) &= \sum_{t=1}^{P_1} \delta_t^j X_t^j (p_t q_t - c_t) - \sum_{t=1}^{P_0} \delta_t^j g^j (p_t q_t - c_t) \\ &\geqslant P_1 \underline{e}^j \underline{B} - P_0 g^j \overline{B} \geqslant 0,\end{aligned}$$

where the last inequality comes from (25). Since V^j and $\overline{V}^j$ are continuous and (12) is verified, the assumptions of Theorem 1 are satisfied and the result follows. □

Proof of Lemma 4.2. Let us denote $\alpha^0 = \alpha$ and $\alpha^1 = 1 - \alpha$. Observe that, since $X^1(P_1) \geqslant a$ and

$B(P_1) \geqslant b$, $V^\alpha(P_1)$ satisfies:

$$V^\alpha(P_1) = \alpha\left(-X_0 I + \sum_{t=1}^{P_1} \delta_t B_t X_t\right) + (1-\alpha)\left(-I + \sum_{t=1}^{P_1} \delta_t B_t\right) \tag{36}$$

$$\geqslant \alpha(-X_0 I + \sum_{t=1}^{P_1} \delta_t b a) + (1-\alpha)(-I + \sum_{t=1}^{P_1} \delta_t b) \tag{37}$$

$$= -I(\alpha X_0 + 1 - \alpha) + \sum_{t=1}^{P_1} \delta_t b(\alpha a + 1 - \alpha) \tag{38}$$

Thus:

$$V^\alpha(P_1) \geqslant -I(\alpha X_0 + 1 - \alpha) + \sum_{t=1}^{P_1} \delta_t b(\alpha a + 1 - \alpha) \tag{39}$$

$$\Rightarrow \frac{V^\alpha(P_1)}{\alpha X_0 + 1 - \alpha} \geqslant -I + \left(\frac{\alpha a + 1 - \alpha}{\alpha X_0 + 1 - \alpha}\right) b \sum_{t=1}^{P_1} \delta_t \tag{40}$$

From Assumption 6, we have $V^\alpha(P_1) \geqslant \overline{V}^\alpha(P_1)$:

$$\frac{V^\alpha(P_1)}{\alpha X_0 + 1 - \alpha} \geqslant N \Rightarrow V^\alpha(P_1) \geqslant \overline{V}^\alpha(P_1) \tag{41}$$

□

Proof of Lemma 4.3. Since $X(P) \geqslant a$ and $B(P) \geqslant b$, $V^1(P)$ satisfies:

$$V^1(P_1) = V^1(T) + \sum_{t=T+1}^{P_1} \delta_t X_t B_t \tag{42}$$

$$\geqslant V^1(T) + \sum_{t=T+1}^{P_1} \delta_t a b \tag{43}$$

From (30):

$$V^1(P_1) \geqslant V^1(T) + ab \sum_{t=T+1}^{P_1} \delta_t \Rightarrow V^1(P_1) \geqslant (1 + r_T^0)^T X_0 I \tag{44}$$

$$V^1(P_1) \geqslant \overline{V}^1(P_1) \tag{45}$$

□

B Simulations' data

The project data used for the simulations in Section 6 stems from CCR NovaDutra, a highway franchise in Brazil that was in place between 1996 and 2020. Yearly revenue, operational cost and investment data in current values can be seen in Table 9.

Table 9: CCR Nova Dutra – Yearly Revenue (pq_t), Investment (I_t) and Cost (c_t)

Year	pq_t	I_t	c_t
1996	82,791	114,950	17,108
1997	202,782	133,364	44,662
1998	225,373	20,2307	53,149
1999	243,209	131,520	128,332
2000	281,570	73,650	85,060
2001	314,398	75,917	114,949
2002	353,779	33,661	157,616
2003	394,573	49,846	62,152
2004	500,107	70,063	81,315
2005	551,542	88,297	65,218
2006	611,791	75,866	79,655
2007	665,577	100,411	86,238
2008	748,324	159,720	92,432
2009	782,241	172,692	160,625
2010	900,052	203,472	79,348
2011	993,056	229,810	83,078
2012	1,050,626	197,592	82,509
2013	1,113,905	219,078	79,813
2014	1,150,439	241,056	77,840
2015	1,168,369	137,177	87,125
2016	1,210,658	89,337	87,664
2017	1,297,371	165,574	84,563
2018	1,350,917	172,908	93,001
2019	1,429,118	39,262	211,450
2020	1,320,042	102,489	143,212
Sum	18,942,610	3,280,019	2,338,114

Source: own elaboration with data from Brazil's Ministry of Infrastructure. Note: To keep notation of investment as a one-time disbursement (I_0), one could consider investment to be zero and include it as part of operational cost c_t.

To perform simulations for each of the eight countries, we use national Consumer Price Index rates as discount factor δ_t and inflation adjusted US Dollar exchange rates X_t. These values are shown in Tables 10 and 11.

Table 10: Inflation rate (i_t) and exchange rate (X_t) by country.

	Brazil		Mexico		Indonesia		Mozambique	
	i_t (%)	X_t	i_t (%)	X_t	i_t (%)	X_t	i_t (%)	X_t
1996	9,6	0,99	27,7	0,13	6,0	0,00042	48,5	0,089
1997	5,2	0,95	15,7	0,14	10,3	0,00023	7,4	0,091
1998	1,7	0,89	18,6	0,14	77,6	0,00023	1,5	0,088
1999	8,9	0,60	12,3	0,15	1,9	0,00027	2,9	0,083
2000	6,0	0,61	9,0	0,16	9,3	0,00021	12,7	0,076
2001	7,7	0,50	4,4	0,17	12,5	0,00021	9,1	0,059
2002	12,5	0,45	5,7	0,17	9,9	0,00026	16,8	0,059
2003	9,3	0,45	4,0	0,15	5,2	0,00029	13,4	0,065
2004	7,6	0,50	5,2	0,15	6,5	0,00027	12,7	0,076
2005	5,7	0,61	3,3	0,15	17,1	0,00029	7,2	0,077
2006	3,1	0,69	4,1	0,16	6,6	0,00033	13,2	0,076
2007	4,5	0,78	3,8	0,16	6,6	0,00032	8,2	0,079
2008	5,9	0,84	6,5	0,16	10,2	0,00030	14,5	0,093
2009	4,3	0,81	3,6	0,14	2,8	0,00035	3,8	0,085
2010	5,9	0,96	4,4	0,15	7,0	0,00039	12,4	0,076
2011	6,5	1,04	3,8	0,15	3,8	0,00039	11,2	0,096
2012	5,8	0,92	3,6	0,15	4,3	0,00037	2,6	0,099
2013	5,9	0,87	4,0	0,15	7,7	0,00031	4,3	0,096
2014	6,4	0,84	4,1	0,15	8,4	0,00033	2,6	0,093
2015	10,7	0,65	2,1	0,13	3,4	0,00031	3,6	0,075
2016	6,3	0,66	3,4	0,11	3,0	0,00032	17,4	0,055
2017	3,0	0,72	6,8	0,12	3,6	0,00032	15,1	0,062
2018	3,8	0,64	4,8	0,12	3,1	0,00030	3,9	0,066
2019	4,3	0,61	2,8	0,12	2,7	0,00032	2,8	0,064
2020	4,5	0,48	3,2	0,11	1,6	0,00031	3,1	0,059

Source: own elaboration with data from the International Financial Statistics (IMF).

Table 11: Inflation rate (i_t) and exchange rate (X_t) by country.

	Chile		South Africa		Peru		Philippines	
	i_t (%)	X_t	i_t (%)	X_t	i_t (%)	X_t	i_t (%)	X_t
1996	6,6	0,0024	9,3	0,23	11,5	0,41	7,5	0,038
1997	6,1	0,0024	6,2	0,23	8,6	0,40	5,6	0,035
1998	4,7	0,0023	9,0	0,20	7,2	0,38	9,2	0,027
1999	2,3	0,0020	2,2	0,18	3,5	0,34	5,9	0,029
2000	4,5	0,0019	7,0	0,17	3,8	0,33	4,0	0,026
2001	2,6	0,0016	4,6	0,14	2,0	0,32	5,3	0,023
2002	2,8	0,0015	13,5	0,12	0,2	0,32	2,7	0,023
2003	1,1	0,0018	-1,6	0,17	2,3	0,32	2,3	0,022
2004	2,4	0,0019	2,2	0,19	3,7	0,33	4,8	0,022
2005	3,7	0,0021	2,0	0,19	1,6	0,34	6,5	0,023
2006	2,6	0,0020	4,8	0,19	2,0	0,33	5,5	0,025
2007	7,8	0,0023	7,6	0,19	1,8	0,35	2,9	0,028
2008	7,1	0,0018	9,3	0,17	5,8	0,38	8,3	0,030
2009	-2,6	0,0022	6,2	0,18	2,9	0,38	4,2	0,029
2010	3,0	0,0025	3,3	0,20	1,5	0,40	3,8	0,032
2011	4,4	0,0022	6,3	0,21	3,4	0,41	4,7	0,034
2012	1,5	0,0024	5,8	0,20	3,6	0,44	3,0	0,035
2013	2,8	0,0022	5,2	0,17	2,8	0,43	2,6	0,035
2014	4,6	0,0020	5,3	0,16	3,4	0,42	3,6	0,034
2015	4,4	0,0018	5,2	0,14	3,4	0,39	0,7	0,034
2016	2,7	0,0019	7,1	0,13	3,6	0,37	1,3	0,032
2017	2,3	0,0021	4,5	0,15	3,0	0,39	2,9	0,030
2018	2,6	0,0018	4,4	0,15	1,5	0,38	5,2	0,030
2019	3,0	0,0017	4,0	0,14	2,3	0,38	2,5	0,031
2020	3,0	0,0018	3,1	0,13	2,0	0,36	2,6	0,032

Source: Own elaboration with data from the International Financial Statistics (IMF).

In Subsection 6.2, we use the rate of return of the project to estimate NPV target functions for each country. This rate (5.88%) is obtained through Equation B, where pq_t, c_t and I_t are shown in Table 9 and δ_t is obtained using Brazil's inflation rate i_t in Table 10.

$$r^d = \left(\frac{\sum_{t=1}^{25} \delta_t (pq_t - c_t)}{\sum_{t=1}^{25} \delta_t I_t}\right)^{\frac{1}{24}} - 1 = 5.88\%$$

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